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Some New Results on Soft n-T_4 Spaces

Orhan GÖÇÜR[1*]

ABSTRACT: Göçür and Kopuzlu showed that any soft T_4 space, may not be a soft T_2 space (also may not be a soft T_3 space). In this case, they described a new soft separation axiom which is called soft n-T_4 space. Then they indicated that any soft n-T_4 space is soft T_3 space also (Göçür and Kopuzlu, 2015b). In the present paper we showed that if (X, τ, E) is a soft $n - T_4$ space, topological space (X, τ_e) is a T_4 space for all $e \in E$. Then we indicated that any Soft Metric space is also soft n-T_4 space. Consequently, we indicated that any Soft Metric space $\Longrightarrow$ Soft n-T_4 space $\Longrightarrow$ Soft T_3 space $\Longrightarrow$ Soft T_2 space $\Longrightarrow$ soft T_1 space $\Longrightarrow$ soft T_0 space also.

Keywords: soft metric space, soft separation axioms, soft set, soft closed set, soft n-T_4 space, soft topological space.

[1] Orhan GÖÇÜR **(Orcid ID:** 0000-0001-7141-118X), Bilecik Şeyh Edebali Üniversitesi, Fen Edebiyat Fakültesi, İstatistik ve Bilgisayar Bilimleri Bölümü, Bilecik, Türkiye

Sorumlu Yazar/Corresponding Author: Orhan GÖÇÜR, e-mail:orhangocur@gmail.com

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INTRODUCTION

Molodtsov defined soft set and gives some properties about it. For this, he thinked that there are many uncertainties to solve complicated problems such as in sociology economics, engineerig, medical science, environment problems, statistics, etc, and there are no deal to solve them successfully. However, there are some theories such as vague sets theory (Gau and Buehrer,1993), fuzzy sets (Zadeh, 1965), probability, intuitionistic fuzzy sets (Atanassov, 1986), rough sets (Pawlak,1982), interval mathematics (Gorzalzany, 1987), etc, but these studies have their own complexities (Molodtsov, 1999). Then Maji et al. (2003) defined some operators for soft sets and gave some properties about it. Up to now, there are many studies on the soft sets and their applications in different fields.

Shabir and Naz defined soft topological spaces which are introduced over an inital universe with constant set of parameters. They indicated a parameterized family of topological spaces mean that also be a soft topological space. They defined the concept of soft open sets, soft closed sets, soft closure, soft separation axioms, etc. Also, they gave some important properties about soft separation axioms. They gave definitions of soft T_i spaces for $i = 0,1,2,3,4$. And they gave some relations about them. For example; they indicated that any soft T_i space is a soft T_{i-1} space for $i = 1,2$ also. And they asserted that any soft T_3 space need not be a soft T_2 space by given an example (Shabir and Naz, 2011). But the example is false. In this case, Won Keun Min indicated that any soft T_3 space is also a soft T_2 space (Min, 2011). After that, Shabir and Naz asserted that if a soft topological space (X, τ, E) is a soft T_1 space, then topological space (X, τ_e) is T_1 space for all $e \in E$ (Shabir and Naz, 2011). But this proposition is false. In this case, Göçür and Kopuzlu showed that if a soft topological space (X, τ, E) is a soft T_1 space, then topological space (X, τ_α) is a T_1 space for which $\alpha \in E$. And also, they showed that if a soft topological space (X, τ, E) is a soft T_0 space, topological space (X, τ_α) is a T_0 space for which $\alpha \in E$ (Göçür and Kopuzlu, 2015a). Shabir and Naz indicated that if a soft topological space (X, τ, E) is a soft T_2 space, then topological space (X, τ_e) is a T_2 space for all $e \in E$ (Shabir and Naz, 2011). And in this case Won Keun Min indicated that if a soft topological space (X, τ, E) is a soft T_3 space, then topological space (X, τ_e) is a T_3 space for all $e \in E$ (Min, 2011).

Shabir and Naz asserted that if a soft topological space (X, τ, E) is a soft T_4 space, then (X, τ, E) may not be a soft T_3 space (Shabir and Naz 2011). This proposition is true but given an example about this is false. For this, Zhang gave correct example about it (Zhang, 2015). After then Göçür and Kopuzlu indicated that if a soft topological space (X, τ, E) is a soft T_4 space, (X, τ, E) may not be a soft T_2 space (also may not be a soft T_3 space). And they indicated that any soft discrete space is soft T_2 space. Then they showed that any soft discrete space may not be soft T_3 space. After that they indicated any soft discrete space is soft T_4 space. In this case, they described a new soft separation axiom which is called soft n-T_4 space. Then they indicated that if a soft topological space (X, τ, E) is a soft n-T_4 space, (X, τ, E) is also a soft T_3 space. Consuquently they showed that any soft n-T_4 space ⇒ soft T_3 space ⇒ Soft T_2 space ⇒ soft T_1 space ⇒ soft T_0 space. In this case, Göçür and Kopuzlu showed that any soft discrete topological space may not be a soft n-T_4 space. So, they introduced a new soft topological space that is called soft single point space. And they indicated that any soft single point space is also soft subspace of soft discrete space. Then they indicated that any soft single point space is soft n-T_4 space. (Göçür and Kopuzlu, 2015b). After that, Göçür introduced soft metric space which is defined over an initial universe set with constant set of parameters. And he gave definitions of soft open ball and soft closed ball in soft metric spaces. Also, he introduced soft metrizable. And he showed that any soft discrete space is soft non-metrizable while soft single point space is soft metrizable. Finally, he indicated that any soft metrizable space is soft n-T_4 (Göçür, 2017).

In the present paper, we show that if a soft topological space (X, τ, E) is a soft n-T_4 space, then topological space (X, τ_e) is a T_4 space for all $e \in E$. Then we show that any Soft Metric space is soft n-T_4 space also. Consuquently, we indicate that any Soft Metric space $\Longrightarrow$Soft n-T_4 space $\Longrightarrow$ soft T_3 space $\Longrightarrow$ Soft T_2 space $\Longrightarrow$ soft T_1 space $\Longrightarrow$ soft T_0 space.

MATERIALS AND METHODS

We will use this terminology for following pages: X denotes an initial universe, E denotes universal set of parameters; A, B, C are subset of E, $P(X)$ denotes the power set of X and then, $(F, E), (G, E), (H, E)$ are soft sets over X.

Definition 1. (F, A) is said to be a soft set over X, where F is a mapping from A to $P(X)$ (Molodtsov, 1999).

Definition 2. (F, E) is a soft subset of (G, E), denoted by $(F, E) \tilde{\subseteq} (G, E), F(e) \subseteq G(e)$, $\forall e \in E$ (Maji et al, 2003).

Definition 3. If $(F_1, E) \tilde{\subseteq} (F_2, E)$ and $(F_2, E) \tilde{\subseteq} (F_1, E)$, it is called (F_1, E) is a soft equal (F_2, E) and it is indicated that $(F_1, E) \cong (F_2, E)$ (Maji et al, 2003).

Definition 4. (F, E) is an empty soft set denoted with $\tilde{\emptyset}$, if $F(e) = \emptyset, \forall e \in E$, (Maji et al, 2003).

Definition 5. (H, E) is called soft union of (F_1, E) and (F_2, E). It is denoted by $(F_1, E) \tilde{\cup} (F_2, E)$ such that $H(e) = F_1(e) \cup F_2(e)$, $\forall e \in E$ (Maji et al, 2003).

Definition 6. (H, E) is called soft intersection of (F, E) and (G, E). It is denoted by $(F_1, E) \tilde{\cap} (F_2, E)$ such that $H(e) = F_1(e) \cap F_2(e)$, $\forall e \in E$ (Feng et al, 2008).

Definition 7. (H, E) is called soft difference of (F_1, E) and (F_2, E). It is denoted by $(F_1, E) \tilde{\setminus} (F_2, E)$ such that $H(e) = F_1(e) \setminus F_2(e)$, $\forall e \in E$ (Shabir and Naz, 2011).

Definition 8. Let $x \in X$. If $\forall e \in E, x \in F(e)$, then $x \tilde{\in} (F, E)$. Note this; if $x \notin F(\alpha)$, $\exists \alpha \in E$, $x \tilde{\notin} (F, E)$ (Shabir and Naz, 2011).

Definition 9. Let $x \in X$. (x, E) is called the soft set if $x(e) = \{x\}$, $\forall e \in E$ (Shabir and Naz, 2011).

Definition 10. $(F, E)'$ is called relative complement of (F, E) if $(F', E) \cong (F, E)'$ where F' is a mapping from E to $P(X)$; $F'(e) = X - F(e), \forall e \in E$ (Shabir and Naz, 2011).

Definition 11. Let τ be the collection of soft sets on X, then τ is called soft topology over X if

1. $\tilde{\emptyset}$, $\tilde{X} \tilde{\in} \tau$,
2. the intersection of any two soft sets in $\tau \tilde{\in} \tau$,
3. the soft union of any number of soft sets in $\tau \tilde{\in} \tau$.

So (X, τ, E) is a soft topological space on X (Shabir and Naz, 2011).

We will use this terminology for following: (X, τ, E) as a soft topological space over X and $x, y, z \in X$.

Definition 12. Given (X, τ, E). The members of τ are called soft open sets in X (Shabir and Naz, 2011).

Definition 13. Given (X, τ, E). (F, E) is called soft closed set in X, if $(F, E)' \tilde{\in} \tau$ (Shabir and Naz, 2011).

Proposition 1. Given (X, τ, E). The collection $\tau_e = \{F(e) | (F, E) \in \tau\}$, $\forall e \in E$, defines a topology on X (Shabir and Naz, 2011).

Proposition 2. Given (X,τ,E). (F,E) be a soft closed set over X, if $F(e)$ is closed set in (X,τ_e), $\forall e\in E$ (Evanzalin and Thangavelu 2017).

Definition 14. Given (X,τ,E), soft open sets (F,E), (G,E) and $x\neq y$. If $x\,\tilde{\in}\,(F,E), y\,\tilde{\notin}\,(F,E)$ or $y\,\tilde{\in}\,(G,E), x\,\tilde{\notin}\,(G,E)$, then (X,τ,E) is called soft T_0 space (Shabir and Naz, 2011).

Remark 1. Let (X,τ,E) be a soft T_0 space. Then there exist soft open sets (F,E) and (G,E) such that $x\,\tilde{\in}\,(F,E), y\,\tilde{\notin}\,(F,E)$ or $y\,\tilde{\in}\,(G,E), x\,\tilde{\notin}\,(G,E)$ from Definition 14. Also we know that for all $e\in E$, (X,τ_e) is a topological space from Proposition 1. Then we can see that clearly, since $x\,\tilde{\in}\,(F,E)$, there exists open set $F(e)$ in τ_e such that $x\in F(e)$ for all $e\in E$; and since $y\,\tilde{\notin}\,(F,E)$, there exists open set F(e_i) in τ_{e_i} such that $y\notin F(e_i)$ for $e_i\in E$, $i\in I$. Or similarly since $y\,\tilde{\in}\,(G,E)$, there exists open set $G(e)$ in τ_e such that $y\in G(e)$ for all $e\in E$; and since $x\,\tilde{\notin}\,(G,E)$, there exist open sets G(e_j) in τ_{e_j} such that $x\notin G(e_j)$ for $e_j\in E$, $j\in I$ (Göçür and Kopuzlu, 2015a).

Theorem 1. Given (X,τ,E) and $x\neq y$ and let $i,j\in I$ such that mentioned in Remark 1, $e\in E$. If (X,τ,E) is a soft T_0 space, then at least one of (X,τ_{e_i}) and (X,τ_{e_j}) are T_0 spaces (Göçür and Kopuzlu, 2015a).

Definition 15. Given (X,τ,E), soft open sets (F,E) and (G,E), $x\neq y$. If $x\,\tilde{\in}\,(F,E)$,$y\,\tilde{\notin}\,(F,E)$ and $y\,\tilde{\in}\,(G,E)$,$x\,\tilde{\notin}\,(G,E)$, (X,τ,E) is said to be soft T_1 space (Shabir and Naz, 2011).

Remark 2. Let (X,τ,E) be a soft T_1 space, then there exist (F,E),$(G,E)\,\tilde{\in}\,\tau$ such that $x\,\tilde{\in}\,(F,E), y\,\tilde{\notin}\,(F,E)$ and $y\,\tilde{\in}\,(G,E), x\,\tilde{\notin}\,(G,E)$ from Definition 15. Also we know that for each $e\in E$, (X,τ_e) is a topological space from Proposition 1. Then we can see that clearly, since $x\,\tilde{\in}\,(F,E)$, there exists open set $F(e)$ in τ_e such that $x\in F(e)$ for all $e\in E$; and since $y\,\tilde{\notin}\,(F,E)$, there exists open set F(e_i) in τ_{e_i} such that $y\notin F(e_i)$ for $e_i\in E$, $i\in I$.

And similarly since $y\,\tilde{\in}\,(G,E)$, there exists open set $G(e)$ in τ_e such that $y\in G(e)$ for all $e\in E$; and since $x\,\tilde{\notin}\,(G,E)$, there exist open sets G(e_j) in τ_{e_j} such that $x\notin G(e_j)$ for $e_j\in E$, $j\in I$ (Göçür and Kopuzlu, 2015a).

Theorem 2. Given (X,τ,E), x,y such that $x\neq y$ and let $i,j\in I$ such that mentioned in Remark 2, $e\in E$. Let $k,l\in I$ such that $e_{i_k}=e_{j_l}$. If (X,τ,E) is a soft T_1 space, then $(X,\tau_{e_{i_k}})$ are T_1 spaces (Göçür and Kopuzlu, 2015a).

Example 1. Let $X=\{a,b\}, E=\{e_1,e_2\}$ and $\tau=\{\emptyset,X,(G_1,E),(G_2,E),(G_3,E)\}$ where

$$G_1(e_1)=X,\ \ G_1(e_2)=\{a\},$$

$$G_2(e_1)=\{b\},\quad G_2(e_2)=\{b\},$$

$$G_3(e_1)=\{b\},\quad G_3(e_2)=\emptyset.$$

We note that (X,τ,E) is a soft T_1 space because there exist soft open sets (G_1,E) and (G_2,E) such that $a\,\tilde{\in}\,(G_1,E), b\,\tilde{\notin}\,(G_1,E)$ and $b\,\tilde{\in}\,(G_2,E), a\,\tilde{\notin}\,(G_2,E)$.

We can see clearly that (X,τ_{e_1}) is not T_1 space because of $\tau_{e_1}=\{\emptyset,X,\{b\}\}$. Also we can see clearly that, τ_{e_2} is a T_1 space because of $\tau_{e_2}=\{\emptyset,X,\{a\},\{b\}\}$.

Definition 16. Given (X,τ,E), soft open sets (F,E), (G,E) and $x\neq y$. If $x\,\tilde{\in}\,(F,E), y\,\tilde{\in}\,(G,E)$ and $(F,E)\,\tilde{\cap}\,(G,E)=\tilde{\emptyset}$, then (X,τ,E) is called soft T_2 space (Shabir and Naz, 2011).

Proposition 3. (X,τ_e) is a T_2 space for all $e\in E$ if (X,τ,E) is a soft T_2 space (Shabir and Naz, 2011).

Definition 17. Given (X,τ,E). Let (H,E) be a soft closed set in X, $x\,\tilde{\notin}\,(H,E)$. If (F,E),$(G,E)\,\tilde{\in}\,\tau$ such that $x\,\tilde{\in}\,(F,E)$, $(H,E)\,\tilde{\subseteq}\,(G,E)$ and

$(F,E)\,\widetilde{\cap}\,(G,E)=\widetilde{\emptyset}$, then (X,τ,E) is said to be soft regular space (Shabir and Naz, 2011).

Definition 18. If (X,τ,E) is also soft regular and soft T_1 space, then it is called soft T_3 space (Shabir and Naz, 2011).

Remark 3. If (X,τ,E) is a soft T_3 space, (X,τ_e) is T_3 space for each parameter $e\in E$ (Min 2011).

Definition 19. Let (F,E) and (G,E) be soft closed sets in (X,τ,E). And let $x\,\widetilde{\in}\,(F,E)$, $(F,E)\,\widetilde{\cap}\,(G,E)\cong\widetilde{\emptyset}$. If there exist soft open sets (F_1,E) and (F_2,E) such that $y\,\widetilde{\in}\,(F_2,E)$, $(F,E)\,\widetilde{\subset}\,(F_1,E)$, $(G,E)\,\widetilde{\subset}\,(F_2,E)$ and $(F_1,E)\,\widetilde{\cap}\,(F_2,E)\cong\widetilde{\emptyset}$, then (X,τ,E) is a soft n-normal space. (Göçür and Kopuzlu, 2015b)

Definition 20. If (X,τ,E) is a soft n-normal space and also soft T_1 space, then (X,τ,E) is a soft n-T_4 space (Göçür and Kopuzlu, 2015b).

Theorem 3. Any Soft n-T_4 space is soft T_3 space (Göçür and Kopuzlu, 2015b).

Corollary 1. Any Soft n-T_4 space $\Rightarrow$ soft T_3 space. $\Rightarrow$ Soft T_2 space $\Rightarrow$ soft T_1 space $\Rightarrow$ soft T_0 space. (Göçür and Kopuzlu, 2015b).

Definition 21. Let $\tilde{X}$ be the absolute soft set i.e., $F(e)=X$, for all $e\in E$, where $(F,E)=\tilde{X}$. Let $\widetilde{\mathbb{R}}$ denotes set of all soft real numbers (briefly SRN). And let $\tilde{r}$ to denote SRN such that $F(e)=r$, for all $e\in E$, where $(F,E)=\tilde{r}$. For instance, $\tilde{0}$ is the SRN where $F(e)=0$, for all $e\in E$ where $(F,E)=(0,E)$ for $0\in\mathbb{R}$. Also for shortly, we use $\tilde{a},\tilde{b},\tilde{x},\tilde{y},\tilde{z}$ instead of (a,E),(b,E), (x,E), (y,E), (z,E) respectively for all $a,b,x,y,z\in X$ and for all $e\in E$.
A mapping $d:\tilde{X}\times\tilde{X}\to\widetilde{\mathbb{R}}$ is called a soft metric, if d satisfies the following:

1. $d(\tilde{x},\tilde{y})\geq\tilde{0}$,
2. $d(\tilde{x},\tilde{y})=\tilde{0}$ if and only if $x=y$,
3. $d(\tilde{x},\tilde{y})=d(\tilde{y},\tilde{x})$,
4. $d(\tilde{x},\tilde{z})\leq d(\tilde{x},\tilde{y})+d(\tilde{y},\tilde{z})$,

The soft set $\tilde{X}$ with a soft metric d on $\tilde{X}$ is denoted by $(\tilde{X},d,E)$ and it is said to be a soft metric space (Göçür, 2017).

We will use this terminology for following: $(\tilde{X},d,E)$ be a soft metric space, $\tilde{r}$ and $\tilde{\epsilon}$ be non-negative SRN.

Definition 22. Given $(\tilde{X},d,E)$ and $\tilde{r}$. For any $a\in X$, open soft ball with centre $\tilde{a}$ and radius $\tilde{r}$ satisfy $d(\tilde{x},\tilde{a})<\tilde{r}$. Thus the open soft ball with centre $\tilde{a}$ and radius $\tilde{r}$ is denoted by $B(\tilde{a},\tilde{r})$. Hence $B(\tilde{a},\tilde{r})=\{x\in X; d(\tilde{x},\tilde{a})<\tilde{r}\}$ (Göçür, 2017).

Definition 23. Given $(\tilde{X},d,E)$ and $\tilde{r}$. For any $a\in X$, closed soft ball with centre $\tilde{a}$ and radius $\tilde{r}$ satisfy $d(\tilde{x},\tilde{a})\leq\tilde{r}$. Thus the closed soft ball with centre $\tilde{a}$ and radius $\tilde{r}$ is denoted by $B[\tilde{a},\tilde{r}]$. Hence $B[\tilde{a},\tilde{r}]=\{x\in X; d(\tilde{x},\tilde{a})\leq\tilde{r}\}$ (Göçür, 2017).

RESULTS AND DISCUSSION

Theorem 4. Given (X,τ,E). If X is a soft n-normal space, then (X,τ_e) is a normal space for all $e\in E$.

Proof Let (X,τ,E) be a soft n-normal space and $x,y\in X$. And let (H_1,E) and (H_2,E) be soft closed sets such that $x\,\widetilde{\in}\,(H_1,E)$ and $(H_1,E)\,\widetilde{\cap}\,(H_2,E)=\widetilde{\emptyset}$. Then, there exist soft open sets (G_1,E) and (G_2,E) such that $y\,\widetilde{\in}\,(G_2,E)$, $(H_1,E)\,\widetilde{\subset}\,(G_1,E)$, $(H_2,E)\,\widetilde{\subset}\,(G_2,E)$ and $(G_1,E)\,\widetilde{\cap}\,(G_2,E)=\widetilde{\emptyset}$ from Definition 19. Here, because (H_1,E) and (H_2,E) be soft closed sets such that $x\,\widetilde{\in}\,(H_1,E)$ and $(H_1,E)\,\widetilde{\cap}\,(H_2,E)=\widetilde{\emptyset}$, there exist closed sets $H_1(e)$ and $H_2(e)$ in (X,τ_e) such that $H_1(e)\cap H_2(e)=\emptyset$, for all $e\in E$ from Proposition 2 and Definition 4. And then there exist open sets $G_1(e)$ and $G_2(e)$ in (X,τ_e) such that $H_1(e)\subset G_1(e)$, $H_2(e)\subset G_2(e)$ and $G_1(e)\cap G_2(e)=\emptyset$, for all $e\in E$ from Definition 2, Proposition 1 and Definition 4. Hence (X,τ_e) is a normal space, for all $e\in E$.

Theorem 5. Given (X,τ,E). If X is a soft n-T_4 space, then (X,τ_e) is a T_4 space, for all $e \in E$.

Proof (X,τ,E) is both soft n-normal space and soft T_1 space from Definition 20. Because (X,τ_e) is normal space for all $e \in E$ from Theorem 4 and (X,τ_e) is T_1 space for all $e \in E$ from Corollary 1, Proposition 3 and we know that any T_2 space is also T_1 space from classical topology, then (X,τ_e) is T_4 space, for all $e \in E$.

Example 2. Let $X = \{a,b,c\}, E = \{e_1,e_2\}$ and

$\tau = \{\emptyset, X, (G_1,E), (G_2,E), (G_3,E), (G_4,E), (G_5,E), (G_6,E)\}$

Where

$$G_1(e_1) = \{a\}, \qquad G_1(e_2) = \{a\},$$
$$G_2(e_1) = \{b\}, \qquad G_2(e_2) = \{b\},$$
$$G_3(e_1) = \{c\}, \qquad G_3(e_2) = \{c\}.$$
$$G_4(e_1) = \{a,b\}, \qquad G_4(e_2) = \{a,b\}$$
$$G_5(e_1) = \{a,c\}, \qquad G_5(e_2) = \{a,c\}$$
$$G_6(e_1) = \{b,c\}, \qquad G_6(e_2) = \{b,c\}$$

Then (X,τ,E) is a soft topological space over X. Here, we can see clearly that (X,τ,E) is soft n-T_4 space and so (X,τ_{e_1}) and (X,τ_{e_2}) are T_4 space.

Theorem 6. Any soft metric space is soft T_1 space also.

Proof Let $(\tilde{X},d,E)$ be a soft metric space; $a,b \in X$ and let $\tilde{\varepsilon}$ be a non-negative SRN such that $d(\tilde{a},\tilde{b}) = \tilde{\varepsilon}$. Then there exist soft open ball $B(\tilde{a},\widetilde{\varepsilon/3})$ such that $b \notin B(\tilde{a},\widetilde{\varepsilon/3})$. And similarly, there exist soft open ball $B(\tilde{b},\widetilde{\varepsilon/3})$ such that $a \notin B(\tilde{a},\widetilde{\varepsilon/3})$. Hence $\tilde{X}$ is soft T_1.

Theorem 7. Any soft metric space is soft n-normal space.

Proof Let $(\tilde{X},d,E)$ be a soft metric space. Let (A,E) and (B,E) be disjoint soft closed soft subsets of $\tilde{X}$. For each $a \in (A,E)$, choose $\widetilde{\varepsilon_a}$, which is non-negative SRN, so that the soft ball $B(\tilde{a},\widetilde{\varepsilon_a})$ does not intersect (B,E). Similarly, for each $b \in (B,E)$, choose $\widetilde{\varepsilon_b}$, which is non-negative SRN, so that the soft ball $B(\tilde{b},\widetilde{\varepsilon_b})$ does not intersect (A,E). Define

$$(U,E) \cong \bigcup_{a\in(A,E)} \widetilde{B(\tilde{a},\widetilde{\varepsilon_a/3})}$$

and

$$(V,E) = \bigcup_{b\in(B,E)} \widetilde{B(\tilde{b},\widetilde{\varepsilon_b/3})}$$

Then (U,E) and (V,E) are soft open sets containing (A,E) and (B,E) respectively.

Also $b \,\tilde{\in}\, (V,E)$. We assert they are disjoint. For if $z \in (U,E) \,\tilde{\cap}\, (V,E)$, then

$$z \,\tilde{\in}\, B(\tilde{a},\widetilde{\varepsilon_a/3}) \cap B(\tilde{b},\widetilde{\varepsilon_b/3})$$

for some $a \in (A,E)$ and $b \in (B,E)$ the triangle inequality applies to show that $d(\tilde{a},\tilde{b}) < (\widetilde{\varepsilon_a} + \widetilde{\varepsilon_b})/3$. If $\widetilde{\varepsilon_a} \le \widetilde{\varepsilon_b}$, then $d(\tilde{a},\tilde{b}) < \widetilde{\varepsilon_b}$, so that the soft ball $B(\tilde{b},\widetilde{\varepsilon_b})$ contains a. If $\widetilde{\varepsilon_b} \le \widetilde{\varepsilon_a}$, then $d(\tilde{a},\tilde{b}) < \widetilde{\varepsilon_a}$, so that the soft ball $\mathrm{B}(\tilde{a},\widetilde{\varepsilon_a})$ contains $\tilde{b}$. Neighter situation is possible.

Theorem 8. Any soft metric space is soft n-T_4 space.

Proof It is obvious that Definition 20, Theorem 6 and Theorem 7.

Corollary 2. Soft Metric Space ⇒ Soft n-T_4 space ⇒ soft T_3 space. ⇒ Soft T_2 space ⇒ soft T_1 space ⇒ soft T_0 space.

Proof It is obvious that Theorem 8 and Corollary 1.

CONCLUSION

In the present paper, we showed that if (X, τ, E) is a soft n-T_4 space, then topological space (X, τ_e) is a T_4 space for all $e \in E$. Then we showed that any Soft Metric space is soft n-T_4 space also. Consuquently, we indicated that any Soft Metric space ⇒Soft n-T_4 space ⇒ soft T_3 space. ⇒ Soft T_2 space ⇒ soft T_1 space ⇒ soft T_0 space.

In this study, our purpose is completing as much as possible that soft separation axioms defined over an initial universe with constant set of parameter. And we hope that researchers investigate soft metric, soft compactness, soft connectedness, soft sequences etc.

REFERENCES

Atanassov K, 1986. Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20:87-96.

Evanzalin E.P and Thangavelu P, 2017. On quasi soft sets in soft topology. GJPAM, 13(12):8343-8359.

Feng F, Jun YB and. Zhao XZ, 2008. Soft semirings. Comput. Math. Appl, 56:2621-2628.

Gau WL and Buehrer DJ, 1993. Vague sets. IEEE Trans. System Man Cybernet, 23(2):610-614.

Gorzalzany MB, 1987. A method of inference in approximate reasoning based on interval valued fuzzy sets. Fuzzy Sets and Systems, 21:1-17.

Göçür O, 2017. Soft single point space and soft metrizable. Ann. Fuzzy Math. Inform, 13(4):499-507.

Göçür O, Kopuzlu A, 2015a. On soft separation axioms. Ann. Fuzzy Math. Inform, 9(5):817-822.

Göçür O, Kopuzlu A, 2015b. Some new properties on soft separation axioms. Ann. Fuzzy Math. Inform, 9(3):421-429.

Maji PK, Biswas R and Roy R, 2003. Soft set theory, Comput. Math. Appl, 45:555-562.

Min WK, 2011. A note on soft topological spaces. Comput. Math. Appl, 62:3524-3528.

Molodtsov D, 1999. Soft set theory first results. Comput. Math. Appl, 37:19-31.

Pawlak Z, 1982. Rough sets. Int. J. Comp. Inf. Sci, 11:341-356.

Shabir M, Naz M, 2011. On soft topological spaces. Comput. Math. Appl, 61:1786-1799.

Zadeh LA, 1965. Fuzzy sets, Information and Control, 8;338-353.

Zhang X, 2013. Further Study on Soft Separation Axioms. Computer Engineering and Applications, 49(5):48-50.