



Magnetoplasma waves on the Graphene with impurity in high magnetic field

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Received: 01.02.2015; Accepted: 05.05.2015

Abstract. The spectrum and damping factor of plasma and magnetoplasma waves in a two-dimensional electron gas at low temperatures on the Graphene are calculated, taking into account local electron states at impurity atoms. It is shown that localization of electrons decreases the frequency of long-wave plasmons and rear ranges the magnetoplasma spectrum in the vicinity of resonant frequencies of electron transitions between the Landau levels and local levels. As a result, the plasma absorption peak is displaced towards low frequencies, and the magnetoplasma peak splits. The characteristics of plasmons and magnetoplasmons are calculated for parameters on the Graphene at the boundary between silicon and silicon dioxide.

Keywords: Plasma waves, magnetoplasma waves, 2DEG, Graphene

1. INTRODUCTION

Two-dimensional plasmons in the inversion layer at the boundary between silicon and silicon dioxide at low temperatures were observed for the first time by Allen *et al.* who excited plasma waves by linearly polarized infrared radiation incident on the grating deposited on the surface of a semitransparent shutter [1,2]. Since the wavelength of the radiation is much larger than the thickness Graphene, the latter can be treated as a conducting plane $z=0$ with a two-dimensional conductivity tensor. The calculations [2] made in the approximation of an infinitely large conductivity of the shutter without taking into account delay effects proved that the plasmon frequency can be written in the form

$$\omega_p(q) \left\{ \frac{4\pi e^2 n q}{m} [\varepsilon_s \coth(qd_s) + \varepsilon_d \coth(qd_d)]^{-1} \right\}^{1/2}, \quad (1)$$

where m and e are the electron mass and charge, n is the electron number density in the Graphene, ε_s and ε_d are the static permittivities of the semiconductor and the insulator, d_s and d_d their thicknesses, and q is the two-dimensional wave vector. Collisionless damping of long-wave plasmons in a two-dimensional degenerate electron gas is not observed. Their damping is mainly determined by collision of electrons with impurity atoms. The damping factor is equal to $\nu/2$, where ν is the collision frequency determined by the potential electron-impurity scattering. The absorption of the electromagnetic radiation by the Graphene is determined by the real component of the quantity $\bar{\sigma}(q, \omega) = \sigma(q, \omega) / \varepsilon(q, \omega)$ where σ and ε are the conductivity and permittivity of the two-dimensional electron gas, which are functions of the wave vector q and frequency ω . In the vicinity of frequency (1), we have

$$\text{Re} \bar{\sigma}(q, \omega) = \frac{e^2 n \gamma_q}{2m} [(\omega - \omega_q)^2 + \gamma_q^2]^{-1} \quad (2)$$

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where ω_q is the spectrum of plasmons (1) and γ_q is the damping factor. This expression shows that plasma waves are manifested in the presence of a clear peak on the curve describing the frequency dependence of absorption, which is superimposed on the high-frequency Drude background [2].

$$\text{Re } \bar{\sigma} = e^2 n \nu / (m \omega^2).$$

In a magnetic field perpendicular to the Graphene, plasma resonance is transformed into magnetoplasma resonance. Such a resonance was observed for the first time by Theis *et al.*[3] and is manifested in the presence of a peak on the dependence of absorption P on the magnetic field strength or radiation frequency. The peak lies at the magnetoplasmon frequency [2].

$$\omega_q^{(0)} = [\omega_c^2 + \omega_c^2(q)]^{1/2}, \quad (3)$$

where ω_c is the cyclotron frequency of electrons. The peak width is given by

$$\gamma_q^{(0)} = \frac{\nu}{2} (1 + \omega_c^2 / \omega_q^{(0)2}). \quad (4)$$

In the vicinity of the peak, the real component of $\bar{\sigma}$ is given by formula (2) as in the absence of the field, but ω_q and γ_q are defined by (3) and (4).

It was noted in Refs. [1-3] that the experimentally observed properties of plasmons and magnetoplasmons in the inversion layer at the Si-SiO₂ boundary are satisfactorily described by the classical formulas (1)-(4) only for a large electron number density. In the case of a low density ($n < 10^{16} \text{ m}^{-2}$) deviations from the classical theory of conductivity of a two-dimensional electron gas become significant. For example, Allen *et al.*[1] noted that the plasma peak on the $P(\omega)$ curve for low electron densities is shifted from the value predicted by formula (1) towards low frequencies. According to Allen *et al.*,[1] such a displacement is due to an increase in the effective mass m or electron localization. Theis *et al.*[3] proved that for small n the magnetoplasma peak splits into two peaks. One of them lies below the peak described by formula (3), while the other peak lies above it. In order to explain this phenomenon, the effects associated with a nonlocal nature of conductivity of a two-dimensional electron gas in a magnetic field were taken into account [2,4]. The conductivity contains a correction of the order of $(ql)[2]$ (l is the magnetic length) with a resonance at a frequency $2\omega_c$. The interaction of a magnetoplasmon with the subharmonic structure of the cyclotron resonance leads to magnetoplasmon peak splitting. It was noted in Refs. [2,4], however, that this interaction is too weak to be responsible for the observed splitting. For this reason, the problem of the plasma absorption line Graphene cannot be regarded as solved completely.

In the present paper, we consider the effect of local electron states in the field of impurity atoms on the spectrum and absorption of plasmons and magnetoplasmons in a two-dimensional electron gas. We used the model and the computational method described in Refs. [5,6]. It will be shown that electron localization leads to a displacement of the plasma peak on the $P(\omega)$ curve towards the low-frequency region. The inclusion of resonant transitions of electrons between Landau levels and local levels causes a rearrangement of magnetoplasmon spectrum in the vicinity of resonant frequencies similar to the crossover in the theory of coupled waves.[7] This leads to the splitting of the magnetoplasmon peak mentioned above.

In Sec. 2, we consider the properties of magnetoplasmons near the frequency $\omega_+ = \omega_c + \omega_0$ of resonant electron transitions from a local level to the Landau level lying above it (ω_0 is the separation between a Landau level and the local level split from it by an attracting impurity). The neighborhood of the frequency $\omega_- = \omega_c - \omega_0$ corresponding to transitions from

a Landau level to a local level will be analyzed in Sec. 3. The spectrum and damping of plasmons taking into account electron localization in zero magnetic field are considered in Sec. 4. The obtained results are summarized in Conclusion, where the theoretical results are compared with experimental data.

2. EFFECT OF LOCAL ELECTRON STATES ON THE PROPERTIES OF TWO-DIMENSIONAL MAGNETOPLASMONS ON THE GRAPHENE

It was proved in Ref. 6 that local electron states in a quantizing magnetic field perpendicular to the Graphene are manifested in the presence of resonant terms in the tensor or high-frequency conductivity of a two-dimensional electron gas. For example, the transverse conductivity σ_{xx} in the vicinity of the frequency $\omega_+ = \omega_c - \omega_0$ corresponding to electron transitions between a local level and a Landau level contains the term

$$\delta\sigma = i \frac{e^2 n \alpha_+ \omega_+}{m \omega} (\omega - \omega_+ + i\Gamma)^{-1}, \quad (5)$$

Where

$$\alpha_+ = \frac{n_i N r_{N-1}}{2\pi m n \omega_0^2 \omega_+ l^4} [1 + (1 + N^{-1})(1 + 2\omega_c / \omega_0)^{-2}] \times [f(\varepsilon_{N-1}^l) - f(\varepsilon_N)] \quad (6)$$

is the oscillator force of a resonant transition and Γ the width of the local level labeled by $N-1$ and participating in transitions. Here n_i is the number density of impurity atoms, r_N the residue of the amplitude of the electron-impurity scattering at the pole $\varepsilon_N^l - i\Gamma$, and $f(\varepsilon)$ the Fermi function; the quantum constant is assumed to be equal to unity. The difference in the Fermi functions in (6) takes into account the Pauli exclusion principle in electron transitions from the local level ε_{N-1}^l to the Landau level ε_N . We assume that the Fermi boundary ε_F of two-dimensional electrons lies between the levels ε_{N-1} and ε_N^l and $|\varepsilon_N^l - \varepsilon_N| \ll 1$. In this case, the expression for the oscillator force contains only one term (6), and spatial dispersion of conductivity (5) can be neglected.

Expression (5) must be taken into account in the dispersion equation for magnetoplasma waves[2]:

$$\varepsilon(q, \omega) = 1 + \frac{4\pi i q}{\omega} \sigma_{xx}(q, \omega) \times (\varepsilon_s \coth qd_s + \varepsilon_d \coth qd_d)^{-1} = 0. \quad (7)$$

This equation can be written in the form

$$1 - A_+ (x^2 - \omega_c^2 / \omega_+^2)^{-1} = \alpha_+ A_+ x^{-2} (x - 1)^{-1}, \quad (8)$$

Where

$$x = \omega / \omega_+,$$

$$A_+ = 4\pi e^2 n q [m \omega_+^2 (\varepsilon_s \coth qd_s + \varepsilon_d \coth qd_d)]^{-1}. \quad (9)$$

In the absence of local levels ($\alpha_+ \rightarrow 0$), the solution of Eq. (8) is the function (3). The inclusion of electron localization leads to a rearrangement of the magnetoplasmon spectrum in the vicinity of frequency ω_+ . As a matter of fact, the straight line $\omega = \omega_+$ intersects the dispersion curve (3) at the point

$$q_0 = \frac{m\omega_0^2}{4\pi e^2 n} \left(1 + 2 \frac{\omega_c}{\omega_0} \right) (\varepsilon_s + \varepsilon_d),$$

where we assume that $q d_s \gg 1$ and $q d_d \gg 1$. Consequently, in the vicinity of this point we have a crossover situation similar to that observed in the spectrum of a lattice with quasilocal vibrations.⁷ Equation (8) has two real positive roots x_1 and x_2 , one of which lies below the magnetoplasmon frequency (3), while the other lies above this frequency. The results of numerical solution of the dispersion equation (8) for values of parameters of the graphene at the Si-SiO₂ boundary in a magnetic field of induction $B = 0.1$ T. The dashed curve describes the function (3), while the lower and upper curves present x_1 and x_2 as functions of the ratio of the wave number q to the Fermi wave number k_F . The following values of parameters are used: $m = 0.2m_0$ (m_0 is the free electron mass), $(n_s/n_e = 0.1, n = 10^{16} \text{m}^{-2}, e_s + e_d = 15, \text{ and } \frac{\omega_c}{\omega_0} = 0.1)$. In this case, $N = 196$, which allows us to neglect the effect of magnetic field on the scattering amplitude residue. In zero field, it is given by [6] $r = 2\pi |\varepsilon_1| / m$, where ε_1 is the position of the local level in the field of an attracting impurity. The residue r is obtained for $\varepsilon_F / |\varepsilon_1| = 2$. For such values of parameters, we have $q_0 = 1.5 \times 10^2 \text{m}^{-1}$ and $\omega_+ = 9.8 \times 10^{10} \text{s}^{-1}$. The damping factor for magnetoplasmons with the spectrum $\omega^{(i)}$ ($i = 1, 2$ is the number of the branch) is given by $\gamma_q^{(i)} = \gamma_q^g + \delta\gamma_q^{(i)}$, where

$$\delta\gamma_q^{(i)} = \frac{\omega_0^4 \alpha_+}{2\omega_+^2} (1 + 2\omega_c / \omega_0)^2 \Gamma [(\omega_q^{(i)} - \omega_+)^2 + \Gamma^2]^{-1} \quad (10)$$

The term (10) in the decrement, which is associated with the local level, has a peak at the frequency ω_+ of electron transitions from the local level ε_{N-1}^1 to the Landau level ε_N . The existence of two roots of the dispersion equation (8) indicates that the plasma peak on the curve describing the frequency dependence of absorption of electromagnetic radiation splits into two peaks. In the vicinity of the z th peak, the absorption is proportional to

$$\text{Re } \bar{\sigma}_i(q, \omega) = \frac{m(\varepsilon_s + \varepsilon_d)^2 \gamma_q^{(i)}}{32\pi^2 e^2 n q^2} (\omega_q^{(i)2} - \omega_c^2)^2 \times [(\omega - \omega_q^{(i)})^2 + \gamma_q^{(i)2}]^{-1}. \quad (11)$$

The position of the peak of (11) is determined by the energy- momentum relation $\omega_q^{(i)}$ for magnetoplasmons, and the peak width is determined by the damping factor $\gamma_q^{(i)}$.

3. MAGNETOPLASMONS IN THE VICINITY OF FREQUENCY

In the vicinity of frequency $\omega_- = \omega_c - \omega_0$, the conductivity of resonant transition of electrons from a Landau level to the neighboring local level differs from (5) in the resonance frequency and oscillator force. The latter quantity is defined as

$$\alpha_- = \frac{n_i N r_N}{2\pi m n \omega_0^2 \omega_-^4} [1 + (1 - N^{-1})(1 - 2\omega_c / \omega_0)^{-2}] \times [f(\varepsilon_{N-1}) - f(\varepsilon_N^1)],$$

where N is the number of the local level participating in transitions. The dispersion equation (7) for magnetoplasma waves in the vicinity of frequency ω_- can be obtained from Eq. (8) by replacing the subscript "+" by "-" This equation has two roots $y_{1,2} = \omega_{1,2} / \omega_-$ lying below and above (3). Figure 2 shows the solution of the dispersion equation for the values of

parameters used in Sec. 2 and for $B = 1$ T. In this case, $N = 20$, and the residue r_N should be calculated by taking into account the magnetic field. For $\omega_0 \ll \omega_c$, it is given by [6] $r = 2\pi(1\omega_0)^2$. The contribution of the local level to the damping factor for magnetoplasmons can be obtained from formula (10) by replacing the subscript "+" by "-" and by changing the sign of ω_c . This leads to a peak against the background of a smooth dependence of the damping factor (4) on the plasmon frequency. The position of the peak is determined by the resonance frequency ω_- , and its width by the broadening of the local level. The ratio of the maximum value of decrement (10) to (4) is equal to 3.3 for all values of parameters indicated in Sec. 2 and for $\nu = \Gamma = 10^{11} \text{ s}^{-1}$. As in Sec. 2, the magnetoplasma peak on the curve describing the frequency dependence of absorption splits into two peaks. The absorption in the vicinity of the peak is proportional to expression (11) in which we must substitute the frequency $\omega_q^{(i)}$ and the damping factor $\gamma_q^{(i)}$ of magneto-plasma waves obtained in this section. The ratio of the maximum absorption to the background

$$\text{Re} \bar{\sigma} \frac{e^2 n \nu}{m} (\omega^2 + \omega_c^2) (\omega^2 - \omega_c^2)^{-2}$$

is $k_1 = 0.02$ for the low-frequency branch and $k_2 = 21.4$ for the high-frequency branch. It should be noted that this ratio for magnetoplasmons in the absence of electron localization is $k_0 = 0.6$. The calculations were made for the above values of parameters and for $q = 10^4 \text{ m}^{-1}$. In this case, $\omega_p = 3.2 \times 10^{10} \text{ s}^{-1}$, $\omega_q^{(0)} = 9.5 \times 10^{11} \text{ s}^{-1}$, and $\omega_- = 8 \times 10^{11} \text{ s}^{-1}$.

4. CONCLUSIONS

In this paper, we considered the effect of electron localization in the field of isolated impurity atoms on the spectrum and damping of plasma and magnetoplasma waves in a two-dimensional electron gas. We also considered the plasma absorption of electromagnetic radiation incident on the electron layer. It was proved that electron localization reduces the frequency of long-wave plasmons as compared to its value in the absence of local levels. Such a decrease was observed by Allen *et al.* [1] who studied plasmons in the inversion layer at the boundary between Si and SiO_2 and explained the freezing out of charge carriers to local levels. But this is not the only effect observed in this case. Ionization of electrons localized at impurities by an electromagnetic field is accompanied by the emergence of a noticeable contribution to the high-frequency conductivity, which must be taken into account in the dispersion equation for plasmons. This also leads to a decrease in the plasmon frequency, which is manifested in a shift of the plasma peak on the curve describing the frequency dependence of absorption towards the low-frequency region.

In a quantizing magnetic field perpendicular to the electron layer, a system of local levels alternating with Landau levels is formed. Electron transitions between these levels, which are induced by the magnetic field, lead to resonant corrections to the conductivity of the two-dimensional electron gas. A rearrangement of the magnetoplasmon spectrum similar to the crossover in the theory of coupled waves is observed in the vicinity of the resonant transition frequencies. As a result, the magnetoplasma peak on the frequency dependence of absorption splits into two peaks. Such a splitting was observed by Theis *et al.* [1] who studied the dependence of absorption on the magnetic field in the inversion layers at the Si- SiO_2 boundary. According to calculations, for a fixed radiation frequency ω (following Ref. 3, we put $\omega = 3.7 \text{ meV}$), the resonant fields B_1 and B_2 for which absorption has the maximum value are

$B_1 = 6.88\text{T}$ and The $B_2 = 5.69\text{T}$ positions of the peaks and their separation $\Delta B = 1.2\text{T}$ are in good agreement with the experimental data [3]. However, the theory gives higher heights of the peaks as in the case of zero magnetic field. This is apparently due to the existence of mechanisms of electron scattering in inversion layers, which were not taken into account in this research.

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