Cumhuriyet Üniversitesi Fen Fakültesi
Fen Bilimleri Dergisi (CFD), Cilt:36, No: 3 Özel Sayı (2015)
ISSN: 1300-1949

Cumhuriyet University Faculty of Science
Science Journal (CSJ), Vol. 36, No: 3 Special Issue (2015)
ISSN: 1300-1949

On Intuitionistic Zero Gradations

Mohammad ABRY[1,*], Jafar ZANJANI[1]

[1]*School of Mathematics and Computer science, university of Damghan, Iran*

Received: 01.02.2015; Accepted: 05.05.2015

Abstract. In this paper, we introduce the concept of an intuitionistic zero gradation in Lowen fuzzy topological spaces. We study some relations between the notion of zero-dimensionality of fuzzy bitopological spaces and intuition- istic zero gradations and then prove that the property of being intuitionistic zero gradation is invariant under a strongly gradation preserving map.

Keywords and phrases: Fuzzy topology, Gradation of openness, Intuitionistic gradation of openness, Intuitionistic zero gradation, Zero-dimension.

1. INTRODUCTION

Fuzzy topological spaces is defined in somewhat different ways by Chang [4], Lowen [12] and by Hutton [10] over the system of fuzzy sets proposed by Zadeh in 1965 [16]. There was no fuzziness involved in the openness or closedness of fuzzy sets in all of them. Since a fuzzy set is a set without distinct boundaries, it is not always suitable to ask whether it is completely open or not. Regarding to this point, Chattopadhyay et al. [5] introduced the concept of fuzzy topology by the notion of gradation of openness as a function $\tau: \mathbb{I}^X \to \mathbb{I}$ (satisfying some axioms) such that for each $r \in [0,1], \tau_r = \{\mu \in \mathbb{I}^X: \tau(\mu) \geq r\}$ is a Chang fuzzy topology on X. In parallel with these studies, Atanassov [3] introduced the concept of intuitionistic fuzzy set that is a generalization of fuzzy set in Zadeh's sense. Applications of intuitionistic fuzzy concepts have already been done by Atanassov and others in knowledge engineering, natural language, neural network, medical diagnosis etc [8]. Coker [6] introduced the concept of intuitionistic fuzzy topological spaces by the notion of intuitionistic fuzzy sets that is a generalization of fuzzy topological spaces in Chang's sense. In [13] Samanta et al. introduced the concept of intuitionistic gradations of openness that is a generalization of the concept of gradation of openness defined by Chattopadhyay.

In this paper, we introduce a concept of intuitionistic zero gradation that is the first basic step to develop the theory of fuzzy inductive dimension on the intuition- istic fuzzy topological spaces. We also prove that the concept of intuitionistic zero gradation is invariant under a strongly gradation preserving map. Some examples of zero- and nonzero gradations are also presented. It is worth pointing out that when we refer to topological dimensions, we use the notation and definitions in [9]. In particular, ind represents the small inductive dimension.

*Corresponding author. *Email address: mabry@du.ac.ir*

Special Issue: The Second National Conference on Applied Research in Science and Technology

2. PRELIMINARIES

A fuzzy set in a nonempty set X is a function (membership function) from X into the closed unit interval $\mathbb{I} = [0,1]$. A fuzzy set μ in X is called crisp if $\mu(X) \subset \{0,1\}$. The family of all fuzzy sets on X is denoted by $\mathbb{I}^X$. For every fuzzy subset $\mu \in \mathbb{I}^X$, 2010 Mathematics Subject Classification. 54A40, 54F45.

the support of μ is defined by $supp(\mu) = \{x \in X: \mu(x) > 0\} = \mu^{-1}(0,1]$. A fuzzy set μ is said to be contained in a fuzzy set η if $\mu(x) \leq \eta(x)$ for each x in X, denoted by $\mu \leq \eta$. The union and intersection of a family of fuzzy sets is defined by $\vee\mu_\alpha = \sup(\mu_\alpha)$ and $\wedge\mu_\alpha = \inf(\mu_\alpha)$, respectively.

Definition 2.1. For every $x \in X$ and every $\alpha \in (0,1)$, the fuzzy set x_α with membership function

$x_\alpha(y) = \begin{cases} \alpha & y = x \\ 0 & y \neq x \end{cases}$ is called a fuzzy point. x_α is said to be contained in a fuzzy set μ, denoted by $x_\alpha \in \mu$, if $\alpha < \mu(x)$ [15].

Any point x_1 is called a crisp point. We denote a constant fuzzy set whose unique value is $c \in [0,1]$ by c_X. Note that any fuzzy set is the union of all points which are contained in it.

Definition 2.2. (See [12].) A fuzzy topology is a family δ of fuzzy sets in X which satisfies the following conditions

(i) $\forall c \in \mathbb{I}, c_X \in \delta$,

(ii) $\forall \mu, \nu \in \delta \Rightarrow \mu \wedge \nu \in \delta$,

(iii) $\forall (\mu_j)_{j\in J} \subset \delta \Rightarrow \vee_{j\in J}\mu_j \in \delta$

δ is called a Lowen fuzzy topology for X, and the pair (X,δ) is called a Lowen fuzzy topological space. Open sets, closed sets and clopens are defined as usual.

In Chang's definition of fuzzy topology the condition (i) should be replaced by (i)$'$ $0_X, 1_X \in \delta$. A base or subbase for a fuzzy space have the same meaning in the classic sense.

Definition 2.3. (See [11].) The triplet (X,T,T^*) is called a fuzzy bitopological space where T and T^* are Lowen fuzzy topologies on X. The ordered pair (T,T^*) is called a fuzzy bitopology on X. A fuzzy bitopological space (X,T,T^*) is said to be inclusive if $T \subset T^*$.

Let $f: (X,T,T^*) \to (Y,S,S^*)$ be a mapping, where (X,T,T^*) and (Y,S,S^*) are two bitopological spaces of fuzzy subsets. Then f is said to be continuous if

$f: (X,T) \to (Y,S)$ and $f: (X,T^*) \to (Y,S^*)$ are continuous [1].

Definition 2.4. (See [5].) A fuzzy topological space is a pair (X,τ), where $\tau: \mathbb{I}^X \to \mathbb{I}$ is a mapping satisfying the following properties

(i) $\tau(0_X) = \tau(1_X) = 1$,

(ii) $\tau(\mu_1 \cap \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$,

(iii) $\tau(\cup_i \mu_i) \geq \wedge_i \tau(\mu_i)$.

The map τ is called a gradation of openness or a fuzzy topology on X. The real number $\tau(\mu)$ is the degree of openness of the fuzzy subset $\mu \in \mathbb{I}^X$ that this degree may range from 0 "completely nonopen set" to 1 "completely open set". For each $r \in [0,1], \tau_r = \{\mu \in \mathbb{I}^X: \tau(\mu) \geq r\}$ is a Chang fuzzy topology on X that is called the r-level Chang fuzzy topology on X with respect to the gradation of openness τ.

Definition 2.5. (See [3].) An intuitionistic fuzzy set A in a set X is an object having the form

$A = \{(x, \mu_A(x), \gamma_A(x)): x \in X\}$,

where the function $\mu_A: X \to \mathbb{I}$ and $\gamma_A: X \to \mathbb{I}$ denote the degree of membership and the degree of nonmembership of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

It should be noted that the concept of the boundary of a fuzzy set is essential in the definition of inductive dimension. Tarres et al. [7] proposed a definition of fuzzy boundary of a fuzzy set. Throughout this paper we use their definition.

Definition 2.6. (See [7].) Let μ be a fuzzy set in a Lowen fuzzy topological space X. The fuzzy boundary of μ, denoted by $Fr(\mu)$, is defined as the infimum of all closed fuzzy sets σ in X with the property $\sigma(x) \geq \bar{\mu}(x)$ for all $x \in X$ for which $\bar{\mu}(x) - \mu^0(x) > 0$.

It is ready to see that a fuzzy set μ is clopen if and only if $Fr(\mu) = 0_X$. If the definition of the Adnadjevic's dimension function [2] is particularized, in the case of zero dimensionality, the following definition is obtained.

Definition 2.7. (See [7].) A Lowen fuzzy topological space (X,δ) is called zero-dimensional and it is denoted by $ind(X) = 0$ if for each fuzzy point x_α in X and every fuzzy open set μ containing x_α, there exists an open fuzzy set σ in X with $Fr(\sigma) = 0_X$ such that $x_\alpha \in \sigma \leq \mu$.

Example 2.8. Let δ be the Lowen fuzzy topology on $X = [0,1]$ with subbase

Where

$\{c_X: c \in [0,1]\} \cup \{\mu\}$,

$$\mu(x) = \begin{cases} 0 & 0 \leq x \leq \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} < x \leq 1 \end{cases}$$

Clearly any non-constant open fuzzy sets has the form

$$\nu(x) = \begin{cases} a & 0 \le x \le \frac{1}{2} \\ b & \frac{1}{2} < x \le 1 \end{cases}; \ 0 \le a \le b \le \frac{1}{3}$$

There exists no fuzzy clopen set σ such that $\sigma \le \mu$ because the constant fuzzy sets are the only clopen fuzzy sets. Thus $\operatorname{ind}(X) \neq 0$.

It is ready to see that for every non empty set X the fuzzy topological space (X, δ) is zero-dimensional, where $\delta = \{c_X : c \in [0,1]\}$. A bitopological space (X, T, T^*) is called zero-dimensional if $\operatorname{ind}(x, T) = \operatorname{ind}(X, T^*) = 0$.

Remark 2.9. Let (X, δ) be a Lowen fuzzy topological space and $Y \subset X$, then the family $\delta_1 = \{\mu|_Y : \mu \in \delta\}$ is a fuzzy topology for Y and (Y, δ_1) is called a subspace of (X, δ). If $\operatorname{ind}(X) = 0$, then $\operatorname{ind}(Y) = 0$. Note that restriction of every clopen subset of X on subspace Y is a clopen set in Y.

3. INTUITIONISTIC ZERO GRADATIONS

In Lowen and Chang's definition of fuzzy topology, fuzziness in the concept of openness of a fuzzy subset has not been considered. The initial request is that the topology be a fuzzy subset of a power set of fuzzy subsets. For this purpose, Chattopadhyay et al. gave an axiomatic definition in [5] so called a gradation of openness and Samanta et al. [13] extended it to the concept of intuitionistic gradations of openness. Here we use the following modified definition.

Definition 3.1. Let X be a nonempty set and $\tau, \tau^* : \mathbb{I}^X \to \mathbb{I}$ be two mappings

satisfying

(i) $\tau(c_X) = 1$ and $\tau^*(c_X) = 0$, for all $c \in [0,1]$,

(ii) $\tau(\mu) + \tau^*(\mu) \le 1$, for all $\mu \in \mathbb{I}^X$,

(iii) $\tau(\mu_1 \cap \mu_2) \ge \tau(\mu_1) \wedge \tau(\mu_2), \tau^*(\mu_1 \cap \mu_2) \le \tau^*(\mu_1) \vee \tau^*(\mu_2), \mu_1, \mu_2 \in \mathbb{I}^X$ (iv) $\tau\left(\bigcup_{i \in J} \mu_i\right) \ge \bigwedge_{i \in J} \tau(\mu_i), \tau^*\left(\bigcup_{i \in J} \mu_i\right) \le \bigvee_{i \in J} \tau^*(\mu_i), \mu_i \in \mathbb{I}^X$

The ordered pair (τ, τ^*) is called intuitionistic gradation of openness or intuition- istic fuzzy topology on X and the triplet (X, τ, τ^*) is called an intuitionistic fuzzy topological space. The mappings τ and τ^* is interpreted as gradation of openness and gradation of nonopenness, respectively. Suppose that (τ, τ^*) and (σ, σ^*) be two intuitionistic fuzzy topologies on a given nonempty set X. If $(\tau, \tau^*) \supset (\sigma, \sigma^*)$ i.e., $\tau(\mu) \ge \sigma(\mu)$ and $\tau^*(\mu) \le \sigma^*(\mu)$ for every $\mathbb{I}^X$, we say that (τ, τ^*) is finer than (σ, σ^*).

Example 3.2. Let $X = [0,1]$, we consider two fuzzy sets η_1 and η_2 in X as follows

$$\eta_1(x) = \begin{cases} 0 & x \in [0, \frac{1}{2}] \\ \frac{1}{2} & x \in (\frac{1}{2}, 1] \end{cases} \text{ and } \eta_2(x) = \begin{cases} a & x \in [0, \frac{1}{2}] \\ b & x \in (\frac{1}{2}, 1] \end{cases}$$

where $0 \leq a \leq b \leq \frac{1}{2}$. define $\tau, \tau^*: \mathbb{I}^X \to \mathbb{I}$ by

$$\tau(\mu) = \begin{cases} 1 & \mu = c_X \\ 0.5 & \mu = \eta_1, \eta_2 \\ 0 & \text{otherwise} \end{cases} \text{ and } \tau^*(\mu) = \begin{cases} 0 & \mu = c_X \\ 0.2 & \mu = \eta_1, \eta_2 \\ 1 & \text{otherwise} \end{cases}$$

Then (τ, τ^*) is an intuitionistic gradation of openness on X.

Definition 3.3. The ordered pair (ξ, ξ^*) of mappings from $\mathbb{I}^X$ to $\mathbb{I}$ is called an intuitionistic gradation of closedness on X if it satisfies:

(i) $\xi(c_X) = 1$ and $\xi^*(c_X) = 0$, for all $c \in \mathbb{I}$,

(ii) $\xi(\mu) + \xi^*(\mu) \leq 1$, for all $\mu \in \mathbb{I}^X$,

(iii) $\xi(\mu_1 \cup \mu_2) \geq \xi(\mu_1) \wedge \xi(\mu_2), \xi^*(\mu_1 \cup \mu_2) \leq \xi^*(\mu_1) \vee \xi^*(\mu_2), \mu_1, \mu_2 \in \mathbb{I}^X$,

(iv) $\xi(\bigcap_{i \in J} \mu_i) \geq \bigwedge_{i \in J} \xi(\mu_i), \xi^*(\bigcap_{i \in J} \mu_i) \leq \bigvee_{i \in J} \xi^*(\mu_i), \mu_i \in \mathbb{I}^X$

Example 3.4. Define the mappings $\xi, \xi^*: \mathbb{I}^X \to \mathbb{I}$ as

$$\xi(\mu) = \begin{cases} 1 & \mu = c_X \\ \alpha & \text{otherwise} \end{cases}, \text{and } \xi^*(\mu) = \begin{cases} 0 & \mu = c_X \\ \beta & \text{otherwise} \end{cases}$$

where $\alpha, \beta \in \mathbb{I}$ and $\alpha + \beta \leq 1$, Then (ξ, ξ^*) is an intuitionistic gradation of closedness on X.

Proposition3.5. For each $r \in (0,1]$, $\tau_r = \{\mu \in \mathbb{I}^X: \tau(\mu) \geq r\}$ and $\tau^*_r = \{\mu \in \mathbb{I}^X: \tau^*(\mu) \leq 1 - r\}$ are Lowen fuzzy topologies on X. Where (X, τ, τ^*) is an intuitionistic fuzzy topological space.

Proof. At the first we show that τ^*_r is a Lowen fuzzy topology on X. Since $\tau^*(c_X) = 0 \leq 1 - r$, so $c_X \in \tau^*_r$. Let μ_1 and μ_2 are two fuzzy sets in τ^*_r. Hence $\tau^*(\mu_1) \leq 1 - r$ and $\tau^*(\mu_2) \leq 1 - r$. Since τ^* is a gradation of nonopenness, so $\tau^*(\mu_1 \cap \mu_2) \leq \tau^*(\mu_1) \vee \tau^*(\mu_2) \leq 1 - r$. Therefore, $\mu_1 \cap \mu_2 \in \tau^*_r$. To check the third condition, let $(\mu_i)_i$ be a family of fuzzy sets in τ^*_r such that $\tau^*(\mu_i) \leq 1 - r$. By hypothesis $\tau^*(\bigcup_i \mu_i) \leq \bigvee_i \tau^*(\mu_i) \leq 1 - r$. Hence, $\bigcup_i \mu_i \in \tau^*_r$. Similarly, τ_r is a Lowen fuzzy topology on X.

It is ready to see that the families $\{\tau_r\}$ and $\{\tau^*_r\}$ are two descending families of Lowen fuzzy topology on X such that $\tau_r = \bigcap_{s<r} \tau_s$, and $\tau^*_r = \bigcap_{s<r} \tau^*_s, \forall r \in (0,1]$. Since for each $r \in (0,1], \tau_r \subset \tau^*_r$, then the ordered pair (τ_r, τ^*_r) is an inclusive

Lowen fuzzy bitopology on X and $(X,\tau_r,{\tau^*}_r)$ will be called an r-level inclusive fuzzy bitopological space.

Proposition 3.6. Let (X,T,T^*) be a Lowen fuzzy bitopological space. Define for each $r\in(0,1]$, two mappings $T^r,(T^*)^r:\mathbb{I}^X\to\mathbb{I}$ by

$$T^r(\mu)=\begin{cases}1 & \mu=c_X\\ r & \mu\in T,\mu\neq c_X,\\ 0 & otherwise\end{cases}$$

and

$$(T^*)^r(\mu)=\begin{cases}0 & \mu=c_X\\ 1-r & \mu\in T^*,\mu\neq c_X\\ 1 & otherwise\end{cases}$$

Then $(T^r,(T^*)^r)$ is an intuitionistic gradation of openness on X such that $(T^r)_r=T$ and $((T^*)^r)_r=T^*$.

Proof. First notice that $T^r(c_X)=1$ and $(T^*)^r(c_X)=0$ and clearly $T^r(\mu)+(T^*)^r(\mu)\leq 1$ for all $\mu\in\mathbb{I}^X$. The rest of the proof is similar to [13, Theorem 2.18].

The ordered pair $(T^r,(T^*)^r)$ is called r-th intuitionistic gradation of openness on X and $(X,T^r,(T^*)^r)$ is called r-th graded intuitionistic fuzzy topological space.

Definition 3.7. Let (τ,τ^*) be an intuitionistic gradation of openness on X and

$Y\subset X$. Define two mappings $\tau_Y,{\tau^*}_Y:\mathbb{I}^Y\to\mathbb{I}$ by $\tau_Y(\mu)=\vee\{\tau(\eta):\eta\in\mathbb{I}^X,\eta|_Y=\mu\}$

and

$${\tau^*}_Y(\mu)=\wedge\{\tau^*(\eta):\eta\in\mathbb{I}^X,\eta|_Y=\mu\}$$

for all $\mu\in\mathbb{I}^Y$. Then $(\tau_Y,{\tau^*}_Y)$ is an intuitionistic gradation of openness on Y. We say that $(\tau_Y,{\tau^*}_Y)$ is a subgradation of (τ,τ^*). It is obvious that $\tau_Y(\mu)\geq\tau(\mu_X),{\tau^*}_Y(\mu)\leq\tau^*(\mu_X)$.

Definition 3.8. Let (X,τ,τ^*) and (Y,σ,σ^*) be two intuitionistic fuzzy topological spaces and $f:X\to Y$ be a mapping. Then f is called a gradation preserving map (strongly gradation preserving map) if for each $\mu\in\mathbb{I}^Y$, $\sigma(\mu)\leq\tau(f^{-1}(\mu))$ and

$$\sigma^*(\mu)\geq\tau^*(f^{-1}(\mu))\ (\sigma(\mu)=\tau(f^{-1}(\mu))\ and\ \sigma^*(\mu)=\tau^*(f^{-1}(\mu))).$$

For example, let (τ,τ^*) and (σ,σ^*) are two intuitionistic fuzzy topologies on a given set X such that (τ,τ^*) is finer than (σ,σ^*). Thus, the identity map $i:(X,\tau,\tau^*)\to(X,\sigma,\sigma^*)$ is a gradation preserving map.

Definition 3.9. Two intuitionistic gradations of openness (τ, τ^*) and (σ, σ^*) on X are called equal and it is denoted by $(\sigma, \sigma^*) \approx (\tau, \tau^*)$ if the identity map be a strongly gradation preserving map from (X, τ, τ^*) to (X, σ, σ^*). In this case $\sigma_r = \tau_r$ and $\sigma^*{}_r = \tau^*{}_r$ for all $r \in \mathbb{I}$.

It is worth noting that the constant functions between Chang fuzzy topological spaces are not necessarily continuous. Therefore, the function f in [13, Theorem 4.3] cannot be constant unlike the next Proposition.

Proposition 3.10. Let (X, τ, τ^*) and (Y, σ, σ^*) be two intuitionistic fuzzy topolog- ical spaces and $f: X \to Y$ be a mapping. Then f is a gradation preserving map if

and only if

$$f: (X, \tau_r, \tau^*{}_r) \to (Y, \sigma_r, \sigma^*{}_r)$$

is continuous for all $r \in (0, 1]$.

Proof. This is straightforward.

Definition 3.11. Let (X, τ, τ^*) be an intuitionistic fuzzy topological space. Then (τ, τ^*) is called an intuitionistic zero gradation if the inclusive fuzzy bitopological space $(X, \tau_r, \tau^*{}_r)$ is zero-dimensional for all $r \in (0. 1]$.

Example 3.12. Let $X = \mathbb{R}$, define the mappings $\tau, \tau^*: \mathbb{I}^X \to \mathbb{I}$ by

$$(\mu) = \begin{cases} 1 & \mu = c_X \\ 0 & otherwise \end{cases} \quad and\ \tau^*(\mu) = \begin{cases} 0 & \mu = c_X \\ 1 & otherwise \end{cases}$$

Since $\tau_r = \tau^*{}_r = \{c_X : c \in \mathbb{I}\}$, it is ready to see that $ind(X, \tau_r) = ind(X, \tau^*{}_r) = 0$, then (τ, τ^*) is an intuitionistic zero gradation.

Example 3.13. Let $(\mathbb{I}, \tau, \tau^*)$ be an intuitionistic fuzzy topological space, where τ and τ^* are the same as Example 3.2. Then (τ, τ^*) is not an intuitionistic zerogradation because the Lowen fuzzy topological spaces $(\mathbb{I}, \tau_{0.4})$ and $(\mathbb{I}, \tau^*{}_{0.4})$ are not zero-dimensional.

Theorem 3.14. Let (X, τ, τ^*) be an intuitionistic fuzzy topological space. Let (τ, τ^*) be an intuitionistic zero gradation and $Y \subset X$. Then the subgradation $(\tau_Y, \tau^*{}_Y)$ of (τ, τ^*) is an intuitionistic zero gradation.

Proof. We show that the Lowen fuzzy topological spaces $(Y, (\tau_Y)_r)$ and $(Y, (\tau^*{}_Y)_r)$ are zero-dimensional for all $r \in (0, 1]$. To prove zero dimensionality of $(Y, (\tau^*{}_Y)_r)^Y$ take an arbitrary fuzzy point y_α in Y and fuzzy open set μ in $(\tau^*{}_Y)_r$ containing y_α. Note that $\tau^*{}_Y(\mu) \leq 1 - r$. Now consider the fuzzy point y_α as a fuzzy point in X.There exists an open fuzzy set μ' in $\tau^*{}_r$ such that $\tau^*(\mu') \leq 1 - r$ and $\mu'|_Y = \mu$. Since

$(X, \tau^*{}_r)$ is zero-dimensional, there exists a fuzzy open set η' such that $Fr(\eta') = 0_X$,

$y_\alpha \in \eta' \leq \mu'$ and $\tau^*(\eta') \leq 1-r$. Put $\eta = \eta'|Y$, then $y_\alpha \in \eta \leq \mu$. Similarly, $(Y, (\tau_Y)_r)$ is zero-dimensional.

Proposition 3.15. The intersection of a family of intuitionistic zero gradations on a given set X is an intuitionistic zero gradation.

Proof. Let $\{(\tau_i, \tau^*_i)\}_i$ be an arbitrary family of intuitionistic zero gradations on X. One can readily check that $\bigcap_i(\tau_i, \tau^*_i) = (\bigwedge_i \tau_i, \bigvee_i \tau^*_i)$ is an intuitionistic fuzzy topology on X. Because for all $r \in (0,1]$ the r-level Lowen fuzzy topologies $(\bigwedge_i \tau_i)_r = \{\mu \in \mathbb{I}^X : \bigwedge_i \tau_i(\mu) \geq r\}$ and

$(\bigvee_i \tau^*_i)_r = \{\mu \in \mathbb{I}^X : \bigvee_i \tau^*_i(\mu) \leq 1-r\}$are subspaces of $(\tau_i)_r$ and $(\tau^*_i)_r$, respectively, we conclude that $\bigcap_i(\tau_i, \tau^*_i)$ is an intuitionistic zero gradation by Remark 2.9.

Note that the converse of the Proposition 3.15 is not necessarily hold. For exam- ple, the intersection of two intuitionistic gradations in Example 3.12 and Example

3.2. Is an intuitionistic zero gradation.

Theorem 3.16. Let $f: (X, \tau, \tau^*) \to (Y, \sigma, \sigma^*)$ be a bijective strongly gradation preserving map. If (τ, τ^*) is an intuitionistic zero gradation, (σ, σ^*) will be an intuitionistic zero gradation.

Proof. First, we show that Lowen space (Y, σ^*_r) is zero-dimensional for all r $\in$ (0, 1].Take a fuzzy point $y_\alpha$in Y and let μ be a fuzzy set containing y_α which $\sigma^*(\mu) \leq 1-r$, for all r $\in$ (0, 1]. $f^{-1}(y_\alpha)$ is the fuzzy point x_α in X where $f^{-1}(y) = x$ and $f^{-1}(\mu)$ is a fuzzy open set in

X respect to τ^*_r. Note that $\tau^*(\mathbf{f}^{-1}(\mu)) = \sigma^*(\mu) \leq 1-r$. Thereis a clopen fuzzy set η'

(X, τ^*_r). Put $\eta = f(\eta')$in τ^* such that $x_\alpha \in \eta \leq \mathbf{f}^{-1}(\mu)$ by zero dimensionality of,

Thus $y_\alpha \in \eta \leq \mu$ and $\eta \in \sigma^*$ because $\sigma^*(\eta) = \tau^*(f^{-1}(\eta)) = \tau^*(\eta') \leq 1-r$ Similarly, (Y, σ_r) is zero-dimensional.

Remark 3.17. The strongly condition for the gradation preserving map $\mathbf{f}$ in the Theorem 3.16 cannot be removed. Consider the identity map i : $(X, \iota, \iota^*) \to (X, \tau, \tau^*)$, where $X = I$, the intuitionistic gradation (τ, τ^*) is the same as Example 3.2 and (ι, ι^*) is defined by the rule $\iota(\mu) = 1$, $\iota^*(\mu) = 0$, $\forall \mu \in I^X$. Because $\tau(\eta_1) \neq$

$\iota(\eta_1)$ and $\tau^*(\eta_2) \neq \iota^*(\eta_2)$, then the identity map i is not a strongly gradation preserving map. Note that (ι, ι^*) is an intuitionistic zero gradation and (τ, τ^*) is

not an intuitionistic zero gradation.

Proposition 3.18. Let (τ, τ^*) be an intuitionistic zero gradation on X and $(\sigma, \sigma^*) \approx$

(τ, τ^*), then (σ, σ^*) is an intuitionistic zero gradation.

Proof. The proof is obvious since $\{\sigma_r : r \in [0,1]\} = \{\tau_r : r \in [0,1]\}$ and $\{\sigma^*_r : r \in [0,1]\} = \{\tau^*_r : r \in [0,1]\}$.

REFERENCES

[1] A.S. Abu Sa.ya, A.A. Fora and M.W. Warner, Higher separation axioms in bifuzzy topological spaces, Fuzzy sets and systems 79 (1996) 367–372.
[2] D. Adnadjevic, Dimension of fuzzy spaces, Fuzzy sets and systems. 26 (1988) 85–92.
[3] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems. 20 (1986), no. 1, 87–96.
[4] C.L. Chang, Fuzzy topological spaces, J.Math.Anal.Appl. 24(1968) 182–190.
[5] K.C. Chattopadhyay, R.N. Hazra and S.K. Samanta, Gradation of openness; Fuzzy topology, Fuzzy sets and systems. 49(1992) 237–242
[6] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and systems. 88 (1997) 81–89.
[7] E. Cuchillo-Ibanez, J. Tarres, On the boundary of fuzzy sets, Fuzzy sets and systems. 89 (1997) 113–119.
[8] S. K. De, R. Biswas, and A. R. Roy, An application of intuitionistic fuzzy sets in medical diagnosis, Fuzzy Sets and Systems, 117 (2001), no. 2, 209–213.
[9] R. Engelking, Dimension Theory, North-Holland, Amsterdam, (1978).
[10] B. Hutton, Products of fuzzy topological spaces, Top. Appl. 11 (1980) 59–67.
[11] Kandil, A.A. Nouh and S.A. El-Sheikh, On fuzzy bitopological spaces, Fuzzy sets and systems. 74 (1995) 353–363.
[12] R. Lowen, Fuzzy topological spaces and fuzzy compactness, J. Math.Anal.Appl. 56 (1976) 621–633.
[13] T. K. Mondal and S. K. Samanta, On intuitionistic gradation of openness, Fuzzy sets and systems. 131 (2002) 323–336.
[14] L. G. Oversteegen and E. D. Tymchatyn, On the dimension of certain totally disconnected spaces, Proc. Amer. Math. Soc. 122 (1994) 885–891.
[15] R. Srivastava, S. N. Lal, and A. K. Srivastava, Fuzzy Hausdorff topological spaces, J.Math.Anal.Appl. 81 (1981) 497–506.
[16] L.A. Zadeh, Fuzzy sets, Information and control. 8 (1965) 338–353.