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A Comparative Review on Operational Modal Analysis Methods

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Abstract. In the present study we investigate development of operational modal analysis (OMA), different OMA techniques as well as relevant issues and experimental studies. Furthermore, we classify previously performed studies have been conducted so far and consider the most important one. So, first we review the fundamental concepts. Then, the history and application of the OMA method will be examined comprehensively to observe their role in the development of the technique. Three basic methods of FDD, SSI, and Next will be discussed and the research works associated to them will be analyzed.

Keywords: OMA Methods, Review, EMA, NExT, SSI, FDD

1. INTRODUCTION

Materials analysis plays an outstanding role in numerous industries and applications. The materials analysis actually is applicable in designing, optimization, health monitoring, vibration monitoring and damage detection of structures, bridges, and buildings and so forth. Currently, along with development of vibration measurement systems and analysis methods, operational modal analysis (OMA) has been replaced with experimental modal analysis (EMA) in fairly all functional programs. The major difference between OMA and EMA exists in the source of sources. Unlike EMA, in which the system is tested under known forces and input forces are then measured, in OMA the system is tested in real operation condition of experimental environment. Consequently, the test proceeds without measuring input forces. That is why this method is also called natural excitation modal analysis, modal ambient analysis or output only modal analysis. Using OMA technique has been initiated since 1990s when the method gained widespread popularity in civil engineering, mechanical engineering, and aerospace engineering. The OMA advantages are as follows:

The ambient test is more economic compared with experimental modal test and does not require boundary condition simulation. Yet, in OMA dynamic characteristics of system have been obtained for whole system rather than some part of it. Due to using real random forces imposed over different places on the structure, one linear model has been formed around operational conditions instead of experimental conditions. Since in this method analysis is basically a type of MIMO analysis, close modes are easily recognized. Therefore, OMA is a good method for complex and difficult structures. Because of its online application, OMA could be applied for vibration monitoring as well as for damage detection and health monitoring of structures. Accordingly, in the present study we make attempt to categorize different OMA methods besides providing a brief history of natural excitation technique (NExT), stochastic subspace identification (SS1), and auto regression moving average (ARMA) among two research groups.

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2. LITERATURE REVIEW

In this section, a brief history of different OMA techniques will be discussed.

2.2 Natural Excitation Technique (NExT):

The NExT was first introduced by [1]. In this method correlation function (COR) obtains from random response of a structure due to ambient excitation can be written down as a set of Reduction sinuses. Each reduction sinuses has a damped natural frequency, damped ratio and mode shape factor. Thus, in time domain EMA techniques, instead of impose response function (IRF) in MIMO systems; we can use COR in order to calculate the parameters of system modal.

Actually, by this work we provide a proper background for development and employing the EMA techniques for OMA. The OMA methods consist of two main stages in terms of NExT. The first step includes calculation of a time response function (TRF) while the second step comprises identification of the modal parameters using one of common TD techniques. In order to compute TRF in OMA, there are two proposed methods as: a) employing COR and b) adopting TDs obtained by random decrement technique. The RD method which is a time averaging method in random time response section was first proposed by [2]. However, the method was first used in modal analysis by [3]. He claimed that the result of RD is free system vibration. Subsequently, [6-7] indicated that [4] claims were incorrect and proved that RD is a product of COR. Adopting RD method, we can calculate COR through random response data of structure and use it to identify the modal parameters of OMA in terms of the NExT technique. This approach therefore leads to onset of a new period for a large number of new investigators to propose their own techniques and to expand OMA knowledge [8]. The emerging scholars have utilized RD method for calculation of cross-correlation function (CCOR) and auto correlation (ACOR). Next, they examined three different methods for identification of modal parameters. Another investigator [9] also made a comparison between speed and accuracy of RD with FFT. According to the comparison, RD was one hundred time faster than FFT. Moreover, examining ACOR, RD showed more accuracy than FFT. However, the FFT methods were more precise than RD in calculation of CCOR. In the modal analysis methods based on RD, if we use either ACOR or CCOR, a higher level of noise in CCOR would result in modal parameters errors. But if we employ ACOR only, the phase data will dismissed and the mode shape will be unidentifiable. In (VRD) the conditions for selection of time start points are defined by a vector. This technique therefore makes contribution to maintenance of data. In the same year, [16] investigated VRD method via simulation of four degrees of freedom of the system as well as conducting an experimental test of a bridge model. In other study [9] worked on FRF estimation through RD and FFT. He concluded that FRF demonstrated less noise and leakage compared with RD in addition to a faster performance.

A new method was introduced by [10] for calculation of variance, accuracy and appropriate time duration for RD in modal analysis. Moreover, [20-22] discussed about how to use RD for OMA besides specification of proper parameters for RD. Shen et al (2002) subsequent to investigating scientific development of NExT, suggested a method for identification of the modal parameters in frequency domain using CCOR as well as typical analysis methods of TD. They employed cross power spectral density function (CPSD) in lieu to FRF in frequency domain poly-reference (FDPR) and highlighted the ability of using CCOR in FDPR through conducting experimental tests on an aircraft model. Expanding RD, using this method will also develop into other modal analysis methods in frequency domain. [26] presented a method for computation of Furrier function of RD instead of time function. Employing this technique, the level of noise decreases due to averaging and leakage reduces because of using sufficient time length in RD, as well. They further assessed their proposed ideas by testing them on an experimental model. Ultimately, [27] reviewed different research articles, compared the methods in fields of frequency domain and time domain. He fully clarified using RD in OMA.

2.2 Stochastic subspace identification method (SSI)

One method for identification of time domain modal in developed OMAs is stochastic subspace based method. In 1990s, in control and systems engineering a new method emerged on the basis of stochastic subspace for identification of spatial status of system, which directly used measured data. Accordingly, in 1993 stochastic subspace identification method (SSI) was provided utilizing calculated random response of system [23, 25]. Peeters et al (1995) after describing SSI provide explanations for the association of the vibration model and random model of the system. So, this relationship became a reference for modal analysis of the structure under ambient forces. Similarly, [11] used the method for testing a plate with a fixed electronic model installed on it, and then they compared the results with the results obtained from FDD method in frequency domain.

Subsequently, because of mathematical complexities,[19] endeavored to provide simpler explanations for SSI. They asserted that a majority of the method stages are similar to other methods in time domain. Thus, [24] suggested a new procedure for processing signals from non-stationary signals named also as empirical model decomposition (EMD). According to this method, a new EMD based SSI method appeared. In this technique the investigators first transfer calculated data to modal response function via EMD, then they apply SSI for extraction of modal parameters. Completing studies conducted by [24] , [28] presented some explanation about how to use this method on the basis of stochastic subspace for extraction of time series of modal parameters. He claimed that this method lacks the restrictions of former method, in which the modal properties were limited to the number of sensors.

2.3 FDD method

FDD is an OMA method in frequency domain. The OMA methods in frequency domains are based on a simple relationship between input and output power spectral density of a random process [29]. The simplest method in frequency domain is peak picking method; in while natural frequency is calculated directly y picking peaks in PSD diagram. If modes are separated appropriately, these methods lead to appropriate estimations [30-31]. The big advantage of this method versus the TD methods is lack of any computational mode as well as simplicity and velocity of this method. However, PSD PP has certain problems. They include lower accuracy of this method particularly for complex structures which is because of dependency of PSD spectrum resolution, extraction of operational deflection shape instead of the system natural mode, lower accuracy in computation of damping ratio, and failure to utilize it for systems with closed modes. By providing a new idea in FD containing all previously mentioned advantages and resolving differences, [12] provided frequency domain decomposition technique. In this method analysis of singular value decomposition from output PSD in different frequencies was used as mode indicator function. In addition to identification of different types of modes especially closed modes, signal and noise spaces are separated. Using the FDD technique, natural frequencies and mode shapes could be computed. But, to calculate damping ratios a new method named as enhanced frequency domain decomposition was proposed by [13]. In this method singular values in adjacent to natural frequencies are transferred to TD through inverse FFT and the damping ratio is calculated using logarithmic reduction techniques. Since in EFDD only one separated segment is transferred to TD, probable bias errors in damping ratios would emerge especially in closed mode. In order to overcome this problem, [31-39] introduced FSDD method. In this technique, the scholars used a PSD was enhanced by a singular vector extracted natural frequency and damping ratio by a degree of freedom curving fitting. Due to simplicity and efficiency of FDD method in OMA, [14-18, 50] proposed auto instructions for identification of modal based on FDD. This instruction can be operated in commercial modal analysis and consequently users' interference in end results will be diminished [13]. Regarding to the relationship between complex model indicator function and FDD, enhanced mode indicator function can be introduced as an alternative to EFDD.

2.4 Challenges facing OMA

One of the OMA methods shortcoming versus the EMA methods is presence of un-normalized mode shape in analysis results its reason is ambiguous excitation forces. In this regard, [41] introduced a normalization method of modes shape via changing the structure volume and duplication of tests. In this technique is also known as normalization of mode shape based on sensivity, modal parameters resulted from OMA are used without using finite element models. Also, [42] utilized this method and normalized mode shapes of a moving sprayer arm and a bridge through the OMA results, respectively. In Parloo method, to obtain normalization factor, mass distribution must remain unchanged. Therefore, in this method mass changes is typically insignificant. [43] introcued a recursive method based on non-linear optimization, in which there exists no limitation in mass changes and so it gives more precise results. also, [44] utilized motion control equations and provided a formula for estimation of normalization index on the basis of frequency changes caused by mass changes they also examined the source of errors and concluded that if mass changes take place so that most of changes occur in frequency and mode shapes encounters with fewer changes, then both random errors and errors of approximation will be in the minimum. [45] employed this method to obtain normalized mode shapes used ¼ of a building model and tested errors via different mass changes. [46] presented two methods for mass changes.

In the first method, the test was duplicated with insignificant mass changes. In the second method yet mass changes were more significant but the test repeated only once [48]. Subsequently, they assessed accuracy of both method either through experimental test or numerical simulation. In the same yea, [48] explained how to use this method for updating the finite element model. recently, [49] have suggested a good strategy for mass changes including the number, amount and status of added mass using natural frequencies and system modes shape to archive the best possible results, they then applied this strategy on a beam in ambient.

3. METHOD

The current research aims to investigate system identification performance (Sys ID). In computed dynamic response of the building (case #1) and dynamic response simulated using a non-linear FE model (case #2) were utilized. The latter response was developed using a 3D non-linear finite element model in structural analysis software OpenSees. In addition to ANOVA, meta-models were also employed for evaluating the impact of effect screening in case #2. The meta-models consist of linear interaction of input factors. First three longitudinal mode shapes along with their corresponding natural frequencies and damping ratios are shown in Tale 1. These modes and natural frequencies experimentally identified in terms of tangent stiffness matrix (after using gravity loads) and ambient vibration data not damaged by experimental structure show a good agreement with each other.

Table 1. Different tested modes.

Mode	$F_{H.z}$	$\zeta_{\%}$
M1	2.25	2.89
M2	10.1	2.48
M3	27.54	5.7

We used a full factorial design of the test in this study leading to a set of identification runs 3×3 10.800 = 100× 2× 3×2×3×3 و 54 = 2× 3× (a set of 100 identification runs per each combination of input factors, Excitation amplitude، Spatial density of sensors ،noise level ،length of measured data ،Model order). For the first and second cases and for every two parametric identification methods SSI and NExT-ERA were used, respectively. For non-parametric method EFDD , the model order is not an input factor. thus, identification runs 100 × 2 × 3 × 3 × 3 و 27 = 3 × 3× 3 5.400 = are conducted for case one and two, respectively.

4. FINDINGS

Three Sys ID methods, a) output only , no data is used for input excitation, were employed for identification of parameters of experimental structure modal in terms of its computed data and/or simulated response data. These methods include NExT-ERA, SSI-Data and EFDD. The measured acceleration responses in Hz 240 were simulated, while simulated acceleration response of FE model were destroyed in Hz 120. In both cases, Nyquist frequencies (HZ 60 or Hz 120) are significantly higher than favorite modal frequencies in the present study (below Hz 25). before using the mentioned Sys ID for measured and simulated data, all time histories of absolute acceleration response were filtered through a finite impulse response filter by passing through 0.5-25 Hz in case #1 and 0.5-30 Hz in case #2 bands of high order 1024. In addition, measuring the absolute horizontal acceleration of the white noise stimulation tests, by reducing the basic /input acceleration was transformed into the relative acceleration. We should mention here that these three methods assume that input excitation of a broadband signal (in ideal condition of white noise). Therefore, rejecting this assumption leads to further errors in estimation of modal parameter.

4.1 The first study

As it was mentioned above, a full factorial design of the test was considered in the present study. consequently, Sys ID 54= 2 × 3 × 3 × 3 was performed adopting both ERA-NExT and SSI-Data methods and 27 = 3 × 3 × 3 in EFDD method based on measured test data for case #1. Fig.1 illustrates development of identified modal parameters (natural frequency, damping ration and MAC values among each identified mode shape and corresponding mode shape which were identified via the following value of input factors. the values are : A- Ambient, S- like identified mode, O= 12, L=180 seconds, for the first longitudinal modes and all three identification methods.

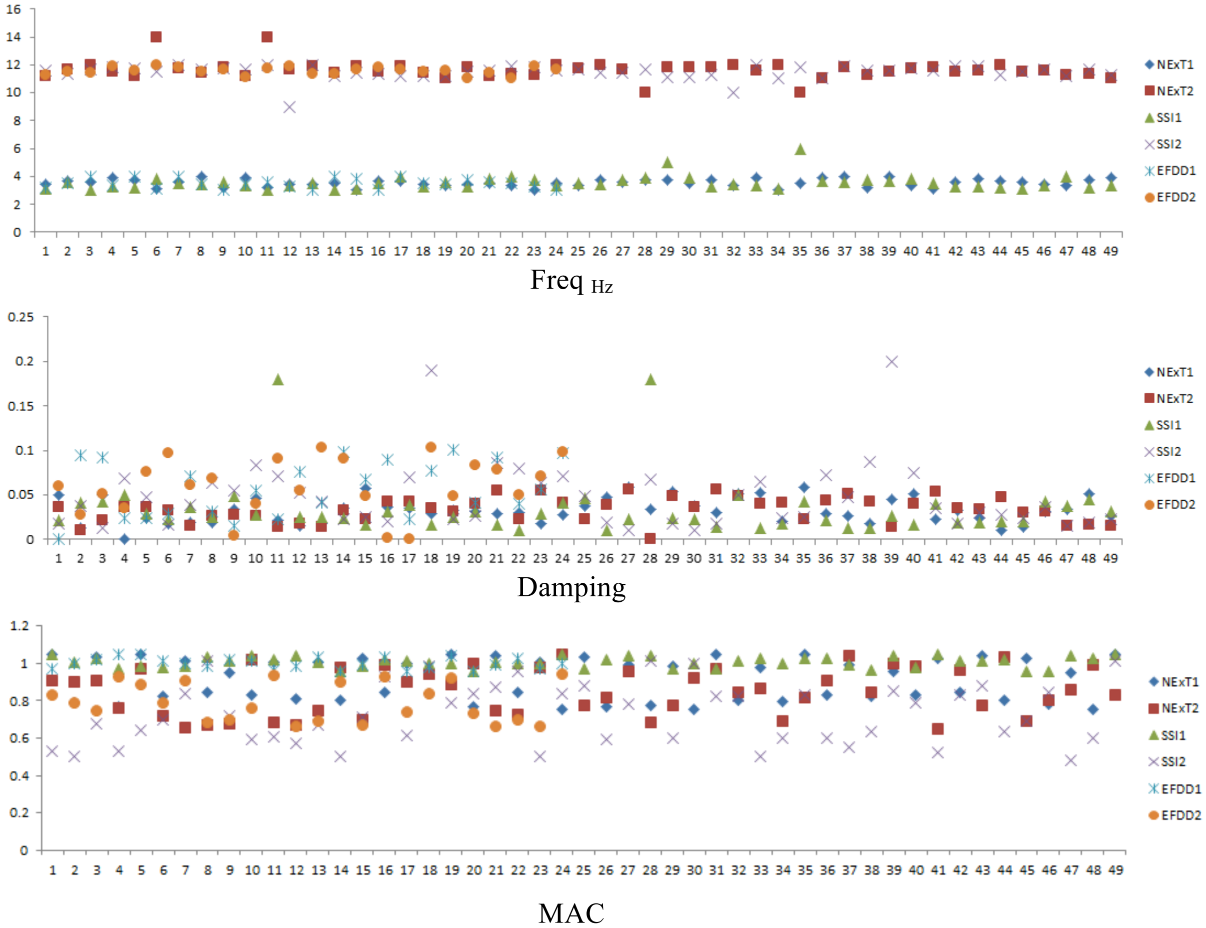
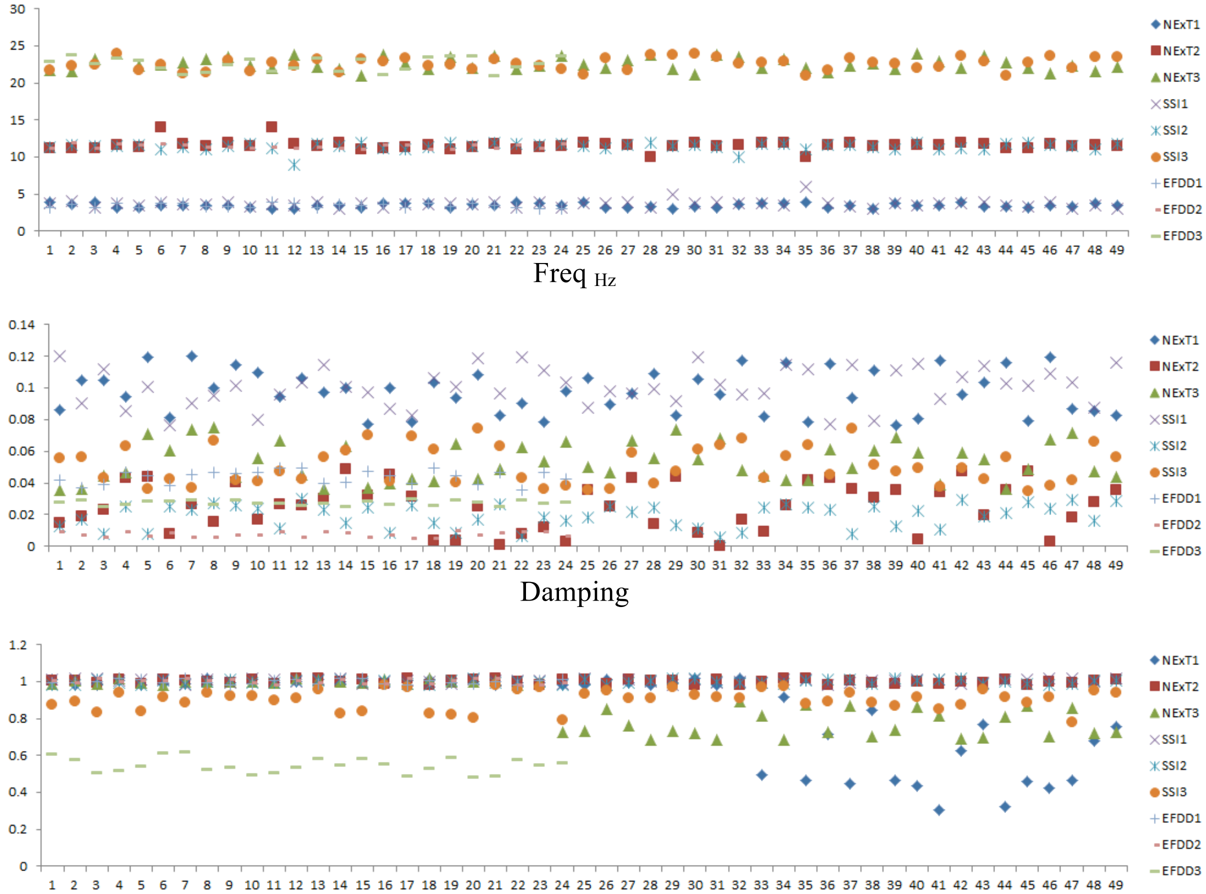


Figure 1. Case 1.

4.2 The second study

Development of modal parameters which are identified based on simulated data in case #2, are the resulted by five varying input factors like excitation amplitude, spatial density of sensors, noise level, length of measured data and model order. So, it lead to 108= 2 × 3× 2 × 3 × combinations for non-parametric method EFDD. Fig. 2 illustrates expansion of statistical mean scores (per each set of 100 identification) of identified modal parameters(natural frequency, damping ration, and MAC values among identified modes and their nominal corresponding computed via FE model) used for first three longitudinal model and each of three identification methods. Therefore, each point on Fig.2 shows mean scores of over 100 identification tests assuming that measurement noise is an independent vector. Minimizing a set of 100 modal identification for each input factors to statistical mean scores is a crude variance reduction technique leading to a decrease in diversity of identified modal parameters. The reason could be because of selection of vector measurement noise process (meaning number of realization noise vector).



MAC

Figure 2. Case 2.

5. CONCLUSION

In the present study we planned to achieve a broad range of different OMA methods and clarify problems associated to them. We examined five different categories of OMA methods including NExT, SSI, FDD, ARMA and random test. Next, three important issues of OMA such as normalization of mode shape, load estimation, and analysis in the presence of harmonic forces were evaluated. After, we discussed about studies on OMA methods based on wavelet transform, using mode indexes in OMA, order and accuracy estimation, methods based on cepstrum, and so forth. Furthermore, we tested one building piece of seven RC shear wall with full index over vibration tale NEES. Three Sys ID methods of output-only were utilized for extraction of modal parameters, i.e. natural frequency, damping ratio, and modes shape taken from the building sample. These methods include: a) natural excitation technique integrated with eigensystem realization algorithm (NExT-ERA), b) data-driven stochastic subspace identification (SSI-Data), and c) enhanced frequency domain decomposition (EFDD). In the current research, an analysis of variability or uncertainty of these is identification methods of system were done in two stages.

The first stage is when these methods are employed on measured response of the structure and the second stage is when these methods are applied on response of the structure simulated using a 3D non-linear finite element model which is calibrated and confirmed. the input factors we considered in the first stage are 1) amplitude of input excitation, A) a level of non-linearity of response, 2) spatial density, S)number of sensors, 3) the length of the structural response data used in the process of data identification (L) and 4) model order utilized in identification methods of parametric system (O).

In the second stage of uncertainty analysis, in addition to four input factors in the first stage such as measurement noise, (N) is also included. Using ANOVA test for system identification results based on experimental measured data I the first stage, we observed that input factor A has the greatest impact on changeability of modal parameters which were identified through using these three methods (especially the natural frequency of the first mode). In the second stage of uncertainty analysis, ANOVA test was used on standard deviation and mean scores (for a set of 100 identification runs for random description of measurement noise)of modal parameters which were identified through finite element simulated data . We understood that variability and mean scores of identified modal parameters (especially natural frequencies) show the greatest sensivitiy towards input factor A for all methods which shows a good agreement with the results of the first stage. Input factors of S and N showed the least possible impact on mean scores of modal parameters which are identified through Next-ERA and EFDD. Although the level of measurement noise (N) greatly contribute (compared with input factors) to variability of standard deviations in identified modal parameters, but it does not work for mean scores of identified modal parameters. Also, meta-models are in good agreement with identified modal parameters in the second stage. According to relative amplitude of coefficients ß (regression) of meta-models, we found out that identified natural frequencies show the highest sensivitiy towards input factor A (as the variance analysis results indicated). Next, input factor L and linear interactions AL revealed the highest sensivitiy. Moreover, we observed that generally modal damping ratios and MAC values showed greater sensivitiy towards input factors S, N, and O against natural frequencies.

Relative amplitudes of coefficients ß relative amplitude of coefficients demonstrate that specified input factors have more significant impact on variability of identified modal parameters of the first mode compared with the parameters of higher order modes. Thus, we conclude that level of accuracy/certainty in system identification results not only depends upon estimation error of used identification methods as well as measurement noise, but it also is a variable of test plans (e.g. excitation domain, sensors spatial density, response length of measurement data, and model order). Consequently, dynamic tests must be designed so that the most influential input factors place in optimum or appropriate levels in order to have more precise and meaningful system identification results.

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