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Free Vibration Analyses of Hexapod Machine tool Set

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Abstract. Hexapod as a parallel robot is mechanism with changeable leg length. From 1950 onwards, parallel robots in a variety of different research and industrial purposes built and have been exploited.in this research, the free vibration analysis of machine tool set based on Stewart mechanisms with legs with the ability to change length and capable of machining complex parts with medium and small sizes have been studied. In earlier works, vibration analysis of hexapod table alone, without considering the structures of Machine tool was implemented. In the present study, body of machine tool as a vibration chain connected to the entire system is analyzed and free vibration analysis of Hexapod Machine tool set includes table and FP4M milling machine head as a complete vibration chain is studied. In order, vibration equations of Hexapod table and connected milling machine and tools is written and extraction program of natural frequencies of the structure in MATLAB software is written. Then in order to verify the theoretical work, modeling of the frame and Hexapod table set is done in the ANSYS software and the natural frequencies obtained were compared with the theoretical results of this software.

Keywords: Free vibration, Hexapod, machine tool

1. INTRODUCTION

Recognition and vibration control or isolation of vibration in different tools and devices to improve performance and increase the efficiency of their work is an important field of study and efforts of engineers and scientists. There is many theoretical studies on direct and inverse kinematics and dynamics, calculation of workspace and singularities, and motion control of Hexapod machine tools in the world but little work has been done on the analysis of the vibrations which are mentioned below.

Dohner and colleagues [1] of various research institutions in the field of active vibration reduction and elimination of Octahedral milling machine researched that led to the vibration control of cutting process. Flint and colleagues [2] working on a variety of parallel mechanisms with the aim of positioning and active vibration isolation, concluded that vibration forecasting and preventing of it in positioning mechanism such as Hexapod table, is better than active vibration control and with the wide range of environmental conditions is more practical. Abuhanieh [3] has investigated uniaxial isolation methods using a variety of sensors and feedback. In his study, regardless of the mass and the depreciation of structures, assuming the mass of the platforms, the natural frequencies of set are obtained. In that system, the vibration model has not the configuration change, position and mass. Mahboobkhah and colleagues [4, 5], have investigated free and forced vibrations of Hexapod table considering flexible components of legs and joints of the system in their study and natural frequency of Hexapod table are gained regardless of the connection of work piece and the tools. In their research, the machine tool construction as a vibration chain attached to the Hexapod table is not addressed that in the present study will be discussed.

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2. VIBRATION EQUATION MODELING OF HEXAPOD MACHINE TOOL SET

The machine tools set examined, includes FP4M milling machines and Hexapod table that modeling of machine tools' body According to dimensional data's of the FP4M milling machine in Tabriz Mashinsazi group's website [6] and the precise measurement of structures and the sample Hexapod table model made at Tarbiat Modarres University, Tehran, trying to preserve maximum detail and complete accuracy of the structure has been done in CATIA software. Figure 1 shows an example of intended Hexapod machine tool set modeled considering all the structural components that are important in natural frequencies and mode shapes of structures.

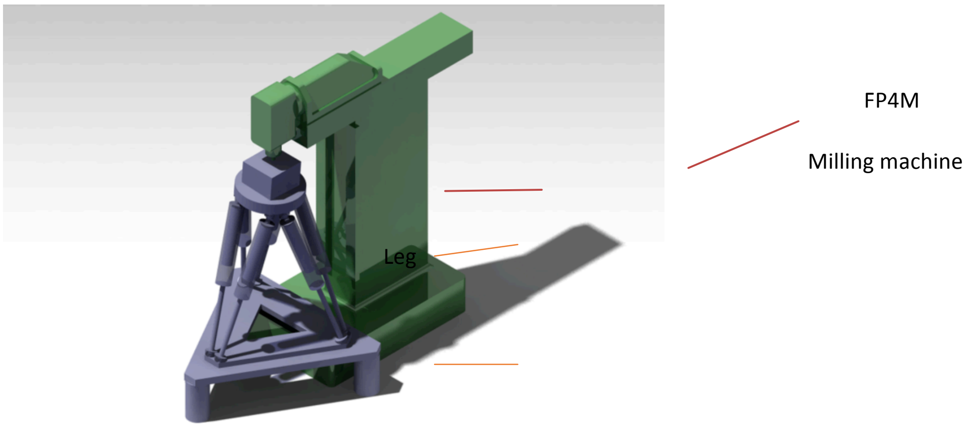

Figure 1. The CAD model of Hexapod machine tool set.

To form the vibration chain of Hexapod Machine tool, the six legs are connected to the Machine tool by moving table. According to Figure 2, each leg are considered equivalent to a spring that is attached to a movable table. Another spring connected to the moving platform by means of tools that is equivalent to spring of Milling Machine body. With the above assumptions, the equations of free vibration of moving platform with respect to the total displacement of the moving platform of caused by stiffness forces acting on the moving platform, can be calculated.

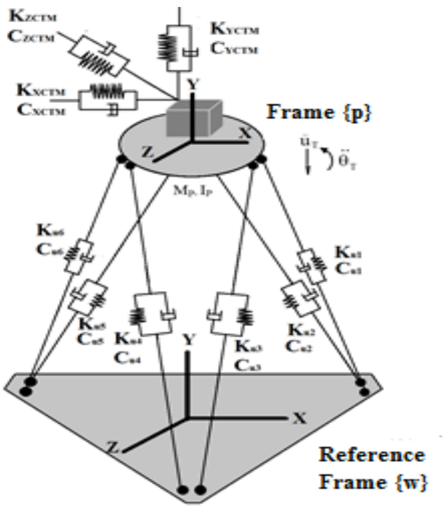

Figure 2. Schematic of the Hexapod machine tool set as equivalent springs.

In Figure 2, m, k and C, respectively, are the symbol of mass, stiffness and damping. K_{ui} and C_{ui} are the stiffness and damping coefficients of the legs and joints and stable platform of Hexapod table. K_{CTM} and C_{CTM} are the total of the stiffness and damping coefficients of the machine tool that imports of the moving platform of the and cutting tool in directions X, Y and Z, which will be calculated in the following. M_p and I_p are the total mass and mass moment of inertia of the moving platform, work piece and fixture. There are three coordinate system is used to define the system movement. Reference coordinate system {W} is located at the geometric center of the lower stable platform. Local coordinate system {P} is connected to the geometric center of the moving platform. Another local system {A}, connected to the geometric center of the moving platform to define the rotation and orientation of the moving platform. This coordinate system moves with the platform but don't rotate with it. Vector $T(x_0, y_0, z_0)$ defined in the {W}, specify the location of the coordinate system {P}. Rotation of coordinate system {P} relative to {A} is shown with the rotation matrix ${}^{\mathbf{W}}\mathbf{R}_{\mathbf{P}}$. Also $\ddot{u}_T$ and $\ddot{\theta}_T$ are the linear and angular acceleration of the platform in reference frame {W} according to the leg's linear acceleration $\ddot{l}_{PI}$ (In all of the following equations the vector's symptoms due to the large number of vectors removed and in its place are shown in bold.).For the centralized mass of moving platform, the equation of the vibration- force (Newton) is obtained as follows:

$$\mathbf{M}_{\mathrm{P}}\ddot{\mathbf{u}}_{\mathrm{T}} + [\sum \mathbf{n}_{\mathrm{i}}\mathbf{F}_{\mathrm{ci}} - {}^{\mathbf{W}}\mathbf{R}_{\mathrm{P}}\mathbf{F}_{\mathrm{CTM}}] + [\sum \mathbf{n}_{i}\mathbf{F}_{\mathrm{Kui}} - {}^{\mathbf{W}}\mathbf{R}_{\mathrm{P}}\mathbf{F}_{\mathrm{KTM}}] = {}^{\mathbf{W}}\mathbf{R}_{\mathrm{P}}\mathbf{F}_{\mathrm{mac}} \tag{1}$$

Where ${}^{W}\mathbf{R}_{P}$ the rotation matrix of the moving platform with local coordinates {P} relative to the reference frame {W} is and $\mathbf{n}_{\mathrm{i}}$ is the unit vector along the legs. Also $\mathbf{F}_{KTM}$ is the equivalent stiffness of FP4M Milling Machine tool that enters the moving table in the joint of the work piece and tool. Momentum equation (Euler) of moving platform is as follows:

$$\begin{aligned}&\mathbf{I}_{\mathrm{P}}\ddot{\boldsymbol{\theta}}_{\mathrm{T}} + \left[\sum \mathbf{q}_{\mathrm{ai}} \times \mathbf{n}_{\mathrm{i}}\mathbf{F}_{\mathrm{cui}} - {}^{\mathrm{P}}\mathbf{M}_{\mathrm{CTM}} - \mathbf{GC} \times {}^{\mathrm{P}}\mathbf{F}_{\mathrm{CTM}}\right] + \left[\sum \mathbf{q}_{\mathrm{ai}} \times \mathbf{n}_{\mathrm{i}}\mathbf{F}_{\mathrm{Kui}} - {}^{\mathrm{P}}\mathbf{M}_{\mathrm{KTM}} - \mathbf{GC} \times {}^{\mathrm{P}}\mathbf{F}_{\mathrm{KTM}}\right] \\ &= {}^{\mathrm{W}}\mathbf{R}_{\mathrm{P}}({}^{\mathrm{P}}\mathbf{M}_{\mathrm{mac}} + \mathbf{GC} \times {}^{\mathrm{P}}\mathbf{F}_{\mathrm{mac}})\end{aligned} \tag{2}$$

By the coupling mentioned equations and simplifying them:

$$\begin{aligned}&\begin{bmatrix}\mathbf{M}_{\mathrm{P}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{\mathrm{P}}\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{u}}_{\mathrm{T}} \\ \ddot{\boldsymbol{\theta}}_{\mathrm{T}}\end{Bmatrix} + \mathbf{J}^{-\mathrm{T}}\mathbf{C}_{\mathrm{us}}.\Delta\dot{\mathbf{l}}_{\mathrm{us}} - \begin{bmatrix}{}^{\mathrm{W}}\mathbf{R}_{\mathrm{P}}.\mathbf{C}_{\mathrm{CTM}}.\dot{\mathbf{u}}_{\mathrm{T}} \\ \mathbf{C}_{\theta\mathrm{CTM}} + \mathbf{GC} \times \mathbf{C}_{\mathrm{CTM}}.\dot{\mathbf{u}}_{\mathrm{T}}\end{bmatrix} + \mathbf{J}^{-\mathrm{T}}\mathbf{K}_{\mathrm{us}}.\Delta\mathbf{l}_{\mathrm{us}} \\ &- \begin{bmatrix}{}^{\mathrm{W}}\mathbf{R}_{\mathrm{P}}.\mathbf{K}_{\mathrm{CTM}}.\mathbf{u}_{\mathrm{T}} \\ \mathbf{K}_{\theta\mathrm{CTM}}.\boldsymbol{\theta}_{\mathrm{T}} + \mathbf{GC} \times \mathbf{K}_{\mathrm{CTM}}.\mathbf{u}_{\mathrm{T}}\end{bmatrix} = \begin{bmatrix}{}^{\mathrm{W}}\mathbf{R}_{\mathrm{P}}.{}^{\mathrm{P}}\mathbf{F}_{\mathrm{mac}} \\ {}^{\mathrm{W}}\mathbf{R}_{\mathrm{P}}\left({}^{\mathrm{P}}\mathbf{M}_{\mathrm{mac}} + \mathbf{GC} \times {}^{\mathrm{P}}\mathbf{F}_{\mathrm{mac}}\right)\end{bmatrix}\end{aligned} \tag{3}$$

With the replacement of the following formula by equation (3),

$$\Delta\mathbf{l}_{\mathrm{us}} = \mathbf{J}^{-1}\begin{Bmatrix}\mathbf{u}_{\mathrm{T}} \\ \boldsymbol{\theta}_{\mathrm{T}}\end{Bmatrix} \qquad \Delta\dot{\mathbf{l}}_{\mathrm{us}} = \mathbf{J}^{-1}\begin{Bmatrix}\dot{\mathbf{u}}_{\mathrm{T}} \\ \dot{\boldsymbol{\theta}}_{\mathrm{T}}\end{Bmatrix} \tag{4}$$

We get:

$$\begin{bmatrix} \mathbf{M}_{\mathrm{P}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{\mathrm{P}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_{\mathrm{T}} \\ \ddot{\boldsymbol{\theta}}_{\mathrm{T}} \end{Bmatrix} + \mathbf{J}^{-\mathrm{T}}\mathbf{C}_{\mathrm{us}}\mathbf{J}^{-1} \begin{Bmatrix} \dot{\mathbf{u}}_{\mathrm{T}} \\ \dot{\boldsymbol{\theta}}_{\mathrm{T}} \end{Bmatrix} - \begin{bmatrix} {}^{\mathrm{W}}\mathbf{R}_{\mathrm{P}}.\mathbf{C}_{\mathrm{CTM}} & 0 \\ \mathbf{GC}\times\mathbf{C}_{\mathrm{CTM}} & \mathbf{C}_{\theta\mathrm{CTM}} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_{\mathrm{T}} \\ \dot{\boldsymbol{\theta}}_{\mathrm{T}} \end{Bmatrix} + \mathbf{J}^{-\mathrm{T}}\mathbf{K}_{\mathrm{us}}\mathbf{J}^{-1} \begin{Bmatrix} \mathbf{u}_{\mathrm{T}} \\ \boldsymbol{\theta}_{\mathrm{T}} \end{Bmatrix}$$

$$- \begin{bmatrix} {}^{\mathrm{W}}\mathbf{R}_{\mathrm{P}}.\mathbf{K}_{\mathrm{CTM}} & 0 \\ \mathbf{GC}\times\mathbf{K}_{\mathrm{CTM}} & \mathbf{K}_{\theta\mathrm{CTM}} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{\mathrm{T}} \\ \boldsymbol{\theta}_{\mathrm{T}} \end{Bmatrix} = \begin{bmatrix} {}^{\mathrm{W}}\mathbf{R}_{\mathrm{P}}.{}^{\mathrm{P}}\mathbf{F}_{\mathrm{mac}} \\ {}^{\mathrm{W}}\mathbf{R}_{\mathrm{P}}\left({}^{\mathrm{P}}\mathbf{M}_{\mathrm{mac}} + \mathbf{GC}\times{}^{\mathrm{P}}\mathbf{F}_{\mathrm{mac}}\right) \end{bmatrix} \tag{5}$$

To find the natural frequencies, should ignore of depreciation and the equation will be as follows:

$$\begin{bmatrix} \mathbf{M}_{\mathrm{P}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{\mathrm{P}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_{\mathrm{T}} \\ \ddot{\boldsymbol{\theta}}_{\mathrm{T}} \end{Bmatrix} + \left(\mathbf{J}^{-\mathrm{T}}\mathbf{K}_{\mathrm{us}}\mathbf{J}^{-1} - \begin{bmatrix} {}^{\mathrm{W}}\mathbf{R}_{\mathrm{P}}.\mathbf{K}_{\mathrm{CTM}} & 0 \\ \mathbf{GC}\times\mathbf{K}_{\mathrm{CTM}} & \mathbf{K}_{\theta\mathrm{CTM}} \end{bmatrix} \right) \begin{Bmatrix} \mathbf{u}_{\mathrm{T}} \\ \boldsymbol{\theta}_{\mathrm{T}} \end{Bmatrix} = 0 \tag{6}$$

Where $\mathbf{K}_{\mathrm{us}}$ is a diagonal matrix whose main diagonal entries are the stiffness coefficients of the spherical joint and the upper part of legs. E_3 is the 3 × 3 unit matrix. **GC** is the position vector of stiffness forces of FP4M machine tool relative to the center of moving platform is usually applied at the junction of the tool to the work piece. Also $\mathbf{J}^{-1}$ transposed Jacobean's matrix on the reverse.

3. CALCULATION OF THE EQUIVALENT STIFFNESS

The equivalent stiffness is the stiffness of the Hexapod legs that the equivalents to the force have been calculated in previous work [4, 5]. The stiffness of the milling machine structure will be calculated in two methods using ANSYS software and stiffness equations as the following is obtained. Actual stiffness coefficient of each member can be determined by practical experiments; however approximate stiffness of each element of the machine tool body is calculated using the formula:

$$K = \frac{AE}{L} \tag{7}$$

Where E Young's modulus (200 GPa for steel) is, A is the cross sectional area and L is the length of the piece. The equivalent stiffness is calculated according to the following equation.

$$\frac{1}{K_T} = \frac{1}{K_1} + \frac{1}{K_2} + \ldots + \frac{1}{K_i} \tag{8}$$

4. FINITE ELEMENT ANALYSIS

FEM modeling of Hexapod machine tool set is done by taking the Structural condition and meshed by Solid element type (Fig. 3).By applying other appropriate material specifications and support conditions in the software ANSYS, the set will be solved using Block Lonczos by finite element analysis method. In Block Lonczos method, the analyst Sparse is used. This method applied for solving large eigenvalues of symmetric and hybrid models made of Shell and Solid elements that are not suitable networked. To obtain more accurate results and considering all boundary conditions for Hexapod table, ten different configurations were considered, and all analyzes were performed for all the ten configurations. The Difference between configurations with each other is the position of the moving platform and its angle with the coordinate axis which leads to differences in the legs length for each configuration that is created relative to the workspace of the system. In addition, the modal analysis of the Hexapod table and machine tools body has done separately for comparison with the results of the modal analysis of Hexapod machine tool set

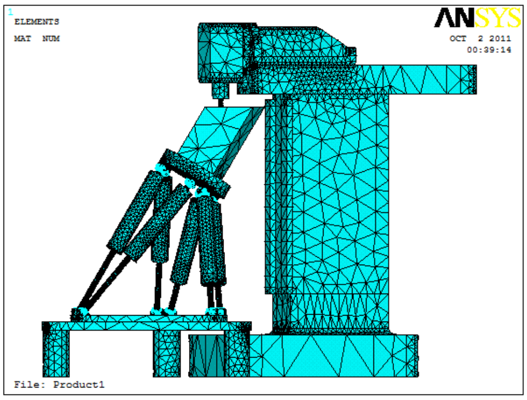

Figure 3. FEM model of Hexapod machine tool set is meshed in the ANSYS software

5. CONCLUSIONS

The first six natural frequencies are displacement in the direction of X, displacement in the direction Y, rotation around the axis Z, moving along Z, rotation around the X and Y. As ANSYS software analysis results, the first six natural frequencies of the machine tool body in Table 1 and Hexapod table in Table 2 are listed. The natural frequencies of the Hexapod machine tool set Obtained by solving vibrational equation by MATLAB software using the two methods of calculation and finite element to obtain stiffness for each configuration is given in Table 3. Finally the natural frequencies of the Hexapod machine tool set obtain from the finite element method for the three types of contact between the joints of legs in three modes of all rigid joints; all flexible joints and upper flexible joints are listed in Table 4. To further understand the results, the comparisons of the results are shown in Figures 4 to 6.

Table 1. The natural frequencies of FP4 milling machine tool.

The FEM model for analysis	Natural frequencies (hertz)					
	1st mode	2nd mode	3rd mode	4th mode	5th mode	6th mode
Structure of milling machine tool	61.845	118.9	131.425	382.9	563.3	682.3

Table 2. The natural frequencies of Hexapod table.

		Natural frequencies (hertz)					
	1st mode	2nd mode	3rd mode	4th mode	5th mode	6th mode	1st mode
Lower 1	94.176	114.175	186.354	301.915	324.756	403.304	94.176
Lower 2	94.17	117.525	178.362	302.102	314.947	400.322	94.17
Lower 3	108.89	113.592	247.353	285.599	324.065	403.864	108.89
Lower 4	139.363	141.281	284.275	287.808	315.938	436.967	139.363
Middle 1	62.684	94.976	159.346	276.051	298.226	347.328	62.684
Middle 2	63.45	95.997	160.432	275.630	300.225	348.999	63.45
Upper 1	94.26	113.613	159.700	338.162	417.987	428.067	94.26
Upper 2	93.296	111.713	156.627	322.156	416.919	426.067	93.296
Upper 3	73.256	111.912	154.652	320.198	413.969	427.067	73.256
Upper 4	52.436	117.255	150.878	316.971	411.87	427.857	52.436

Table 3. The natural frequencies of the machine tool body using the calculated stiffness (right column) and finite element method (left column)

Configuration name	Natural frequencies (hertz)											
	1st mode		2nd mode		3rd mode		4^{th} mode		5^{th} mode		6^{th} mode	
Lower1	76.31	78.708	124.95	124.979	156.54	157.771	261.13	260.751	320.28	320.365	485.83	486.503
Lower2	79.788	80.229	116.928	118.462	192.978	193.923	222.191	223.282	324.433	324.080	486.07	476.795
Lower3	97.348	101.773	110.785	108.858	239.931	240.061	336.238	336.003	370.783	370.661	464.37	365.584
Lower4	110.19	114.857	121.690	119.423	264.530	264.487	390.051	369.05	399.774	399.817	446.69	447.686
Middle1	65.351	73.675	105.911	105.809	181.204	181.662	321.364	321.417	527.145	527.025	607.02	607.640
Middle2	76.063	76.668	107.746	111.262	183.258	185.690	309.536	309.869	533.623	533.459	594.62	595.567
Upper1	51.741	57.917	96.019	94.376	160.446	158.526	399.473	399.588	622.658	623.599	688.92	689.300
Upper2	51.113	59.110	95.450	99.137	169.832	163.710	411.266	403.733	634.128	642.633	673.83	685.865
Upper3	40.764	64.745	89.142	81.743	179.919	180.084	506.664	507.158	641.107	641.020	664.32	665.543
Upper4	47.538	69.874	94.442	86.937	189.177	189.180	537.937	539.999	646.983	646,868	647.78	647.876

Table 4. The natural frequencies of the Hexapod machine tool set using finite element method

Configuration name	Joints Contact Type	Natural frequencies (hertz)					
		1st mode	2nd mode	3rd mode	4^{th} mode	5^{th} mode	6^{th} mode
Lower 1	Flexible Joints	78.3	116	161.3	265.1	333.3	494.5
	flexible top joints and rigid down joints	78.6	118.3	163	267.2	333.7	495.7
	Rigid joints	78.8	124	164.2	268.1	334.8	497.2
Lower 2	Flexible Joints	82.38	118.99	199.63	229.19	325.57	481.34
	flexible top joints and rigid down joints	84.12	119.3	200.21	231.06	327.84	483.54
	Rigid joints	85.32	119.56	201.43	232.65	327.99	485.89
Lower 3	Flexible Joints	101.94	109.23	242.56	338.77	370.99	466.73
	flexible top joints and rigid down joints	105.45	112.3	243.77	340.91	372.54	468.83
	Rigid joints	109.619	118.978	247.65	344.07	379.21	471.69
Lower 4	Flexible Joints	112.45	119.75	243.76	370.65	399.09	449.38
	flexible top joints and rigid down joints	115.52	120.98	244.77	372.7	401.98	452.70
	Rigid joints	115.94	122,32	247.32	376.99	406.21	459.62
Middle 1	Flexible Joints	73.86	106.36	185.76	326.76	533.82	612.55
	flexible top joints and rigid down joints	74.75	107.32	189.53	328.99	536.09	614.41
	Rigid joints	78.90	109.89	194.33	331.32	541.31	617.99
Middle 2	Flexible Joints	77.32	110.43	189.76	312.61	536.77	600.83
	flexible top joints and rigid down joints	78.10	111.67	192.37	318.19	538.55	605.35
	Rigid joints	78.90	113.99	195.94	322.88	540.99	616.69
Upper 1	Flexible Joints	57.63	101.22	158.04	389.98	624.94	685.55
	flexible top joints and rigid down joints	59.42	103.23	160.43	394.96	626.76	689.44
	Rigid joints	60.71	108.75	165.92	399.98	629.53	695.56
Upper 2	Flexible Joints	59.63	109.37	165.54	505.72	644.99	686.64
	flexible top joints and rigid down joints	60.21	103.89	166.49	406.92	647.36	689.52
	Rigid joints	62.43	103.73	169.39	412.28	653.35	693.33
Upper 3	Flexible Joints	64.93	82.56	182.44	502.42	645.24	670.33
	flexible top joints and rigid down joints	65.27	83.45	185.93	505.65	648.93	679.82
	Rigid joints	66.48	88.88	187.77	510.32	653.66	687.99
Upper 4	Flexible Joints	68.54	112.4	190.57	532.098	651.88	688.33
	flexible top joints and rigid down joints	71.23	115.54	198.78	540.98	663.78	694.42
	Rigid joints	74.12	124.12	203.89	549.75	670.66	699.67

6. ACKNOWLEDGMENT

The difference in stiffness values obtained using the finite element software and calculation method is related to ignorance of the complex aspects of the calculation in the computation of the stiffness and the absence of gravity that these dimensions are computed carefully and with the force of gravity on the finite element method is obtained. So the stiffness results obtained from the finite element method is more accurate than calculation method and the results were used in further comparisons. The natural frequencies are defined in the case of rigid joints, is more than the state where the joints are flexible in legs. In the cases that difference between the two configurations is related to the difference in center of the moving platform, there is a greater difference between the frequencies obtained. But when the difference between the two configurations, is related the angle of the moving platform to the coordinate axes, the difference in natural frequencies obtained are very low. For different configurations are defined for the Hexapod table, the more the table is away from the steady state, the lower natural frequencies range comes. Hexapod table's instability state, increases by increasing the leg's length in upper position of the moving platform configurations. Also displacing the moving platform, and rotating it relative to the when the rotation angle of the axis is zero to three axis of coordinate system, made the table instable. We can describe the Reduction of mass by increasing natural frequency values mobile platform and its accessories and reduce the stiffness of the series according to the following formula

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