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Finding a Suitable Benchmark in DEA-R

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Abstract. In this paper, it is tried to suggest an appropriate model for inefficient units when the data are relative using DEA-R models. In an organization, taking manager's decision into account, reducing the number of ratios, setting priorities to reduce some of ratios in order to find a suitable model and implementing these ideas on mathematical models are of great importance. In this paper, a new model is suggested by using a non-radial model with constant returns to scale DEA based on the fractional analysis. At the end, the proposed model is solved and analyzed for companies in the stock market.

Keywords: Data Envelopment Analysis, DEA, DEA-R

1. INTRODUCTION

Suitable benchmark-finding is a continuous process of evaluation and comparison of trends in a business with leading organizations with the purpose of providing more information and as a result the development of the organization. [1] A convenient and efficient tool in evaluating the performance of organizations is DEA utilized as a nonparametric method to calculate the efficiency of decision-making. DEA-R models were proposed by Despic et al. (2007) with combining DEA and fractional analysis [2]. The linear models for evaluating the performance of DMUs were presented by Wei et al in 2001 [3]. Consideringly, the present article is to look for a proper model for DMUs using DEA-R models. Generally, taking the manager's opinion into account in the proposed model and using relative data are the main purposes of this article. In the second section an overview of the concept of DEA-R is presented. In the third section, the suitable benchmark -finding model in DEA-R model is recommended with the manager's opinion. And in the fourth section a numerical example is presented and finally conclusions are stated.

2. AN OVERVIEW OF THE BASIC CONCEPTS OF DEA-R

Suppose Xj = (x1j, ..., xmj) as m inputs and Yj = (y1j, ..., ysj) as s outputs. With integration of DEA and efficiency, Despic et al. (2007) studied arithmetic, geometric and harmonic scale efficiencies and Wei et al. (2011) presented a linear model which resolve some problems of DEA such as zero weights using weight limits and the pseudo inefficiency.

The radial envelopment model in DEA-R model is as follows.

Min θ

$$\sum_{j=1}^{n} \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) \le \theta \left(\frac{x_{io}}{y_{ro}}\right) \qquad i = 1,..m,\ r = 1,\dots,s$$

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$$\sum_{j=1}^{n}\lambda_j = 1 \qquad (1)$$

$\lambda_j \geq 0$ j=1,…,n

Model (1) is a linear programming problem for evaluation of DMU0 with n+1 variables and (m.s+1) constraints.

3. FINDING PATTERN IN DEA-R

Four constraints are assumed:

In the first constraint the manager's opinion is effective. In the second, third and fourth constraints, there are relative data which by considering the purpose it is needed to reduce them, remain constant or unchanged and at the end for the fourth constraint it is needed to reduce it, but to a lesser extent than the second constraint, respectively.
Therefore, the following model which is a non-radial model is suggested:

$$\text{Min} \quad \alpha_1 \sum_{i=1}^{m}\sum_{r=1}^{s}\theta_{ir} + \alpha_2 \sum_{i=1}^{m}\sum_{r=1}^{s}\beta_{ir} \qquad \alpha_1 > \alpha_2$$

$$\sum_{j=1}^{n}\lambda_j\left(\frac{x_{ij}}{y_{rj}}\right) \leq \theta_{ir}\left(\frac{x_{io}}{y_{ro}}\right) \qquad i \in I_1, o \in O_1$$

$$\sum_{j=1}^{n}\lambda_j\left(\frac{x_{ij}}{y_{rj}}\right) = \frac{x_{io}}{y_{ro}} \qquad i \in I_2, o \in O_2 \qquad (2)$$

$$\sum_{j=1}^{n}\lambda_j\left(\frac{x_{ij}}{y_{rj}}\right) \leq \beta_{ir}\left(\frac{x_{io}}{y_{ro}}\right) \qquad i \in I_3, o \in O_3$$

$$\sum_{j=1}^{n}\lambda_j = 1$$

λ_j≥0 j=1,…,n
In this relation α_1 and α_2 imply the manager's opinion.
The pattern of units is as follows:

$$\left(\sum_{j=1}^{n}\lambda_j^* x_{ij} \quad i \in I_1, \sum_{j=1}^{n}\lambda_j^* y_{rj} \quad r \in O_1\right)$$

$$\left(\sum_{j=1}^{n}\lambda_j^* x_{ij} \quad i \in I_2, \sum_{j=1}^{n}\lambda_j^* y_{rj} \quad r \in O_2\right) \qquad (3)$$

$$\left(\sum_{j=1}^{n}\lambda_j^* x_{ij} \quad i \in I_3, \sum_{j=1}^{n}\lambda_j^* y_{rj} \quad r \in O_3\right)$$

4. THE NUMERICAL EXAMPLE

The data of 22 companies in the stock market are presented in Tables 1 and 2 [4].

Table 1. The input data of 22 companies in the stock market.

	i_1	i_2	i_1	i_2	i_1	i_2
DMU1	5	4	13	12	25	3
DMU2	6	5	14	18	22	5
DMU3	4	5	18	19	28	5
DMU4	8	5	11	35	23	3
DMU5	5	6	15	98	21	6
DMU6	8	3	11	23	26	3
DMU7	4.4	4.4	41	23	27	4.4
DMU8	2.6	8	13	41	26	3
DMU9	3.4	8	33	24	24	8
DMU10	3.6	4.4	65	35	21	5
DMU11	2	7	12	17	23	2
DMU12	3	7	13	28	24	1
DMU13	3	5.6	23	31	14	6.8
DMU14	2.6	5	44	16	24	1
DMU15	4	4	33	22	17	3
DMU16	5	3.2	31	22	11	10
DMU17	6	4	77	56	11	4
DMU18	4	3.5	14	61	17	7
DMU19	7	3	34	54	18	3
DMU20	6	2.5	12	50	19	2.5
DMU21	8	2	25	16	23	2
DMU22	9	2	17	47	27	2

Table 2. The output data of 22 companies in the stock market.

	O1	O2	O3
DMU1	1	3	4
DMU2	1	8	4
DMU3	1	9	4
DMU4	1	7	4
DMU5	1	4	4
DMU6	1	3	4
DMU7	1	2	4
D*MU8*	1	9	4
DMU9	1	11	4
DMU10	1	10	4
DMU11	1	8	4
DMU12	1	7	4
DMU13	1	3	4
DMU14	1	8	4
DMU15	1	5	4
DMU16	1	7	4
DMU17	1	8	4
DMU18	1	4	4
DMU19	1	3	4
DMU20	1	2	7
DMU21	1	7	9
DMU22	1	4	1

Table 3. The scale efficiency of 22 companies in the stock market.

DMU1	0.906454
DMU2	0.850164
DMU3	1.00000
DMU4	0.815335
DMU5	0.692333
DMU6	0.826918
DMU7	0.857672
DMU8	1.00000
DMU9	0.729975
DMU10	1.00000
DMU11	1.00000
DMU12	0.835515
DMU13	0.880266
DMU14	1.0000
DMU15	0.992415
DMU16	1.00000
DMU17	0.811128
DMU18	1.00000
DMU19	0.860047
DMU20	1.00000
DMU21	1.00000
DMU22	0.936112

In Table 3, the scale efficiency of decision making units is shown with respect to Model 2. The scale efficiency of companies 3, 8, 10 and 11, 14, 16, 18, 20 and 21 is one and companies are then considered efficient. Other units are not efficient.

Also, the radial model based on the covariates is suggested as follows:

$$
\begin{aligned}
& Min\left(1-\frac{1}{f}\sum_{j\in I_1}\sum_{r\in O_1}\frac{S_{ir}}{\frac{x_{io}}{y_{ro}}}\right)+\left(1-\frac{1}{k}\sum_{i\in I_3}\sum_{r\in O_3}\frac{t_{ir}}{\frac{x_{io}}{y_{ro}}}\right) \\
& \quad \sum_{j=1}^{n}\lambda_j\left(\frac{x_{ij}}{y_{rj}}\right)+s_{ir}=\frac{x_{io}}{y_{ro}} \qquad i\in I_1\ ,\ o\in O_1 \\
& \quad \sum_{j=1}^{n}\lambda_j\left(\frac{x_{ij}}{y_{rj}}\right)=\frac{x_{io}}{y_{ro}} \qquad i\in I_2\ ,\ o\in O_2 \\
& \quad \sum_{j=1}^{n}\lambda_j\left(\frac{x_{ij}}{y_{rj}}\right)+t_{ir}=\frac{x_{io}}{y_{ro}} \qquad i\in I_3\ ,\ o\in O_3 \\
& \quad \sum_{j=1}^{n}\lambda_j=1 \\
& \quad f=|I_1||O_1| \\
& \quad k=|I_3||O_3|
\end{aligned}
\tag{4}
$$

Based on covariates, model 4 calculates the amount of inefficiency. When the data are relative, the first and third constraints increase the covariate, respectively, with regard to the manager's opinion. In general, model 4 with n + 2 (m × s) variables and linear constraints is a linear programming problem proposed to find a Suitable benchmark for decision-making units. The results of model 2 and replacing it in model 3 for 22 companies are shown in tables 4 and 5.

Table 4. The input results of model 2.

	I_1		I_2		I_3	
	i_1	i_2	i_1	i_2	i_1	i_2
DMU1	4.92	3.24	29.63	24.76	11.63	9.73
DMU2	2.94	6.05	14.78	18.36	25.24	3.43
DMU3	4	5	18	19	28	5
DMU4	2.29	6.71	12.86	17.29	23.71	2.43
DMU5	3.83	3.69	17.75	61	17.88	6.25
DMU6	7.92	2.02	24.49	17.33	22.84	2.02
DMU7	3.31	4.24	28.75	61	20.44	4.05
DMU8	2.60	8	13	41	26	3
DMU9	2.01	6/98	12.33	17.45	23.1	1.99
DMU10	3.6	4.40	65	35	21	5
DMU11	2	7	12	17	23	2
DMU12	2.05	6.82	14.86	20.93	23.09	1.91
DMU13	2.6	5	44	61	24	1
DMU14	2.6	5	44	61	24	1
DMU15	3.38	4.53	60.42	40.67	21.65	4.13
DMU16	5	3	31	22	11	10
DMU17	4.68	3.3	25.58	34.42	12.91	9.04
DMU18	4	3	14	61	17	7
DMU19	6.62	2.35	16.01	39.52	20.23	2.35
DMU20	6	2/5	12	50	19	2
DMU21	8	2	25	16	23	2
DMU22	8	2	25	16	23	2

For example, in the third company the first and second inputs for I_1 category are 4 and 5, for I_2 category they are 18 and 19 and for I_3 category they are 28 and 25, respectively, which are presented as a Suitable benchmark. In the fourteenth company, for I_1 category the first and second inputs are respectively 6.2 and 5, for I_2 category the first and second inputs are 44 and 61 and for I_3 category the first and second inputs are 24 and 1 which are presented as the pattern.
In the twenty-first company, for I_1 category the first and second input are, respectively, 8 and 2, for I_2 category the first and second input are 25 and 16, and for I_3 category the first and second input are 23 and 2, which are presented as the pattern.

Table 5. The output results of model 2.

	O1	O2	O3
DMU1	1	6.81	4
DMU2	1	8.42	4
DMU3	1	9	4
DMU4	1	8.14	4
DMU5	1	4.5	4
DMU6	1	6.8	8.92
DMU7	1	5.97	4
D*MU8*	1	9	4
DMU9	1	8	4
DMU10	1	10	4
DMU11	1	8	4
DMU12	1	8	4
DMU13	1	8	4
DMU14	1	8	4
DMU15	1	9.56	4
DMU16	1	7	4
DMU17	1	6.04	4
DMU18	1	4	4
DMU19	1	3.54	7.62

DMU20	1	2	7
DMU21	1	7	9
DMU22	1	7	9

As can be seen, DMUs of 8, 10, 11, 14, 16, 18, 20 and 21 are efficient, and they themselves are introduced as patterns.

The outputs of the third company are 1 and 9 and 4, respectively, which is presented as the pattern. Because the decision-making unit number 3 is efficient, it is introduced as the pattern. The outputs of the fourteen company are 1, 8 and 4, respectively, which are presented as the pattern. The outputs of the twenty first company are 1, 7 and 9, respectively, which are introduced as the pattern.

Like the third company, the fourteenth and twenty-first companies are efficient and they themselves are introduced as their patterns.

5. CONCLUSION

In DEA, suitable benchmark-finding of inefficient units is of great importance. In this paper, when the data are relative a model is presented in which first of all the manger's opinion is taken into account, secondly reduction of inputs and increase of outputs is prioritized and thirdly, when the data are relative they are responsive to the pattern of decision-making units. For future research determining the returns to scale and using models for density and determining scale efficiency are recommended.

REFERENCES

[1] Lancaster H.Managing your career. Wall Street Journal, Tuesday, April 28, B1, 1998.

[2] Despic, O., Despic, M. and Paradi, J.C. (2007) DEA-R: Ratio-based comparative efficiency model, its mathematical relation to DEA and its use in applications. Journal of Productivity Analysis. 28: 33–44

[3] Wei, C.K., Chen, L.C., Li, R.K. and Tsai, C.H. (2011) Using the DEA-R model in the hospital industry to study the pseudo-inefficiency problem.Expert Systems with Applications. 38: 2172–2176.Udwadia, F. E. and Trifunac, M. D. (1973),"Ambient Vibration Test of Full Scale Structures," Proc. of the 5th World Conf. On Earthquake Engineering, Rome, pp

[4] Sungmook Lim and partners, Use of DEA Cross-Efficiency Evaluation in Portfolio Selection:An application to Korean Stock Market, European Journal of Operational Research (2013), doi.