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A review of one-dimensional unsteady friction models for transient pipe flow

Hamid SHAMLOO[1], Reyhaneh NOROOZ[2,*] and Maryam MOUSAVIFARD[3]

[1]*Associate Professor, Dept. of Civil Engineering, K.N. Toosi University of Technology, Tehran, Iran, 19697*

[2]*Research student, Dept. of Civil Engineering, K.N. Toosi University of Technology, Tehran, Iran, 19697*

[3]*PhD Student, Dept. of Civil Engineering, K.N. Toosi University of Technology, Tehran Tehran, Iran, 19697*

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Abstract. This paper reviews a quasi-steady model and four unsteady friction models for transient pipe flow. One of the factors which may affect the accuracy of the one-dimensional models of transition flow is the friction coefficient. This coefficient can be estimated as steady, quasi-steady, and unsteady. In the steady approach, a constant value of the Darcy-Weisbach friction factor is used. In the quasi-steady approximation, friction losses are estimated by using formula derived for steady-state flow conditions. The fundamental assumption in this approximation is that the head loss during transient conditions is equal to the head loss obtained for steady uniform flow with an average velocity equal to the instantaneous transient velocity. During transient conditions the shear stress at the wall is not in phase with the mean velocity. In addition, the velocity profile can be completely different from a uniform flow profile. Therefore friction losses computed by using steady-state relationships are inaccurate in transient laminar and turbulent flow. To cope with this problem, for both laminar and turbulent flows, it is possible to algebraically add unsteady-flow terms to the quasi-steady resistance term of one-dimensional models. Unsteady models are divided into two groups. The first group includes those models which instantaneous wall shear stress is the sum of the quasi-steady value plus a term in which certain weights are given to the past velocity changes. Three models of this group are presented in this paper: Zielke, Vardy & Brown, and Trikha. The second group of models assumes the wall shear stress due to flow unsteadiness is proportional to the variable flow acceleration. Brunone model from this group is presented in this paper. Numerical results from the quasi-steady friction model and the Zielke, Vardy & Brown, Trikha and the Brunone unsteady friction models are compared with results of laboratory measurements for water hammer cases with laminar and low Reynolds number turbulent flows. The computational results clearly show that Zielke model yields better conformance with the experimental data

Keywords: Pipelines, transition flow, one-dimensional models of transition flow, unsteady friction models

1. INTRODUCTION

The assumption in accordance to which the unsteady liquid flow in a long closed conduit (pipeline) may be treated as a one-dimensional flow is commonly accepted. In such an approach it is also common practice to assume the velocity profile during transient flows in a closed conduit to remain the same as under the steady-state flow conditions featured by the same mean velocity. However, it is generally known that the approach based on the quasi-steady friction losses hypothesis is one of the basic reasons for differences between experimental and computational results obtained according to the one-dimensional flow theory. Unsteady friction models and the relevant computational methods are the subject of various research projects at the research centers all over the world. Multiyear effort of numerous researchers has resulted in developing miscellaneous models of hydraulic transients with the unsteady hydraulic resistance taken into account. The most widely used models consider extra friction losses to depend on a history of weighted accelerations during unsteady phenomena or on instantaneous flow acceleration.

Development of the first group of models was initiated in 1968 by Zielke (Zielke, 1968). In his model the instantaneous wall shear stress (which is directly proportional to friction losses) is the sum of the quasi-steady value plus a term in which certain weights are given to the past

*Corresponding author. *Email address: reyhaneh_blueparto@yahoo.com*

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velocity changes. This approach is assigned for transient laminar flow cases. The model developed by Zielke is based on solid theoretical fundamentals and the multiple experimental validation tests have shown good conformity between calculated and measured results. It is, therefore, no wonder that numerous researchers have followed Zielke's scent, giving rise to a number of proposals for describing friction losses in unsteady turbulent flows. From among these proposals, known also as the multi-layer models, those of Vardy and Brown (Zielke,1968) (Zarzycki,2000) (two-layer model) or Zarzycki (Zielke,1968) (Zarzycki,2000) (four-layer model) may be distinguished. In general, the models mentioned are based on experimental data on the distribution of the turbulent viscosity coefficient in the assumed flow layers. The efforts aimed at enhancement of these models are still in progress—numerous papers emphasize the need to develop and validate experimentally the models that can be used in a wide range of frequencies and Reynolds numbers. The second group of models mentioned assumes the wall shear stress due to flow unsteadiness is proportional to the variable flow acceleration. This approach has been introduced by a group of researchers headed by Daily (Zielke,1968). The proportionality coefficient has been established based on the experimental measurements carried out by Cartens and Roller (Zielke,1968). Modifications of this model have been the subject of numerous subsequent studies, including those of Brunone et al. (Zarzycki, 2000). Easy applicability to numerical computations is a significant advantage of this approach. However, the need to determine the empirical coefficient mentioned is an obvious demerit. There appeared suggestions that the friction coefficient in this model depends also on the velocity derivative of the second or even higher order.

A quasi-steady model and four distinct unsteady friction models, the Zielke, Vardy & Brown, Trikha and Brunone models, are investigated in this paper in detail. Computational results from the numerical models are compared with the experimental results.

2. GOVERNING EQUATIONS

The following assumptions were made to derive equations describing unsteady liquid flow in closed conduits (water hammer):

– The flow in closed conduits is one-dimensional, which means that the characteristic quantities are cross-section averaged;
– The flow velocity is small compared to the pressure wave celerity (speed);
–The liquid is a low-compressible fluid—it deforms elastically under pressure surges with insignificant changes of its density;
–The pipeline wall is deformed by pressure surges according to the elastic theory of deformation.

According to the above assumptions, the following equations are used for mathematical description of unsteady liquid flows in closed conduits (Zielke,1968):

$$\frac{\partial Q}{\partial t} + gA\frac{\partial H}{\partial x} + \frac{fQ|Q|}{2DA} = 0 \tag{1}$$

$$\frac{\partial H}{\partial t} + \frac{c^2}{gA}\frac{\partial Q}{\partial x} = 0 \tag{2}$$

where x = distance along the pipe, ρ = density of liquid, c = liquid (elastic) wave speed, t = time, f = Darcy-Weisbach friction factor, D = internal pipe diameter, g = gravitational acceleration, Q = discharge, H = pressure head and A = cross-sectional flow area. At a boundary (reservoir, valve), the boundary equation replaces one of the water hammer compatibility equations (Ghidaoui, 2002)..

The method of characteristics has been applied in order to solve the system of Eqs. (1) and (2) (Almeida, 1992). According to this method, Eqs. (1) and (2) are transformed into the following two pairs of ordinary differential equations (for positive and negative characteristics C+ and C−) (Ghidaoui,2002):

$$Q_{i,n} - Q_{i-1,n-1} + \frac{gA}{c}\left(H_{i,n} - H_{i-1,n-1}\right) + \frac{f}{2DA}\Delta t Q_{i-1,n-1}\left|Q_{i-1,n-1}\right| = 0 \tag{3}$$

$$Q_{i,n} - Q_{i+1,n-1} - \frac{gA}{c}\left(H_{i,n} - H_{i+1,n-1}\right) + \frac{f}{2DA}\Delta t Q_{i+1,n-1}\left|Q_{i+1,n-1}\right| = 0 \tag{4}$$

The staggered grid in applying the method of characteristics is used in this paper. A constant value of the Darcy-Weisbach friction factor f (steady state friction factor) is used in most of commercial software packages for water hammer analysis. As an alternative the friction term can be expressed as the sum of the unsteady part f_u and the quasi-steady part f_q:

$$f = f_q + f_u \tag{5}$$

3. QUASI-STEADY FRICTION MODEL

Friction modeling according to the quasi-steady flow hypothesis assumes that the unsteady part f_u equals zero. The quasi-steady friction factor f_q is based on updating the Reynolds number for each new computation. Calculation of the quasi-steady friction coefficient (Darcy friction coefficient):

• for laminar flows depends only on flow characteristics (Re) according to the Hagen-Poiseuille law:

$$f_q = \frac{64}{\mathrm{Re}} \tag{6}$$

•For turbulent depends on flow characteristics (Re) and absolute pipe-wall roughness (ε/D) according to the Colebrook-White formula

$$\frac{1}{\sqrt{f_q}} = -2\log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f_q}}\right) \tag{7}$$

During transient conditions the shear stress at the wall is not in phase with the mean velocity. In addition, the velocity profile can be completely different from a uniform flow profile. Therefore friction losses computed by using steady-state relationships are inaccurate in transient laminar and turbulent flow (Zarzycki,2000) (Ghidaoui,2002).In fact, the velocity profiles in unsteady-flow conditions show greater gradients, and thus greater shear stresses, than the corresponding values in steady-flow conditions (Zarzycki, 2000). As a consequence, one-dimensional models in which the energy dissipation is computed by a relation between energy slope and mean velocity valid for steady-flow conditions (quasi-steady model) underestimate the friction forces and overestimate the persistence of oscillations following the first one (Zarzycki, 2000). To cope with this problem, for both laminar and turbulent flows, it is possible to algebraically add unsteady-flow terms to the quasi-steady resistance term of one-dimensional models.

4. UNSTEADY FRICTION MODELS

The Brunone model [**Hata! Yer işareti tanımlanmamış.**] relates unsteady friction part f_u to the instantaneous local acceleration $\partial V/\partial t$ and instantaneous convective acceleration $\partial V/\partial x$:

$$f = f_q + \frac{kD}{V|V|}\left(\frac{\partial V}{\partial t} - c\frac{\partial V}{\partial x}\right) \tag{8}$$

in which k = Brunone's friction coefficient and x = distance. Vítkovský in 1998 conducted some research on the original Brunone model for different flow situations. He reached that Eq. (8) is unable to predict a correct sign of the convective term $-c\partial V/\partial x$ for particular flow and wave directions in acceleration and deceleration phases. For instance, Eq. (8) fails to predict the correct sign in case of closure of the upstream end valve in a simple pipeline system with initial flow is in positive x direction. The original Brunone formulation performs correctly in case of closure of the downstream end valve (Zarzycki, 2000).

Vítkovský proposed a new formulation of Eq. (8):

$$f = f_q + \frac{kD}{V|V|}\left[\frac{\partial V}{\partial t} + c\,sign(V)\left|\frac{\partial V}{\partial x}\right|\right] \tag{9}$$

In which sign(V) = (+1 for $V \geq 0$ or -1for $V < 0$). Eq. (9) gives the correct sign of convective term for all possible flow and water hammer wave movement directions for either the acceleration or deceleration phases.

Originally, the Brunone's friction coefficient was established to match computational and experimental results in an acceptable level. Vardy and Brown proposed the following empirical relationship to derive this coefficient analytically:

$$k = \frac{\sqrt{C^*}}{2} \tag{10}$$

4.1. The Vardy's shear decay coefficient *C** from (Zarzycki, 2000) is:

- laminar flow:

$$C^* = 0.00476 \tag{11}$$

- turbulent flow:

$$C^* = \frac{7.41}{\mathrm{Re}^{\log\left(14.3/\mathrm{Re}^{0.05}\right)}} \tag{12}$$

In which Re = Reynolds number (Re=VD/υ).

The original version of Zielke's model (Chaudhry, 1979) is used in this paper. The model was analytically developed for transient laminar flow. The unsteady part of friction term is related to the weighted past velocity changes at a computational section:

$$(f_u)_{i,k} = \frac{32\upsilon}{DV_{i,k}|V_{i,k}|}\sum_{j=1}^{k-1}\left(V_{i,j+1} - V_{i,j-1}\right)W((k-j)\Delta t) \tag{13}$$

$$\tau > 0.02 : W(\tau) = \sum_{i=1}^{5} e^{-n_i \tau} \tag{14}$$

$$\tau \leq 0.02 : W(\tau) = \sum_{i=1}^{6} m_i e^{(i-2)/2} \tag{15}$$

$$\tau = \frac{4\tau}{D^2}\left(k - j\right)\Delta t \tag{16}$$

in which j and k = multiples of the time step Δt, W = weights for past velocity changes, υ = kinematic viscosity, τ = dimensionless time, and coefficients $\{n_i, i = 1, \ldots, 5\}$ = {-26.3744, -70.8493, -135.0198, -218.9216, 322.5544} and $\{m_i, i = 1, \ldots, 6\}$ = {0.282095, -1.25, 1.057855, 0.937500, 0.396696, -0.351563}.

The velocity profile analyses for turbulent unsteady flows allow Vardy *et al*. to state that the relation (Eq. 13) proposed by Zielke is correct for turbulent unsteady flows if only a weighting function W would be related to the Reynolds number (Zarzycki, 2000). The Vardy and Brown obtained Re-dependent weighting function W is:

$$W_{app} = \frac{A^*.e^{-\tau/C^*}}{\sqrt{\tau}} \tag{17}$$

with $A^* = \frac{1}{2\sqrt{\pi}} = 0.2821; c^* = \frac{12.86}{\mathrm{Re}^{\kappa}}; \kappa = \log_{10}\left(\frac{15.29}{\mathrm{Re}^{0.0567}}\right)$

According to the authors this model is valid for the initial Reynolds numbers Re<10^8 and for smooth pipes only.

The simplified inertance expression in the Trikha model takes the following form (Bergant *et al.*, 2001):

$$W(\tau) = \sum_{i=1}^{3} m_i.e^{n_i \tau},\ m_i = \{40.0; 8.1; 1\},\ n_i = \{8000; 200; 26.4\} \tag{18}$$

The Trikha model presents a simplification of the Zielke model. Numerical codes based on this model, because of approximation of weighting function used, allow saving a lot of computing power and time needed for making the calculations. Such a solution initiated the development of models with efficient approximations of weighting functions (Suzuki,1995). However, the most important is that Trikha for the first time proposed to apply this approach to calculate turbulent unsteady flows.

5. EXPERIMENTAL APPARATUS

The computational results are compared with the results of experimental studies conducted by Bergant and Simpson (Bergant *et al.*, 2001) which were carried out using a long horizontal pipe with length of 37.20 m and inner diameter of 0.0221 m that connects upstream and downstream reservoirs (see Figure 1). The water hammer wave speed was experimentally determined as c = 1319 m/s. A transient event is initiated by a rapid closure of the ball valve. A comparison is made for the rapid closure of a downstream end valve. The performance of the friction models is investigated for three water hammer cases with steady state flow velocities of V_0 = {0.10, 0.20, 0.30} m/s (laminar and low Reynolds number turbulent flow).

5.1. Comparison of numerical models

As mentioned before, two approaches have been employed for modeling the transient flow: quasi-steady and unsteady models of Brunone, Zielke, Vardy & Brown, and Trikha. In order to investigate the performance of the models, the numerical and experimental results are compared in three runs: for the laminar flow (V_0 = 0.1 *m/s*) and turbulent flow (V_0 = 0.2, 0,3 *m/s*).

5.2. The Comparison of Computational and Experimental Results for V_0 = 0.1 *m/s*

The computational results for the first run with initial velocity V_0 = 0.1 *m/s* for all models are presented in Fig. 2. As it can be seen, computational results of five models agree with the experimental results till 0.4 s. The discrepancies between the experimental results are magnified later times and phase shift occurs. The quasi-steady model overestimates the maximum heads. In addition, it has not become successful to predict the wave shape properly. Maximum head in the Brunone model has been estimated greater than the other unsteady models and shows more discrepancies with the experimental results in comparison with other unsteady models and the wave shape has not been simulated well. Zielke, Vardy & Brown, and Trikha models have quite similar results and when it comes to the maximum heads and wave shape, they give an accurate prediction. All models slightly overestimate the maximum heads.

5.3. The Comparison of Computational and Experimental Results for V_0 = 0.2 and 0.3 *m/s*

The computational results for the second and third runs with initial velocity V_0=0.2 and 0.3 *m/s* for all models are presented in Fig. 3 and 4. As it can be seen, computational results of five models agree with the experimental results till 0.4 s. The discrepancies between the experimental results are magnified later times and phase shift occurs. The quasi-steady model overestimates the maximum heads. In addition, it has not become successful to predict the wave shape properly. Maximum head in the Brunone model has been estimated greater than the other unsteady models and shows more discrepancies with the experimental results in comparison with other unsteady models and the wave shape has not been simulated well. The Zielke, Vardy & Brown, and Trikha models have quite similar results and when it comes to the maximum heads and wave shape, they give an accurate prediction. All models slightly overestimate the maximum heads. The Zielke model has better agreement in terms of simulating the maximum head in both second and third runs. It is worth noting that all models produce less discrepancy with experimental results in the laminar flow (V_0 = 0.1 *m/s*) than turbulent flow (V_0 = 0.3 *m/s*).

6. CONCLUSION

- Computational results of five models agree with the experimental results till 0.4 s. The discrepancies between the experimental results are magnified later times and phase shift occurs.
- The quasi-steady model overestimates the maximum heads. In addition, it has not become successful to predict the wave shape properly.
- Maximum head in the Brunone model has been estimated greater than the other unsteady models and shows more discrepancies with the experimental results in comparison with other unsteady models and the wave shape has not been simulated well.
- The Zielke, Vardy & Brown, and Trikha models have quite similar results and when it comes to the maximum heads and wave shape, they give an accurate prediction.
- All models slightly overestimate the maximum heads.
- The Zielke model has better agreement in terms of simulating the maximum head in both second and third runs.
- It is worth noting that all models produce less discrepancy with experimental results in the laminar flow (V_0 = 0.1 *m/s*) than turbulent flow (V_0 = 0.3 *m/s*).

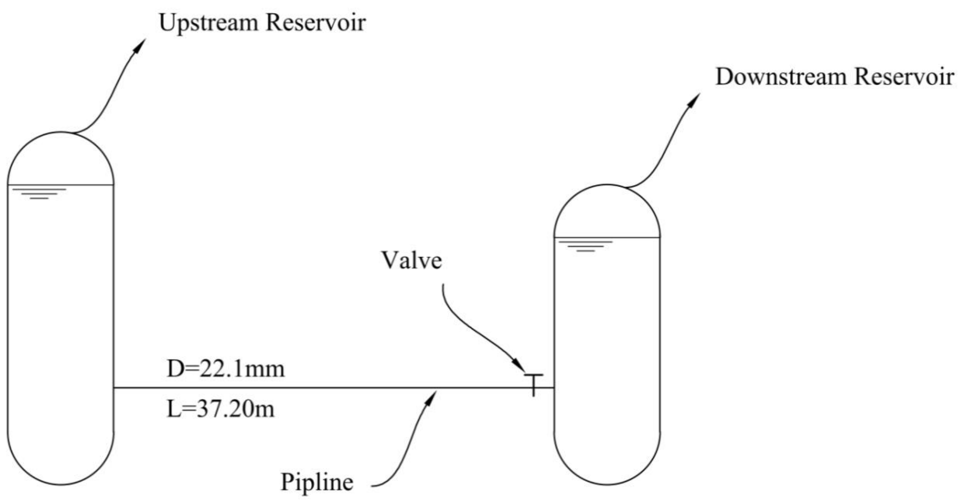

Figure 1. Experimental set up.

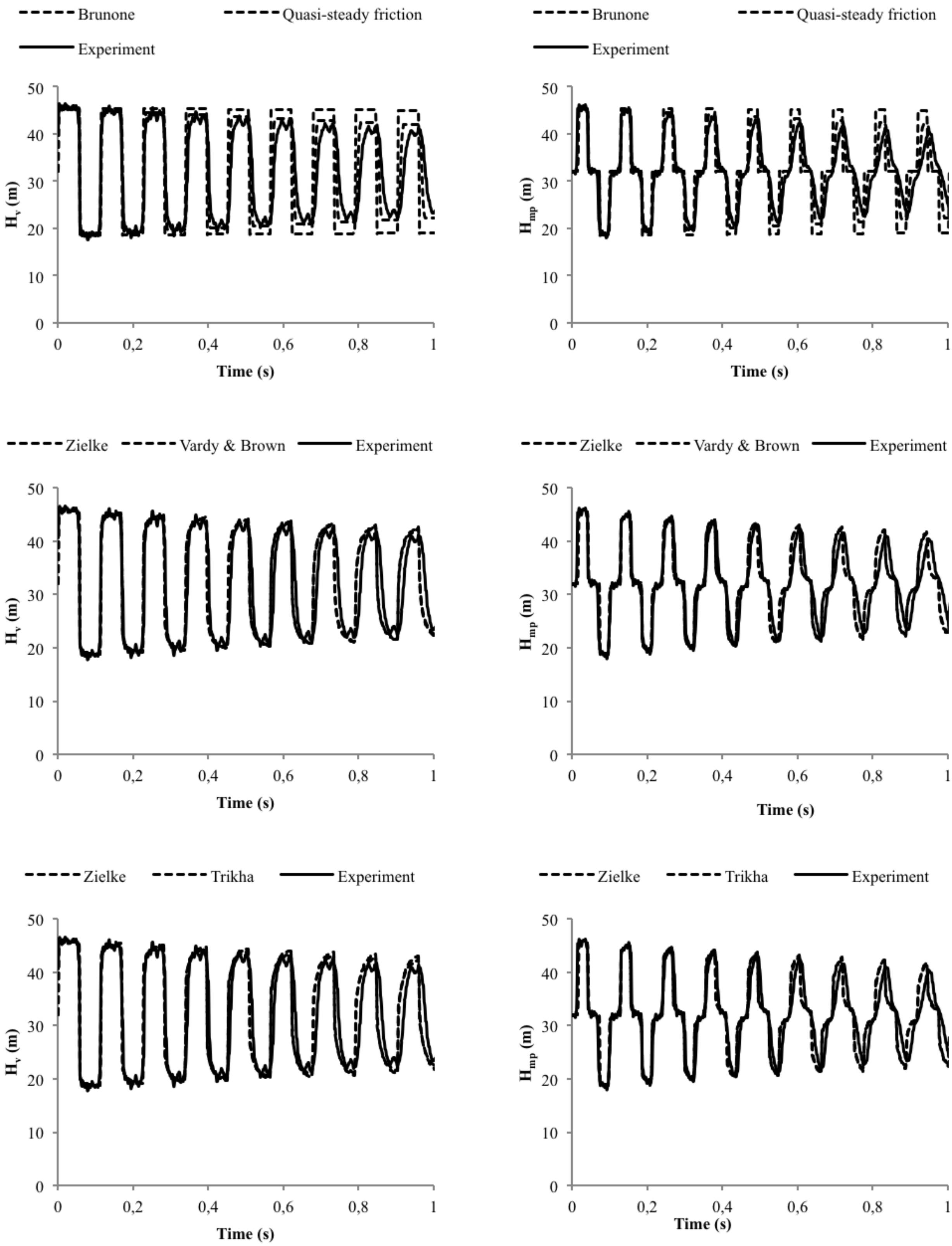

Figure 2. Comparison of heads at the valve (H_v) and at the midpoint (H_{mp}); $v_0 = 0.1\text{m/s},\ Nx = 16$

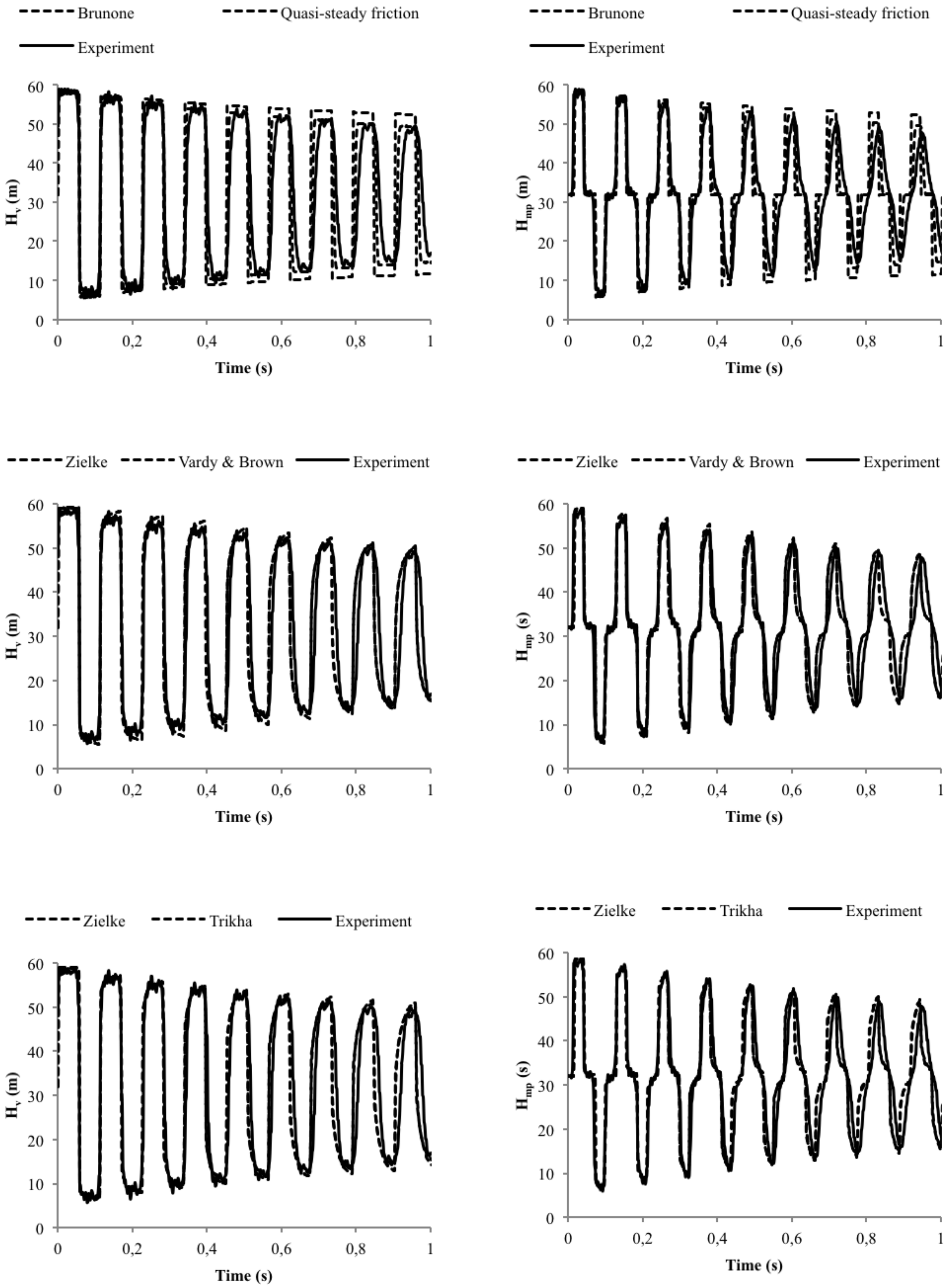

Figure 3. Comparison of heads at the valve (H_v) and at the midpoint (H_{mp}); $v_0 = 0.2 m/s,\ Nx = 16$

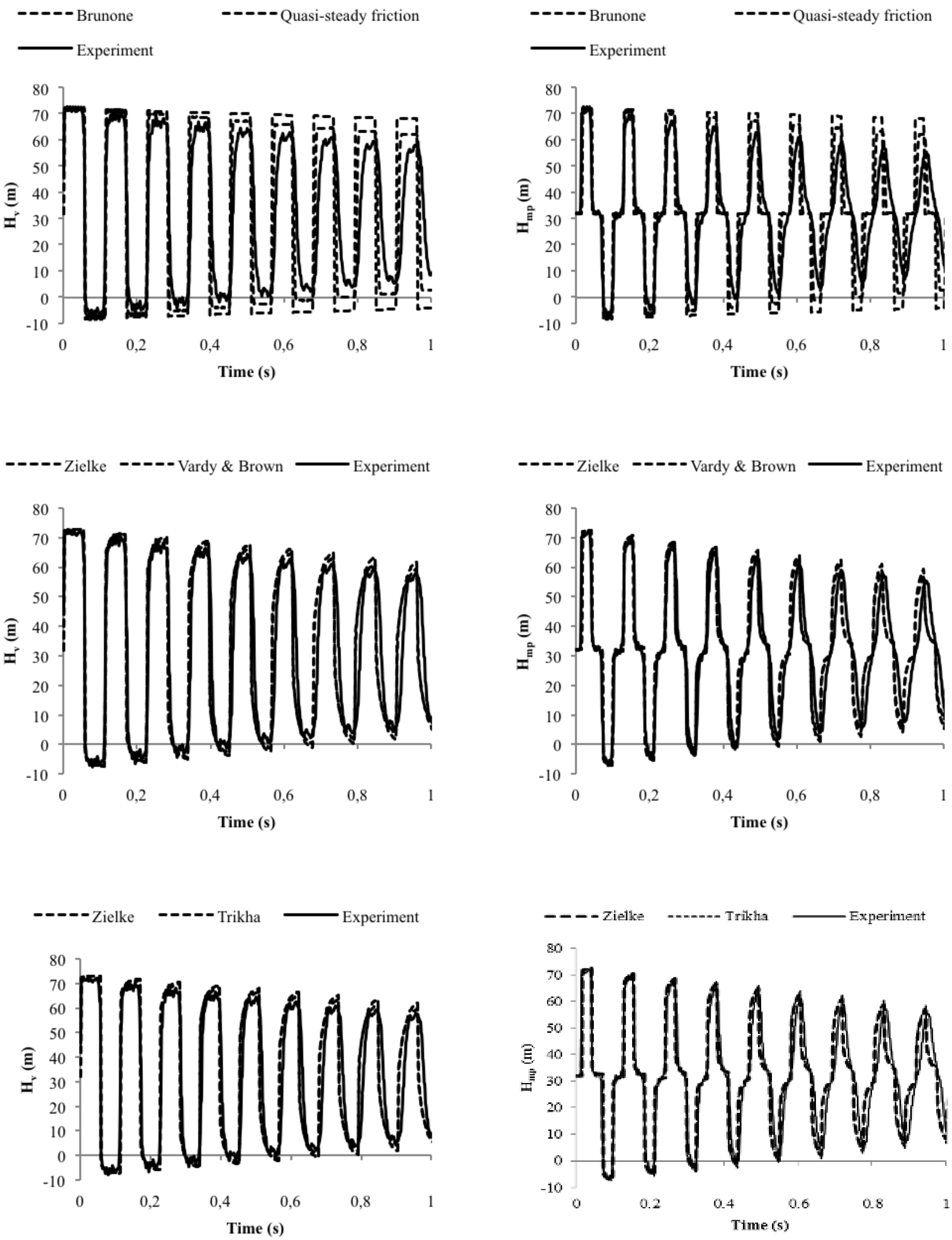

Figure 4. Comparison of heads at the valve (H_v) and at the midpoint (H_{mp}); $v_0 = 0.3 m/s$, $Nx = 16$

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