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Simulation of Groundwater Flow in Porous Media Using the Finite Element Method (Case Study: Kabudarahang Aquifer)

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Abstract. Uncertainty in groundwater flow models is mainly due to the errors in the simulation of model values. In order to generate a mathematical model for groundwater flow, first, the input information in PMWIN software code was gathered and preceded. The obtained model was calibrated in two regimes under permanent and non-permanent flows. After conducting all these processes, the groundwater flow model of the Kabudarahang city were developed. In the calibrated model, parameters such as bedrock, K, S_y, and T were corrected. Lowest and highest corrections were made on the amount of hydraulic conductivity and the bedrock, respectively.

Keywords: Mathematical Model; Finite Elements; Simulation; PMWIN

1. INTRODUCTION

The Kabudarahang plain is one of the study areas of Qara-Chai River basin with an area of 8443 square kilometers located in the north of Hamedan province. This area is connected with Zanjan and Qazvin Plains from the north, Bahar and Qahavand Plains from the south, Razan from the east, and the study area of Gultapeh-Zarinabad from the west. The area of the aquifer is 632 square kilometers and is located between 282000 and 306000 eastern longitudes and 3880000 and 3888000 northern latitudes (figure 1). For each of the time steps, the partial differential equation in saturated porous media is written and solved using the Finite Element Method. The matrix $(n\times n)$ solution method in PMWIN software has been operated in two repeated ways, through which the level of groundwater of each of the grids in the model and at any time period is calculated. In this way, three different loops are executed in each simulation. A loop for pumping periods, the second loop for time steps, and the final loop for iterative method calculations (anonymous, 2004). In fact, the mathematical model solves the mathematical form of balance equation in an area, and generally, the mathematical model is built considering the continuity of the environment (continuum approach). These equations are written for an area of the plain and then are generalized to adjacent areas. Under these conditions, the Bilan's equation turns into a partial differential equation. Every sentence in this equation represents specific amount of a parameter per unit area, volume, or time which includes a wide range of values. Using these values and through mathematical solutions, the unknown points for other time periods are obtained which has been reported BT Anderman et al. (2001). In short, a mathematical model which is used to simulate the groundwater flow is a set of numerical values of various parameters in the balance equation (Khalghi, 2003).

1.1. Basic Equations of Groundwater Systems

Barlebo et al. (2004), reported that in order to be able to form basic equations in an aquifer system, first, an element of Representative Elementary Volume (REV), with dimensions of Δx, Δy (horizontal), and Δz (vertical) is considered. Then, the continuity equations are written and

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integrated with the momentum equations. After integrating the abovementioned equations, and taking into account the principle of conservation of matter, save changes are equal to input minus output and, accordingly, the basic groundwater flow equation is obtained. Barlebo et al. (2001) stated in their study that, in the steady state and considering the save changes at the time of zero, the equation is written as follows (Anderman et al., 2003):

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(T_{zz}\frac{\partial h}{\partial z}\right)=0$$

Where h is the hydraulic potential of the aquifer (L), and x, y, and z are directions (L), $T = T_{ij}$ is the tensor transfer capabilities coefficient of the aquifer (L^2T^{-1}). In a homogeneous and isotropic and assuming $T_{xx} = T_{yy} = T_{zz}$, the above equation becomes as follows:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0$$

Obviously, to solve the above equation, the situation in the system borders should be considered. The solution to groundwater equation and unstable state (transient) is as follows:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=\frac{S_y}{T}\frac{\partial h}{\partial t}\pm\frac{R}{T}$$

Where:

S_y: Save coefficient (without dimensions),

T: Capability coefficient,

$\left(\frac{L^2}{T}\right)$: Aquifer transport,

R: Aquifer supply or discharge $\left(\frac{L}{T}\right)$, and

t: Time (T).

If we consider the approximation of water level changes over time, with progressive difference scheme, as follows:

$$\frac{\partial h}{\partial t}=\frac{h_{i,j}^{k+1}-h_{i,j}^{k}}{\Delta t}$$

Mehl et al. (2001) reported that

The groundwater equation in the unstable state of K is as follows:

$$\frac{h_{i-1,j}^{k+1}-2h_{i,j}^{k+1}+h_{i+1,j}^{k+1}}{(\Delta x)^2}+\frac{h_{i-1,j}^{k+1}-2h_{i,j}^{k+1}+h_{i+1,j}^{k+1}}{(\Delta y)^2}=\frac{s}{T}\frac{h_{i,j}^{k+1}-h_{i,j}^{k}}{\Delta t}\frac{R}{T\,\Delta x\,\Delta y}$$

1.2. Information Processing, the Conceptual Model, and Importing the Data into the Model

The purpose of aquifer modeling is to simulate the natural aquifer using a series of mathematical relationships and achieving results for the management of the aquifer. If we can successfully simulate an aquifer and compare it with natural situation, we can easily investigate the effects of exploitation from the aquifer by changing the location, amount, and time of withdrawal. A groundwater model is in fact a simplified form of a real groundwater system which can, approximately, indicate the correlation between hydrodynamic action and reaction in a system (Khalghi, 2003). The first step after setting the goal, is preparing the conceptual model of the aquifer system. During this process, natural complexities of the aquifer are simplified and the observed data is analyzed, so that the system can be investigated easily and more quickly (Etebari, 2004). The Kabudarahang aquifer have been assumed as a free and single layer aquifer. Within the area of the aquifer, the statistics for 46 piezometers in the statistical period of the 2006-2007 water year are available (figure 2). Due to the very large area of the plain and in order to accurately determine hydrodynamic coefficients of the aquifer, a large number of pumping tests must be conducted. The number of pumping tests in the plan have been very low and it was not possible to accurately determine hydrodynamic coefficients of the aquifer. In this modeling, given the extent of the aquifer and available statistics and information, a rectangular grid was considered with length and width of 24000 and 18000 meters, respectively. In this way, a grid with 120 columns and 90 rows in the x and y directions and 10770 cells were generated for the Kabudarahang aquifer. 8795 of these cells were active. Rotating the model grid, which is of great importance in this study, is one of the special cases in modeling. Because this rotation can cover all the wells in the study area. To this end, the procedure of lineaments of aquifer, geology of the area, plain strain, flow system pattern have been used. Generally, the initial conditions of the model are fluctuations in water level in the same year and groundwater balance. Also, the time step depends on the data and available information. According to the regional piezometers, the maximum water level occur in Farvardin and Ordibehesht and the minimum occur in Shahrivar (figures 4 & 5). Time discretization was conducted according to the different stages of modeling. In the steady stage, duration of model (time step) were considered to be a month consisting of 30 days. Then, in the unsteady stage, time step were chosen to be a year which was divided into 12 stress periods. The considered stress periods were based on months of a year and has been started from September (Mehr), 2006. The duration of each stress period, depending on the month, varies between 29 and 31 days. Of these stress periods, the last period (Shahrivar) were used for validation and the other 11 periods were used for calibration (table 1). When calibrating the model, feed information and also depletion of wells were corrected.

1.3. DESIGNING THE GROUNDWATER FLOW MODEL

The conceptual model is prepared for modeling in an appropriate form. This stage includes designing the grid, selecting the stress periods and time steps, and determining the boundary and initial conditions. In fact, in this stage the available data are entered into the chosen coding environment. The overall extent of the study area is 262 square kilometers with an irregular polygon shape which is located in a 24000×18000 meters rectangular environment. Data such as elevations numbers (topography), statistics for the water level in piezometers, maps capable of being transferred, aquifer bedrock map, the amount of depletion from plain's wells, the flow rate of rivers, etc., were generated and evaluated, and finally were applied to the flow model in the following steps.

2. DISCUSSION AND CONCLUSION

2.1. The Quantitative Model Calibration in the Permanent Regime

Anderman et al. (2003) and Chiang (1996), reported that the quantitative model consists of writing a computer program or code or selecting the appropriate code. The code PMWIN 5/3 has been used in this study. Calibration is minimizing the differences between calculated and observed value. In this model, first we tried to directly calibrate the model. However, given the limited time and high volume of the project and the fact that modeling is a time consuming process, we used the trial and error method to calibrate the model. The result of calibration of the model in the permanent regime is hydraulic conductivity of the aquifer (K). The highest amount of K was in the KurijanDu area at a rate of 6.25 meters per day, and the lowest amount of K was seen in the Khanabad area with a rate of 1.03 meters per day. In the beginning, in order to calibrate hydraulic conductivity, modeling in the permanent regime is very necessary. In this stage, we have to choose a steady state of the aquifer as the starting time for modeling.

2.2. Calibrating the quantitative model in the non-permanent flow regime

Tiederman et al. (2004), in their study, indicated that the results of calibration of the model in non-permanent flow regime, is the Specific Yield (S_y) of aquifer. Among the input data in the model, K and Specific Yield are of great importance and sensitivity in the calibration of the model. In the more advanced stages (unsteady flow) and in the central parts of the plains, through corrections levels of S_y in different points, adjustment of pump flow rate values, feeding, penetration from the bottom of rivers, and underground input values during 12 time steps, the simulated water level came very close to the observed values. The desired result was achieved after about 622 iteration of flow model.

2.3. Investigating the degree of error

There are different methods for evaluating the degree of error in the performed calibration which will not be explained in order to prevent an increase in the volume of presented content. NASH, RMSE, and MAE indicators were used to determine the degree of error and the obtained results were 1.009, 0.94, and 0.992, respectively.

2.4. Sensitivity analysis of parameters

Tiedeman et al. (2004), reported that the purpose of sensitivity analysis is to demonstrate the reaction of quantitative model of the aquifer to an uncertain input parameter. The response of the model toward the change in input parameter can be high or too low. The flow model of the Kabudarahang aquifer is very sensitive to changes in hydrodynamic coefficients and hydrologic tensions, but shows lower sensitivity toward bedrock.

Table 1. Temporal discretization for the flow model.

Modeling stages	Time step	During the period (day)
Calibration	1 (September, 2006)	0-30
	2 (October, 2006)	30-60
	3 (November, 2006)	60-90
	4 (December, 2006)	90-120
	5 (January, 2006)	120-150
	6 (February, 2006)	150-179
	7 (March, 2007)	179-210
	8 (April, 2007)	210-241
	9 (May, 2007)	241-272
	10 (June, 2007)	272-303
	11 (July, 2007)	303-334
Verification	12 (August, 2007)	334-365

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