Natural Convection in a Square Cavity Using Lattice Boltzmann Method for numbers Riley less than 10^6

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Abstract. Since the phenomenon of free convection in a closed container, have a lot of engineering applications.The heat transfer in solar collectors, Equipment designed for cooling, heat transfer in double-glazed windows and applications of this branch of engineering optimization problems.therefore for The analysis easier and more practical for such applications in this paper, the cavity is filled up with air and water would have to rely on Lattice Boltzmann Method simulation. Results for the basis fluid, air and water are provided. In this paper, the Pacific and the four Rayleigh number (Ra =10^3, 10^4,10^ 5, 10 ^ 6) is presented.Single-phase method was used to solve this fluid flow.The main objective of the present study is demonstrate the ability of Lattice Boltzmann Method for solving fluid flow in the long cavity.Lattice Boltzmann results with previous work has been validated.This results have a good agreement with the results of earlier studies.Boundary conditions in the shield to wall upper and lower is adiabatic,and wall of the left and right is temperature constant.

Keywords: Natural Convection Heat Transfer, Lattice Boltzmann Method, Cavity.

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1. INTRODUCTION

The phenomenon of free convection in a closed container, have a lot of engineering applications. Heat transfer in solar collectors, design cooling equipment, heat transfer in double-glazed windows and engineering optimization problems such applications branches**.**In the last decade, a new way to network as the Boltzmann methods in comparison with conventional methods in computational fluid dynamics, heat transfer, and fluid flow to the attention of many researchers is located**.**For example, studies by Refai and Yovanovich [1] on the effect of temperature on the heat transfer phenomenon of free convection in a square cavity filled with air that had been performed**.**Nelson [2] as well as in vitro studies on the free convection heat transfer in the cooling water tanks did. Oliveski [3] the flow field and heat transfer in free convection, the storage tanks numerically and experimentally investigated**.**Mohamad et al [4] using the Lattice Boltzmann method of heat transfer in an enclosure opening natural convection to have action. Applying the boundary condition is discussed in an open container**.**The average Nusselt number, model and the isothermal lines, the number of various Riley and has investigated**.**Pak and Cho [5] to the experimental study of mixed convection heat transfer of Nano fluidsoxide aluminum- water and oxide titanium-water paid. Results Nano fluids increases with increasing volume ratio of nanoparticles Nusselt number and Reynolds number indicated**.**Khanafer et al [6] were first to numerically simulate the flow of Nano fluids. They mix the natural convection of water and copper in a square cavity with a finite volume method were studied**.**Their results show that the heat transfer and fluid flow rate Nano fluids compared to pure flow to due increased heat conductivity and random movement of Nano-particles increases. Nemati et al [7] Lattice Boltzmann method adopted to forcedconvection flow in a square cavity with a fluid containing water-based Nano fluids resolve**.**Different Riley numbers and different volumetric ratio, have different effects on the flow of the research reviewed. They

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observed that increasing the Rayleigh number decreases the effect of nanoparticles on the base fluid**.**

1.1. The process of creating the Boltzmann equation

In 1940von Neumann with idea cellular machinery, which was leading to the emergence of the primer argues, was the primary model for gas network. A cellular system that is made up of smaller parts. This system will be established to determine the state of Lha cells. Each cell usually by several bits (for gas grid method), or by some probability of a particle (for Lattice Boltzmann Method) is determined. In these systems, a simple algorithm, locally recurrent and lead to the renewal of cells and Lha system.

Lattice Boltzmann method (LBM) of an older technique called automatic gas network (LGA) is taken. Gas Automatic network method, first by Hardy et al [8] was presented in 1976. They showed that this method is a complete discrete model for the fluid in a square grid; it can be used to simulate the flow equation. Qian [9] Chen et al [10] were able to use the linearized Boltzmann operator and the relaxation time (the first time in 1954by Bhatnagaret al [11] presented) lattice Boltzmann method known today to gain.

1.2. The equations of Boltzmann method

Bahatnagar et al [12] using the default parameter is defined as the absence of a system of balance and quiet time, Boltzmann collision function is introduced as follows:

$$C(f) = -\frac{1}{\tau}(f - f^{eq}) \tag{1}$$

In relation f^{eq} (1) local equilibrium distribution function and τ is the relaxation time. Maxwell-Boltzmann equation where the equilibrium distribution function is known, in general, a function of the macroscopic properties of the system, is defined as follows:

$$f^{eq} = \rho(2\pi\theta)^{-D}/_2 \, exp\left[\frac{(v-u)^2}{2\theta}\right] \tag{2}$$

In respect of the (2) $\theta = {k_B T}/{m} K_B$, D, ‹T‹ m و ؋Dimensionless temperature of the unit mass, then the system is the Boltzmann constant, temperature, mass and density is.

Boltzmann equation is approximated BGK (Bhatnagar, Gross and Krook) and in the absence of external forces is:

$$\frac{\partial f}{\partial t} + v.\nabla f = -\frac{1}{\tau}(f - f^{eq}) \tag{3}$$

Is a hyperbolic partial differential equation. The connection between the microscopic and macroscopic analysis of this equation is the fact that the flow.And Navier-Stokes equations can be gained from it.Analytical solution of this equation with the boundary conditions are applied to real problems and are not possible to be solved numerically.

1.3. Discretization of the Boltzmann equation

Equation (3) to the Boltzmann equation in the most general case shows without external forces. For the numerical solution, the Boltzmann equation must be discrete. The equation for a limited set of speeds VI corresponding distribution function fi (x, t), in the absence of external forces and the BGK approximation can be written as follows:

$$\frac{\partial f_i}{\partial t} + v_i.\nabla f_i = -\frac{1}{\tau}\left(f_i - f_i^{eq}\right) \quad i = 0,1,2,\dots,b \tag{4}$$

B to have a hyperbolic equation. B for possible speeds vi network shows. Two-dimensional networks of several discrete Boltzmann equations given that most of them are D2Q 9and D2Q 7 (Fig. 1)
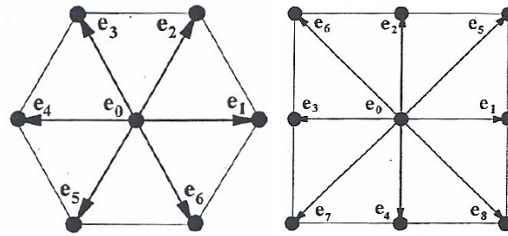

Figure 1. Metod D2Q 7(right channel) and the left D2Q9metod. [13]

After selecting the type of grid to the following equation (4) pay.

$$\frac{\partial f_i}{\partial t} + e_i.\nabla f_i = -\frac{1}{\tau\varepsilon}\left(f_i - f_i^{eq}\right) \tag{5}$$

In equation (5) for the later of the following reference values are used.

$$\hat{t} = t\frac{U}{H} \quad \hat{\nabla} = L\nabla \quad e_i = \frac{v_i}{U} \tag{6}$$

$$\bar{\tau} = \frac{\tau}{t_c} \quad e_i = \frac{\delta\hat{x}}{\delta\hat{t}} \quad \varepsilon = \frac{\delta\hat{x}}{\delta\hat{t}} \tag{7}$$

The final equation (5) as follows after discretization.

$$f_i(x + e_i\Delta t, t + \Delta t) - f_i(x,t) = -\frac{1}{\tau_v}\left(f_i - f_i^{eq}\right) \tag{8}$$

In relation to (8), the equilibrium distribution function to a simpler expression. A simplified form of the equilibrium distribution function can be obtained as follows.[13]

$$f_i^{eq} = \rho w_i\left[1 + \frac{(e_i.u)}{c_s^2} + \frac{1}{2}\frac{(e_i.u)^2}{c_s^2} - \frac{1}{2}\frac{u^2}{c_s^2}\right] \quad i = 0,1,\dots,8 \tag{9}$$

In the above equation u = u (u, v) the macroscopic velocity of the fluid, cc2=1/3 ‹e_iI is the particle velocity and path coefficient WI discrete distribution function and balance for the values of (2 19) we have:

$$W_i = \begin{cases} \frac{4}{9} & i = 0 \\ \frac{1}{9} & i = 1,2,3,4 \\ \frac{1}{36} & i = 5,6,7,8 \end{cases} \tag{10}$$

That is to say, the current practice is that each particle after the collision to the nearest neighbor to move your character, such as those in Figure 2 is shown. The motion of the boundary issue as basic premise is transmitted to all nodes examined and gain for all parts of the velocity distribution function.
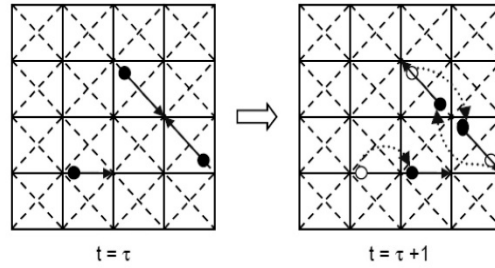
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Figure 2. Shows how the lattice Boltzmann flow distribution [13]

Finally, in a D2Q9lattice Boltzmann method, network, macroscopic properties such as local density ρ and the velocity u can be defined as follows

$$\rho = \sum_{i=0}^{8} m_i f_i \tag{11}$$

$$u = \frac{1}{\rho} \sum_{i=0x}^{8} m_i f_i e_i \tag{12}$$

After solving the velocity distribution function (f) and obtain the macroscopic properties of density and speed, you need to obtain the temperature distribution, the probability distribution function (g) represents the probability of thermal energy particle at position (x, v) in the phase space at time t is. All existing relationships for the velocity distribution function, except for a few cases; the temperature distribution function is established.

Normally, a fluid particle in the infinite heat transfer is permitted. Discrete Boltzmann equation for heat transfer equation is as follows:

$$g_i(x + e_i \Delta t, t + \Delta t) - g_i(x, t) = -\frac{1}{\tau_c}\left(g_i - g_i^{eq}\right) \tag{13}$$

To obtain a function of temperature such that the behavior we go. After the collision, the nearest neighbor of each particle to travel to your specifications.And the values obtained for all parts.Finally, in a network, D2Q 9lattice Boltzmann method, the macroscopic properties of the temperature T is defined as follows:

$$T = \sum_{i=0}^{8} g_i \tag{14}$$

It can be shown that the selection function is introduced to balance the equation (9). Lattice Boltzmann Method for conservative mass and momentum equations (Navier-Stokes) with Assuming incompressible fluid viscosity and density of the fluid and the pressure will come in the form below.

$$v = \frac{2\tau - 1}{6} \tag{15}$$

$$P = \frac{1}{3}\rho \tag{16}$$

1.4. Solving the natural movement using Lattice Boltzmann Method

The numerical simulations and assumptions used are as follows.The process is quite static and in the uniform temperature of the average temperature of the hot and cold wall is located.Then Viscosity and incompressible flows, using the assumption Boussinesq inside a closed box next two values of the initial conditions of the problem starts. Four non-slip wall socket and is assumed to be rigid.Thus, using assumption to resolve the address mentioned.

Geometry is studied in this research, rectangular cavity where the heating occurs through the side walls. Handling natural cavity orthorhombic by rotating it recognizable as if the walls of the left wall warmer, the rotational motion of the fluid in the direction clockwise will be (Figure 3) in the diet slowly, three characteristic flow structure inside the cavity the next are the following:

Vertical boundary layer along the wall of the left and right:

1) Horizontal boundary layer along the top and bottom walls

2) The stable and relatively stationary center cavity
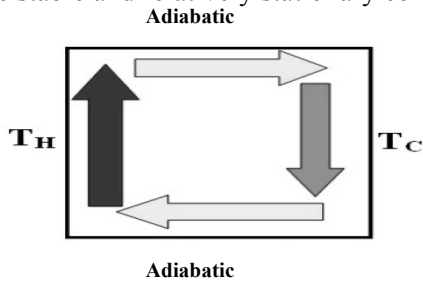
Figure 3. Is a view of the shape and motion of the fluid flow in the event of natural movement

Lattice Boltzmann method of boundary conditions directly affect the accuracy of the final result effect leaves [14]. First look at the network used to throw the unknown parameters to be obtained by the boundary conditions, are specified Here is described only D2Q 9network (other networks and three-dimensional case is also similar to this case).The problem for the distribution function of the boundary condition on the nodes is used ricochet (bounceback). In this boundary condition,after the particle flux at the wall moves to hitting a solid wall, in the opposite direction of his early return to the area occupied by the fluid.This boundary condition, the precision is of the first order (while you carefully time the Lattice Boltzmann Method) to correct this inaccuracy, various methods have been proposed so far.Of all these methods, the way in which Bounce over the middle link (Bounce Back on Links) called, simple, yet accurate time is two to flat walls.The boundary condition of a simple boundary condition at the same time it is widely used in complex problems such as porous surfaces. In this way the nodes is sufficient to consider a solid boundary and to calculate the unknown nodes on the boundary conditions used.For example, the southern boundary is shown in Figure 4for the distribution function can be written as follows:

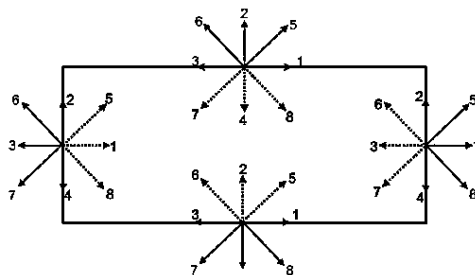$$f_2 = f_4 , \qquad f_6 = f_8 \qquad , f_5 = f_7 \tag{17}$$

Figure 4. Boundary conditions for the network D2Q9

1.5. Temperature distribution function equation and boundary conditions

For the northern and southern boundaries of the cavity that is adiabatic condition on the links, we will ricochet. For example, the northern border is:

$$g_4 = g_2 , g_8 = g_6 , g_7 = g_5 \tag{18}$$

For vertical walls where the temperature is constant from under the conditions used. For example, to walls with higher temperature (on the left wall of the cavity) we have:

$$g_{5,1,j} = T_{west}\big(w(7) + w(5)\big) - g_{7,1,j} \tag{19}$$

$$g_{1,1,j} = T_{west}\big(w(3) + w(1)\big) - g_{3,1,j} \tag{20}$$

$$g_{8,1,j} = T_{west}\big(w(6) + w(8)\big) - g_{6,1,j} \tag{21}$$

In this section the results of the natural movement of the fluid flow in a rectangular cavity with lattice Boltzmann method is presented. In this study, the laminar flow of fluid to the aspect ratio (length to width ratio) constant A = L / H = 1is investigated.In this study, we examined the weather is fluid. It should be noted that thermo physical properties of these particles is fixed and cannot be changed with different variations.In Lattice Boltzmann method to value the viscosity guess it depends on the amount of light that certain relationships and the amount networks and Mach number is obtained. These relationships can be expressed in the following equation[15] .

$$\upsilon = \sqrt{\frac{Ma^2 M^2 Prc^2}{Ra}} \tag{22}$$

The relation (4 4) Ma represents Mach number, M number of network used, and c represents the speed of the particle is equal to the constant c = 1/ √ 3is. It should be noted that the Mach number must be smaller than 0.3, which does satisfy the condition of incompressibility.

2. NUMERICAL SOLUTION AND VALIDATION

In this section we compare the results obtained in this study in order to validate the work program was written to be explored.For this purpose, the program is able to solve the problem with the air flowing through the Markatos et al[16] have been validated. This comparison is presented in Table1.

Table 1. Compares the results obtained in this research work Markatos

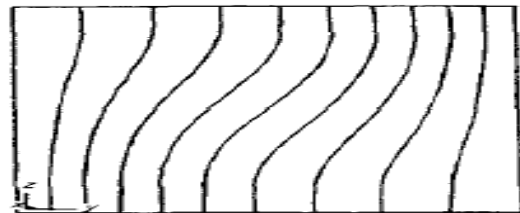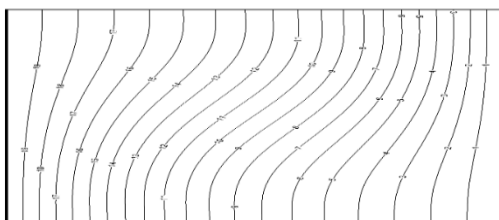| Rayleigh number	Network size	Average Nusselt Markatos review	Average Nusselt present paper
10^3	100*100	1.118	1.128
10^4	100*100	2.243	2.245
10^5	100*100	4.519	4.470
10^6	100*100	8.799	8.898

Figure 5. Compares the temperature profile in the intermediate hole section, the results Markatos et al in pr=0.71and Ra=3*10^3
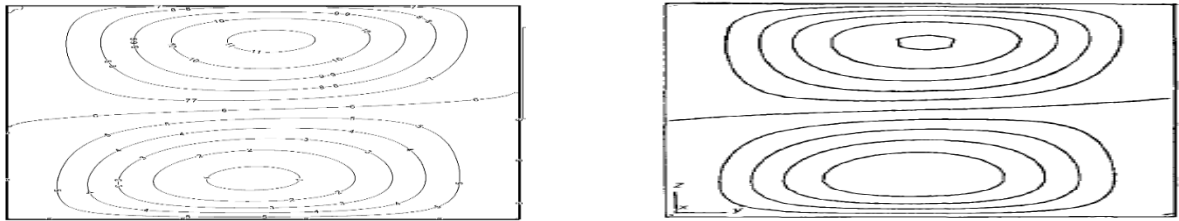

Figure 6. Compares the results of the horizontal velocity and colleagues Markatos in pr=0.71andRa=3*10^5
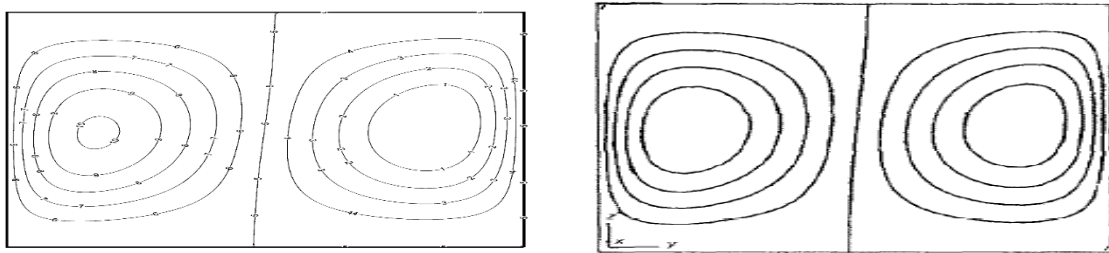

Figure 7. Shows the vertical velocity profile and colleagues Markatos results in pr=0.71 andRa=10^6

Given the above, namely;Cut a hole in the middle of the temperature profile in Figure 5, the horizontal velocity profiles in Figure 6and Figure 7, the vertical velocity profile is specified Which is very similar profiles Markatos et al, which you can verify the code written in the article concluded. Now the verification code in this was written, the more we studied the flow of water. In Figure 8, the temperature profile in the intermediate hole section of the water flow is plotted for different numbers Riley, who initially Ra = 103and along the temperature gradient, heat transfer along the hot wall is more.With increasing Rayleigh number, in Figure 8-B, since the flow has been rotation, the heat transfer convection is dominant in Figure 8(c), 8(d), the maximum temperature gradient in the hot and cold walls has moved the show most of the heat transfer in this range Temperature changes have little hole in the middle line, which represents the effect of heat transfer convection is smaller. Since the temperature profile shown in Figure 8 (d), is symmetric, it can show the effect of thermal conductivity is dominant.
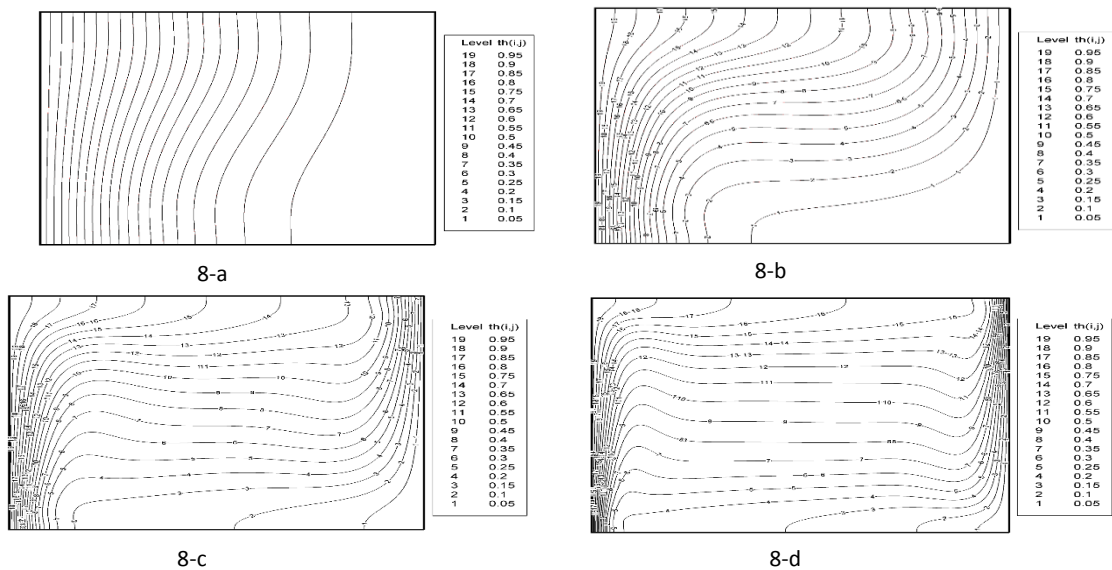

8-a

8-b

8-c

8-d

Figure 8. Shows the temperature profile in the middle cut holes for fluid flow for pr = 6and Rayleigh number (Ra); b) Ra = 10^3, b) Ra = 10^4, c) Ra = 10^5, d) Ra =10^6.
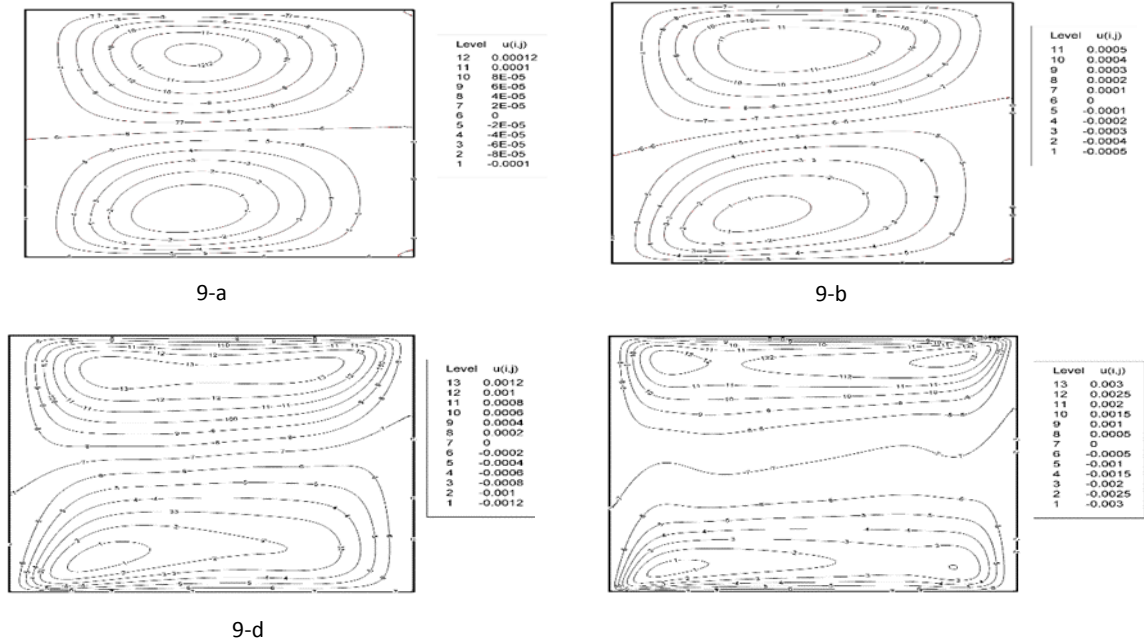
9-a

9-b

9-d

Figure 9. Shows the horizontal velocity profile of the fluid flow for pr =6 and Rayleigh number (Ra) for: a) Ra =10^3, b) Ra = 10^4, c) Ra =10^5, d) Ra = 10^6.

Figure 9, the horizontal velocity for different numbers indicate Riley, who has been Normalizing Contour until Rayleigh$10^6$and have been close to the hot wall. But for the more this is no longer true. The subject of much articles in this field has also been confirmed, for example, an article Mr. Markatos and colleagues[16] .
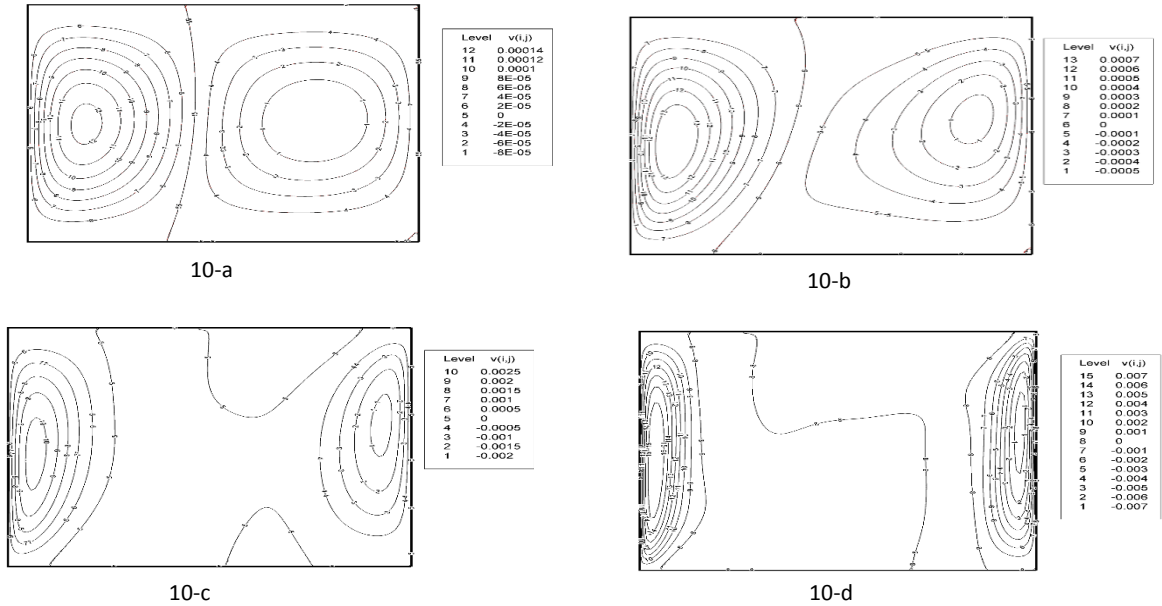

10-a

10-b

10-c

10-d

Figure 10. Shows the vertical velocity profile of the fluid flow for pr =6and Rayleigh number (Ra); b) Ra =10^3, b) Ra=10^4, b) Ra = 10^5, d) Ra =10^6.

Contours of vertical velocity for different Rayleigh numbers were presented in Figure 10. As for the horizontal contours are normalized to 106Rayleigh number. But not for larger Riley[16] .

As is known, the maximum vertical velocity with increasing Rayleigh number is close to the hot wall, and finally moves towards the center of hot wall.

3. DISCUSSION AND CONCLUSIONS

In this article the natural movement of the fluid in the high dimensional holes in the laminar flow regime for A = 1 aspect ratios,Fully for air and water base fluid was studied in the range of numbers Riley$10^3 \leq Ra \leq 10^6$.The equations were used by Lattice Boltzmann Method. After the validation Article ID, different contours for temperature, speed, vertical and horizontal velocity was presented. In general, the greatest effect on the heat transfer fluid in Rayleigh$Ra = 10^4$Obtained, That the results of the study can be used to further industrial design or at least check out the design, speed and accuracy of this method can be used.

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