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Hypergeometric transforms in subclasses of univalent functions

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Abstract. In the present paper, we obtain certain sufficient conditions for special analytic functions to be in the class of normalized analytic functions satisfying the condition $\mathrm{Re}(f'(z)) > \beta|zf''(z)|$ for | z |< 1, where β is a given real number. The purpose of the present paper is to investigate various mapping and inclusion properties involving subclasses of analytic and univalent functions for a linear operator defined by means of Hadamard product with the Gaussian hypergeometric function.

Keywords: Starlike; convex; hypergeometric functions; univalent functions.

1. INTRODUCTION

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1-1}$$

which are analytic in the open unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$, and $\mathcal{S}$ denote the class of analytic and univalent functions in Δ.

A function $f \in \mathcal{A}$ is said to be starlike of order β $(0 \le \beta < 1)$, if and only if $Re\left\{\frac{zf'(z)}{f(z)}\right\} > \beta$, $z \in \Delta$.

This class is denoted by $\mathcal{S}^*(\beta)$, with $\mathcal{S}^*(0) = \mathcal{S}^*$.

A function $f \in \mathcal{A}$ is said to be convex of order β $(0 \le \beta < 1)$, if and only if

$Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \beta$, $z \in \Delta$. This class is denoted by $\mathcal{C}(\beta)$, with $\mathcal{C}(0) = \mathcal{C}$.

Note that $f \in \mathcal{S}$ is convex in Δ, if and only if zf' is starlike in Δ.

A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{UCV}$ of uniformly convex functions in Δ if and only if it has the property that, for every circular arc γ contained in the unit disk Δ, with center η also in Δ, the image curve $f(\gamma)$ is a convex arc.

The class $\mathcal{UCV}$ describes geometrically the domain of values of the expression $1 + \frac{zf''(z)}{f'(z)}$, $z \in \Delta$ to lie in a parabolic region $\Omega = \{w \in \mathbb{C} : (Im\, w)^2 < 2Re\, w - 1\}$.

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Ronning [1] defined the classes $\mathcal{UCV}$ and $\mathcal{S}_p$ as

$$\mathcal{UCV} = \left\{ f \in \mathcal{A}:\ Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \left|\frac{zf''(z)}{f'(z)}\right|, \qquad z \in \Delta\right\}.$$

$$\mathcal{S}_p = \left\{ f \in \mathcal{A}:\ Re\left\{\frac{zf'(z)}{f(z)}\right\} > \left|\frac{zf'(z)}{f(z)} - 1\right|, \qquad z \in \Delta\right\}.$$

A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{UCD}(\beta)$, $\beta \in \mathbb{R}$, if

$$Re(f'(z)) \geq \beta|z f''(z)|, \quad z \in \Delta.$$

The class $\mathcal{UCD}(\beta)$ is introduced by Breaz [2].

A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{R}^t(A, B)$ if

$$\left|\frac{f'(z) - 1}{t(A - B) - B(f'(z) - 1)}\right| < 1, \qquad (-1 \leq B < A \leq 1, t \in \mathbb{C} - \{0\}, z \in \Delta). \tag{1-2}$$

Clearly, a function f belongs to $\mathcal{R}^t(A, B)$ if and only if there exists a function *w* regular in Δ satisfying

$w(0) = 0$ and $|w(z)| < 1$ such that

$$1 + \frac{1}{t}(f'(z) - 1) = \frac{1 + Aw(z)}{1 + Bw(z)}, \quad z \in \Delta. \tag{1-3}$$

The class $\mathcal{R}^t(A, B)$ was introduced by Dixit and Pal [3]. By giving specific values to *t*, *A* and *B* in (1.2), we obtain the following subclasses studied by various researchers in earlier works:

(i) For $t = e^{-i\eta}\cos\eta \ \left(\eta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), A = 1 - 2\beta, 0 \leq \beta < 1\right) \ and\ B = -1$, we obtain the class of functions f satisfying the condition:

$$\left|\frac{e^{-i\eta}(f'(z) - 1)}{2(1 - \beta)\cos\eta + e^{i\eta}(f'(z) - 1)}\right| < 1, \qquad z \in \Delta. \tag{1-4}$$

In this case, the class $\mathcal{R}^t(A, B)$ is equivalent to the class $\mathcal{R}_\eta(\beta)$ which is studied by Ponnusamy and Ronning

[4]. Here, $\mathcal{R}_\eta(\beta)$ is the class of functions $f \in \mathcal{A}$ satisfying the condition:

$$Re\left(e^{i\eta}(f'(z) - \beta)\right) > 0, \quad \left(\eta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), (0 \leq \beta < 1), z \in \Delta\right).$$

(ii) For $t = e^{-i\eta}\cos\eta \ \left(\eta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right)$, we obtain the class of functions $f \in \mathcal{A}$ satisfying the condition:

$$\left|\frac{e^{-i\eta}(f'(z) - 1)}{Be^{i\eta}f'(z) - (A\cos\eta + iB\sin\eta)}\right| < 1, \qquad z \in \Delta. \tag{1-5}$$

which was studied by Dashrath [5].

(i) For $t = 1, A = \beta, (0 \le \beta < 1)$ and $B = -\beta$, we obtain the class of functions f satisfying the condition:

$$\left|\frac{f'(z)-1}{f'(z)+1}\right| < \beta, \qquad 0 \le \beta < 1, z \in \Delta.$$

which was studied by Caplinger and Cauchy [6] and Padmanabhan [7].

Let F(a,b;c;z) be the Gaussian hypergeometric function defined by

$$F(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n (1)_n} z^n \tag{1-6}$$

Where $c \ne 0, -1, -2, \ldots$ and $(\theta)_n$ is the pochhammer symbol defined by

$$(\theta)_n = \begin{cases} 1 & n = 0 \\ \theta(\theta+1)(\theta+2)\ldots(\theta+n-1) & n \in \mathbb{N} \end{cases}$$

We note that $F(a,b;c;1)$ converges for $Re(c-a-b) > 0$ and is related to the Gamma function by

$$F(a,b;c;1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \tag{1-7}$$

Let $f(z) = \sum_{n=2}^{\infty} a_n z^n$, $g(z) = \sum_{n=2}^{\infty} b_n z^n$. Then the Hadamard product or convolution of f(z) and g(z)

written as $(f * g)(z)$ is defined by

$$(f * g)(z) = \sum_{i=2}^{\infty} a_n b_n z^n, \qquad z \in \Delta.$$

For $f \in \mathcal{A}$, we define the operator $I_{a,b,c}(f)$ by

$$I_{a,b,c}(f)(z) = zF(a,b;c;z) * f(z) \tag{1-8}$$

where $*$ denotes the usual Hadamard product (or convolution) of power series.

2. MAIN RESULT

To prove the main result, we need the following lemmas.

Lemma 2.1 [2] A function $f(z)$ of the form (1-1) is in class $\mathcal{UCD}(\beta)$ if

$$\sum_{n=2}^{\infty} n[1+\beta(n-1)]\,|a_n| \le 1.$$

Lemma 2.2 [3]

(i) Let a function $f(z)$ of the form (1-1) be in $\mathcal{R}^t(A,B)$. Then

$|a_n| \leq \dfrac{(A-B)|t|}{n}$.

(ii) Let a function $f(z)$ of the form (1-1) be in $\mathcal{A}$. If

$$\sum_{n=2}^{\infty}(1+|B|)n\ |a_n| \leq (A-B)|t|, \qquad (-1 \leq B < A \leq 1, t \in \mathbb{C}, z \in \Delta).$$

Then $f \in \mathcal{R}^t(A,B)$.

Lemma 2.3 [8] Let w(z) be regular in the unit disk Δ with w(0)=0. Then, If $|w(z)|$ attains its maximum value on the circle $|z| = r < 1$ at a point z_0, then

$$z_0 w'(z_0) = cw(z_0), \qquad (c \geq 1).$$

Lemma 2.4 [4]

(i) For $a, b \in \mathbb{C} - \{0,1\}$ and $c \in \mathbb{C} - \{1\}$ with $c > \max\{0, a+b-1\}$,

$$\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n(1)_{n+1}} = \frac{1}{(a-1)(b-1)}\left[\frac{\Gamma(c)\Gamma(c-a-b+1)}{\Gamma(c-a)\Gamma(c-b)} - (c-1)\right].$$

(ii) For $a, b \in \mathbb{C} - \{0\}$ with $a > 0$ and $b > 0$ and $c > a + b + 1$,

$$\sum_{n=0}^{\infty}\frac{(n+1)(a)_n(b)_n}{(c)_n(1)_n} = \left(\frac{ab}{c-a-b-1} + 1\right)\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}.$$

Lemma 2.5 [9] A function $f(z)$ of the form (1.1) is in class $\mathcal{UCV}$ if

$$\sum_{n=2}^{\infty} n(2n-1)\ |a_n| \leq 1 .$$

Theorem 2.6 Let $a, b \in \mathbb{C} - \{0\}$ and $c \in \mathbb{R}$ such that c > |a|+|b|+2. Let $f \in \mathcal{A}$ and be of the form (1.1). If the

hypergeometric inequality

$$\frac{\Gamma(c)\Gamma(c-|a|-|b|-2)}{\Gamma(c-|a|)\Gamma(c-|b|)}[(c-|a|-|b|-2)(c-|a|-|b|-1) + \beta|ab|(1+|a|)(1+|b|)$$
$$+ (1+2\beta)|ab|(c-|a|-|b|-2)] \leq 2,$$

is satisfied, then $zF(a,b;c;z) \in \mathcal{UCD}(\beta)$.

Proof. The function $zF(a,b;c;z)$ has the series representation given by

$$zF(a,b;c;z) = z + \sum_{n=2}^{\infty}\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}\ z^n.$$

In view of Lemma 2.1, it suffices to show that

$$S(a,b,c,\beta) := \sum_{n=2}^{\infty} n(1+\beta(n-1)) \left| \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}} \right| \leq 1.$$

From the fact that $|(a)_n| = (|a|)_n$, we observe that, since c is real and positive, under the hypothesis

$$S(a,b,c,\beta) \leq \sum_{n=2}^{\infty} n(1+\beta(n-1)) \frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-1}}.$$

Writing n[1 + β (n − 1)] as, 1 + (1 + 2 β)(n − 1) + β (n − 1)(n − 2) we get

$$S(a,b,c,\beta) \leq \sum_{n=2}^{\infty} \frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-1}} + (1+2\beta) \sum_{n=2}^{\infty} (n-1) \frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-1}} +$$

$$\beta \sum_{n=2}^{\infty} (n-1)(n-2) \frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-1}} = \sum_{n=2}^{\infty} \frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-1}} +$$

$$(1+2\beta) \sum_{n=2}^{\infty} \frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-2}} + \beta \sum_{n=3}^{\infty} \frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-3}}.$$

Using the fact that $(a)_n = a(a+1)_{n-1}$, it is easy to see that,

$$S(a,b,c,\beta) \leq \sum_{n=2}^{\infty} \frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-1}} + (1+2\beta) \frac{|ab|}{c} \sum_{n=2}^{\infty} \frac{(1+|a|)_{n-2}(1+|b|)_{n-2}}{(1+c)_{n-2}(1)_{n-2}} +$$

$$\beta \frac{|ab|(1+|a|)(1+|b|)}{c(1+c)} \sum_{n=3}^{\infty} \frac{(2+|a|)_{n-3}(2+|b|)_{n-3}}{(2+c)_{n-3}(1)_{n-3}}.$$

From (1-6), we have

$$S(a,b,c,\beta) \leq F(|a|,|b|;c;1) - 1 + (1+2\beta) \frac{|ab|}{c} F(1+|a|,1+|b|;1+c;1) +$$

$$\beta \frac{|ab|(1+|a|)(1+|b|)}{c(1+c)} F(2+|a|,2+|b|;2+c;1).$$

The proof of Theorem 2.6 follows now by an application of the Gauss summation theorem

$$F(a,b;c;1) = \sum_{n=0}^{\infty} \frac{(a)_n(b)_n}{(c)_n(1)_n} = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \quad Re(c-a-b) > 0 \ . \blacksquare$$

Theorem 2.7 Let $a,b \in \mathbb{C} - \{0\}$ and $c \in \mathbb{R}$ such that c > |a|+|b|+1. If $f \in \mathcal{R}^{\tau}(A,B)$ and If the inequality

$$\frac{\Gamma(c)\Gamma(c-|a|-|b|-1)}{\Gamma(c-|a|)\Gamma(c-|b|)} [\beta|ab| + (c-|a|-|b|-1)] \leq \frac{1}{(A-B)|\tau|} + 1, \qquad (2\text{-}1)$$

is satisfied, then $I_{a,b,c}(f) \in \mathcal{UCD}(\beta)$.

Proof. Let f be of the form (1-1) belong to the class $\mathcal{R}^{t}(A,B)$. In view of Lemma 2.1, it suffices to show that

$$\sum_{n=2}^{\infty} n(1+\beta(n-1))\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}a_n\right| \leq 1. \tag{2-2}$$

Taking into account the inequality (i) of lemma (2.2) and the relation $|(a)_{n-1}| = (|a|)_{n-1}$, we deduce that

$$\sum_{n=2}^{\infty} n(1+\beta(n-1))\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}a_n\right| \leq (A-B)|t|\sum_{n=2}^{\infty}(1+\beta(n-1))\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}\right| \leq$$

$$(A-B)|t|\sum_{n=2}^{\infty}\frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-1}} + \beta(A-B)|t|\sum_{n=2}^{\infty}\frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-2}}.$$

the inequality (2-2) now follows by applying the Gauss summation theorem and (2-1). ■

Corollary 2.8 Let $a,b \in \mathbb{C}-\{0\}$. Suppose that |b|=|a|. Further let $c \in \mathbb{R}$ such that c >2 |a |+1. If

$f \in \mathcal{R}^{t}(A,B)$ and If the inequality

$$\frac{\Gamma(c)\Gamma(c-2|a|-1)}{(\Gamma(c-|a|))^2}[\beta|a|^2+(c-2|a|-1)] \leq \frac{1}{(A-B)|t|}+1, \tag{2-3}$$

is satisfied, then $I_{a,b,c}(f) \in \mathcal{UCD}(\beta)$.

In the special case when b = 1, Theorem 2.7 immediately yields a result concerning the Carlson–Shaffer operator

$$\mathcal{L}(a,c)(f) := I_{a,1,c}(f).$$

Corollary 2.9 Let $a \in \mathbb{C}-\{0\}$. Also, let $c \in \mathbb{R}$ such that c > |a |+2. If $f \in \mathcal{R}^{t}(A,B)$ and If the inequality

$$\frac{\Gamma(c)\Gamma(c-|a|-2)}{\Gamma(c-|a|)\Gamma(c-1)}[\beta|a|+(c-|a|-2)] \leq \frac{1}{(A-B)|t|}+1, \tag{2-4}$$

is satisfied, then $\mathcal{L}(a,c)(f) \in \mathcal{UCD}(\beta)$.

Theorem 2.10 Let $f \in \mathcal{A}$. *If*

$$\left|\left(I_{a,b,c}f(z)'\right)-1\right|^{1-\beta}\left|\frac{z\left(I_{a,1,c}(f)(z)\right)''}{\left(I_{a,1,c}(f)(z)\right)'}\right|^{\beta} < \frac{1}{2^{\beta}}, \qquad 0 \leq \beta < 1, z \in \Delta. \tag{2-5}$$

then $I_{a,b,c}(f)$ sunivalent in Δ.

Theorem 2.11 Let $a,b \in \mathbb{C}-\{0\}$ and c > |a|+|b|. Suppose that $f \in \mathcal{R}^{t}(A,B)$ and satisfy the condition

$$\frac{\Gamma(c)\Gamma(c-|a|-|b|)}{\Gamma(c-|a|)\Gamma(c-|b|)} \leq \frac{1}{1+|B|}+1. \qquad (2\text{-}6)$$

Then the operator $I_{a,b;c}(f)$ maps $\mathcal{R}^{t}(A,B)$ into $\mathcal{R}^{t}(A,B)$.

Proof .Let $a,b\in\mathbb{C}-\{0\}$ and c > |a|+|b|. Suppose that $f(z)=z+\sum_{n=2}^{\infty}a_n z^n\in\mathcal{R}^{t}(A,B)$. Then, By (ii) of Lemma (2.2), it suffices to show that

$$\sum_{n=2}^{\infty}(1+|B|)n\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}a_n\right| \leq (A-B)|t|\,.$$

From (i) of Lemma (2.2) and the fact that $|(a)_n|\leq(|a|)_n$, we have

$$\sum_{n=2}^{\infty}(1+|B|)n\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}a_n\right| \leq \sum_{n=2}^{\infty}(A-B)\,|t|\,(1+|B|)\,\frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-1}}\times$$

$$(A-B)|t|(1+|B|)\left(\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n(1)_n}-1\right).$$

Using the formula (1-7) and the assumption, we find that

$$\sum_{n=2}^{\infty}(1+|B|)n\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}a_n\right| \leq (A-B)|t|(1+|B|)\left(\frac{\Gamma(c)\Gamma(c-|a|-|b|)}{\Gamma(c-|a|)\Gamma(c-|b|)}-1\right)$$
$$\leq (A-B)|t|,$$

which implies that the operator $I_{a,b;c}(f)$ maps $\mathcal{R}^{t}(A,B)$ into $\mathcal{R}^{t}(A,B)$. ■

Theorem 2.12 Let $a,b\in\mathbb{C}-\{0\}$, c > |a|+|b|+1 and $f\in\mathcal{R}^{t}(A,B)$. Suppose that

$$(A-B)|t|\left[\frac{\Gamma(c)\Gamma(c-|a|-|b|)}{\Gamma(c-|a|)\Gamma(c-|b|)}\left(\frac{2|ab|}{c-|a|-|b|-1}+1\right)-1\right] \leq 1. \qquad (2\text{-}7)$$

Then the operator $I_{a,b;c}(f)$ maps $\mathcal{R}^{t}(A,B)$ into $\mathcal{UCV}$.

Proof .Let $a,b\in\mathbb{C}-\{0\}$ and c > |a|+|b|+1. Suppose that $f(z)=z+\sum_{n=2}^{\infty}a_n z^n\in\mathcal{R}^{t}(A,B)$. Then,

By Lemma (2.5), it suffices to show that

$$\sum_{n=2}^{\infty}n\,(2n-1)\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}a_n\right| \leq 1\,.$$

Then, from (1-7) and $|(a)_n|=|a|(|a|)_{n-1}$, we have

$$\sum_{n=2}^{\infty}n\,(2n-1)\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}a_n\right| \leq (A-B)|t|\left[\sum_{n=1}^{\infty}(2n+1)\frac{(|a|)_n(|b|)_n}{(c)_n(1)_n}\right]=$$

$$(A-B)|t|\left[2\sum_{n=1}^{\infty}\frac{(|a|)_n(|b|)_n}{(c)_n(1)_{n-1}}+\sum_{n=0}^{\infty}\frac{(|a|)_n(|b|)_n}{(c)_n(1)_n}-1\right]=$$

$$(A-B)|t|\left[\frac{2|ab|}{c}\times\frac{\Gamma(c+1)\Gamma(c-|a|-|b|-1)}{\Gamma(c-|a|)\Gamma(c-|b|)}+\frac{\Gamma(c)\Gamma(c-|a|-|b|)}{\Gamma(c-|a|)\Gamma(c-|b|)}\right]=$$

$$(A-B)|t|\left[\frac{\Gamma(c)\Gamma(c-|a|-|b|)}{\Gamma(c-|a|)\Gamma(c-|b|)}\left(\frac{2|ab|}{c-|a|-|b|-1}+1\right)-1\right]\leq 1,$$

by (2-7), which completes the proof of Theorem 2.12.

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