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# Fault Analysis in Multi-Phase Power Systems Considering Symmetrical Components and Phase Coordinates Methods

Erdin GÖKALP, Said Mirza TERCAN*

*Yıldız Technical University, Electrical Engineering Department, İstanbul, TURKEY

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* Corresponding author.
E-mail address:
stercan@yildiz.edu.tr

**ABSTRACT**

In this study, fault calculations in multi-phase power system (MPPS) are examined. Three-phase power system (3-PPS) and twelve-phase power system (12-PPS) are modelled. It includes modeling of 12-PPS components such as generator, transformer, and line. In order to analyze faults in MPPS, symmetrical components method is used. Load flow analysis in MPPS is made by using phase coordinates method. Proposed fault calculation method for different types of faults such as single line, single line to ground, two line to ground, etc. are provided. Fault calculations were done for all possible faults in 12-PPS. All symmetrical and asymmetrical fault currents in 12-PPS were calculated and listed. The fault location and type compared with other different locations and types of faults in 12-PPS. In addition, 3-PPS fault currents compared with MPPS examined in this study.



**Nomenclature:**

| | |
|---|---|
| $E_i$ | Phase-neutral voltage for ith phase of sending end of a line |
| $V_i$ | Phase-neutral voltage for ith phase of receiving end of a line |
| $[Z_p]$ | Phase impedance matrix |
| $Z_{ii}$ | Self-phase impedance for ith phase of a line |
| $Z_{ij}$ | Mutual phase impedance between ith phase and jth phase of a line, |
| $Z_{ss}$ | Self-phase impedance of transposed transmission line, |
| $Z_{sm}$ | Mutual impedance of transposed transmission line. |

## 1. Introduction

Power systems were as one-phase and neutral at beginning. With the increasing demand, phase adding idea came out. Thus, three-phase power system (3-PPS), which is a multi-power system (MPPS), was introduced , and then 3-PPS is widely used in todays. Continuous demand increase leads to the idea of voltage increase. Nowadays, the maximum voltage levels are 750, 1000 kV AC and ± 800 kV DC. Due to voltage rise, the noise and the corona losses are increased in transmission line [1]. Therefore, the idea of phase adding instead of voltage rise is still a research topic [2]–[11]. In this study, symmetrical and asymmetrical faults in MPPS investigated.

In part 2, how symmetrical components method applied to 12-PPS is described. Symmetrical components of 3-PPS is also provided. Part 3 includes fault calculation methods in 12-PPS. Fault calculation methods in 3-PPS is also provided. Results and discussion are provided in part 4. Conclusion is the last part.

## 2. Symmetrical Components in MPPS

Fortescue's symmetrical components method allow n buses asymmetrical power system calculation in n buses symmetrical power system. According to symmetrical component method, symmetrical components of 3-PPS are as follows.

3-PPS symmetrical components

Positive components compound 3 phasor that have same magnitude and 120° phase difference between phases in the same phase sequence of original system.

Negative components compound 3 phasor that have same magnitude and 120° phase difference between phases in the counter phase sequence of original system.

Zero components compound three phasor that have same magnitude and no phase difference between phases.

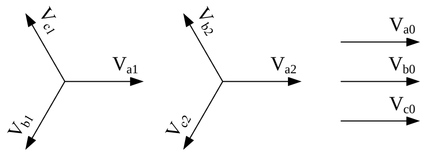


**Figure 1.** Positive, negative and zero sequences of symmetrical components of unbalanced system

$$\begin{aligned}\mathbf{V_a} &= \mathbf{V_{a1}} + \mathbf{V_{a2}} + \mathbf{V_{a0}} \\ \mathbf{V_b} &= \mathbf{V_{b1}} + \mathbf{V_{b2}} + \mathbf{V_{b0}} \\ \mathbf{V_c} &= \mathbf{V_{c1}} + \mathbf{V_{c2}} + \mathbf{V_{c0}}\end{aligned} \tag{1}$$

An operator "a" that rotates a phasor 120° counter clockwise is $1\angle 120° = 1e^{j2\pi/3} = -0{,}5 + j0{,}886$. Eq. 1 can be written in matrix form using operator "a" given as Eq. 2. If symmetrical components of one-phase is known, the unbalanced phase voltages of 3-PPS can be written as in Eq. 2.

$$\begin{bmatrix} \boldsymbol{V_a} \\ \boldsymbol{V_b} \\ \boldsymbol{V_c} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \boldsymbol{a^2} & \boldsymbol{a} \\ \mathbf{1} & \boldsymbol{a} & \boldsymbol{a^2} \end{bmatrix} \begin{bmatrix} \boldsymbol{V_{a0}} \\ \boldsymbol{V_{a1}} \\ \boldsymbol{V_{a2}} \end{bmatrix} \tag{2}$$

If unbalanced phase voltages of 3-PPS is known, symmetrical components can be written as in Eq. 3.

$$\begin{bmatrix} \boldsymbol{V_{a0}} \\ \boldsymbol{V_{a1}} \\ \boldsymbol{V_{a2}} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \boldsymbol{a} & \boldsymbol{a^2} \\ \mathbf{1} & \boldsymbol{a^2} & \boldsymbol{a} \end{bmatrix} \begin{bmatrix} \boldsymbol{V_a} \\ \boldsymbol{V_b} \\ \boldsymbol{V_c} \end{bmatrix} \tag{3}$$

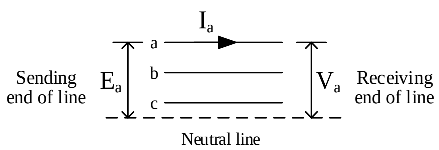


**Figure 2.** 3-PPS

Phase impedance matrix using Figure 2 can be written in Eq. 4.

$$[Z_p] = \begin{bmatrix} \boldsymbol{Z_{aa}} & \boldsymbol{Z_{ab}} & \boldsymbol{Z_{ac}} \\ \boldsymbol{Z_{ba}} & \boldsymbol{Z_{bb}} & \boldsymbol{Z_{bc}} \\ \boldsymbol{Z_{ca}} & \boldsymbol{Z_{cb}} & \boldsymbol{Z_{cc}} \end{bmatrix} \tag{4}$$

After transmission line transposition impedance matrix changed into in Eq. 5.

$$[Z_p] = \begin{bmatrix} \boldsymbol{Z_{ss}} & \boldsymbol{Z_{sm}} & \boldsymbol{Z_{sm}} \\ \boldsymbol{Z_{sm}} & \boldsymbol{Z_{ss}} & \boldsymbol{Z_{sm}} \\ \boldsymbol{Z_{sm}} & \boldsymbol{Z_{sm}} & \boldsymbol{Z_{ss}} \end{bmatrix} \tag{5}$$

Voltage drop of transposed lines can be written as in Eq. 6.

$$\begin{bmatrix} \boldsymbol{E_a - V_a} \\ \boldsymbol{E_b - V_b} \\ \boldsymbol{E_c - V_c} \end{bmatrix} = \begin{bmatrix} \Delta \boldsymbol{V_a} \\ \Delta \boldsymbol{V_b} \\ \Delta \boldsymbol{V_c} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z_{ss}} & \boldsymbol{Z_{sm}} & \boldsymbol{Z_{sm}} \\ \boldsymbol{Z_{sm}} & \boldsymbol{Z_{ss}} & \boldsymbol{Z_{sm}} \\ \boldsymbol{Z_{sm}} & \boldsymbol{Z_{sm}} & \boldsymbol{Z_{ss}} \end{bmatrix} \begin{bmatrix} \boldsymbol{I_a} \\ \boldsymbol{I_b} \\ \boldsymbol{I_c} \end{bmatrix} \tag{6}$$

Eq. 6 can be written for symmetrical components as Eq. 7.

$$\begin{bmatrix} \Delta \boldsymbol{V_{a0}} \\ \Delta \boldsymbol{V_{a1}} \\ \Delta \boldsymbol{V_{a2}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0 - V_{a0}} \\ \boldsymbol{E_{a1} - V_{a1}} \\ \boldsymbol{0 - V_{a2}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z_0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Z_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{I_{a0}} \\ \boldsymbol{I_{a1}} \\ \boldsymbol{I_{a2}} \end{bmatrix} \tag{7}$$

Considering there is a source in only positive sequence, symmetrical component of voltage at the end of the line can be written as follows in Eq. 8.

$$\begin{bmatrix} \boldsymbol{V_{a0}} \\ \boldsymbol{V_{a1}} \\ \boldsymbol{V_{a2}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{E_{a1}} \\ \boldsymbol{0} \end{bmatrix} - \begin{bmatrix} \boldsymbol{Z_0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Z_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{I_{a0}} \\ \boldsymbol{I_{a1}} \\ \boldsymbol{I_{a2}} \end{bmatrix} \tag{8}$$

$$Z_0 = Z_{ss} + 2Z_{sm},\ Z_1 = Z_2 = Z_{ss} - Z_{sm}$$

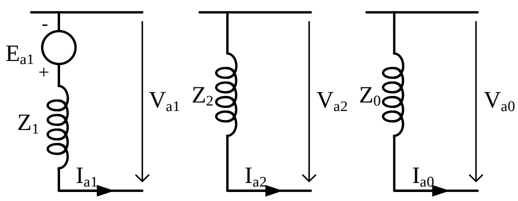


**Figure 3.** Positive, negative and zero sequence circuits (respectively) (3-PPS)

According to fault type, connections of sequence circuits, which are given in Figure 3, are given in Part 4.

*A. 12-PPS symmetrical components*

Symmetrical components of 12-PPS have twelve sequences, because phase number is twelve. First sequence is positive sequence, eleventh sequence is negative sequence, and twelfth sequence is zero sequence. Other sequences are called by their order. Symmetrical components of 12-PPS is given in

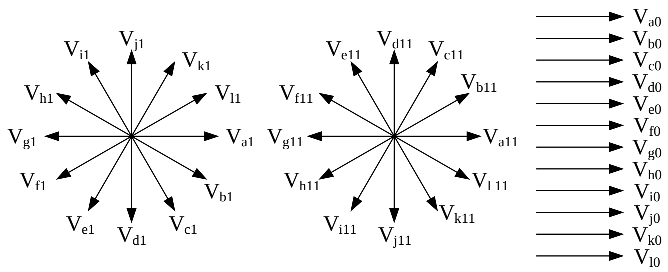

Figure 4.

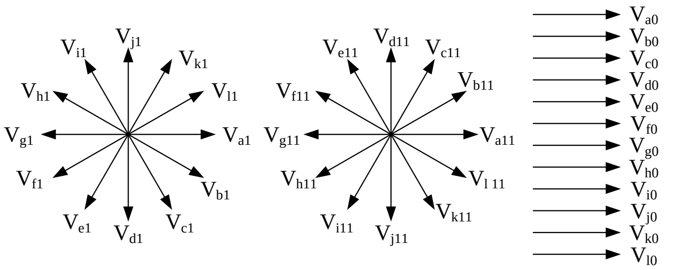


**Figure 4.** Positive, negative and zero sequence components of 12-PPS.

According to Fortescue's method, one phase voltage is equal to summation of all symmetrical components of this phase. It is provided in in Eq. 9 that an example of symmetrical components of one phase voltages in 12-PPS. Further information can be found in [12].

$$\mathbf{V_a} = \mathbf{V_{a1}} + \mathbf{V_{a2}} + \mathbf{V_{a3}} + \mathbf{V_{a4}} + \mathbf{V_{a5}} + \mathbf{V_{a6}} + \mathbf{V_{a7}} + \mathbf{V_{a8}} + \mathbf{V_{a9}} + \mathbf{V_{a10}} + \mathbf{V_{a11}} + \mathbf{V_{a0}} \tag{9}$$

An operator "**c**" that rotates a phasor 30° counter clockwise is $1\angle 30° = 1e^{j2\pi/12} = 0{,}886 + j0{,}5$. If symmetrical components of one-phase is known, the unbalanced phase voltages of 12-PPS can be written as in Eq. 10.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \\ V_g \\ V_h \\ V_i \\ V_j \\ V_k \\ V_l \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \mathrm{c}^{11} & \mathrm{c}^{10} & \mathrm{c}^9 & \mathrm{c}^8 & \mathrm{c}^7 & -1 & \mathrm{c}^5 & \mathrm{c}^4 & \mathrm{c}^3 & \mathrm{c}^2 & \mathrm{c} \\ 1 & \mathrm{c}^{10} & \mathrm{c}^8 & -1 & \mathrm{c}^4 & \mathrm{c}^2 & 1 & \mathrm{c}^{10} & \mathrm{c}^8 & -1 & \mathrm{c}^4 & \mathrm{c}^2 \\ 1 & \mathrm{c}^9 & -1 & \mathrm{c}^3 & 1 & \mathrm{c}^9 & -1 & \mathrm{c}^3 & 1 & \mathrm{c}^9 & -1 & \mathrm{c}^3 \\ 1 & \mathrm{c}^8 & \mathrm{c}^4 & 1 & \mathrm{c}^8 & \mathrm{c}^4 & 1 & \mathrm{c}^8 & \mathrm{c}^4 & 1 & \mathrm{c}^8 & \mathrm{c}^4 \\ 1 & \mathrm{c}^7 & \mathrm{c}^2 & \mathrm{c}^9 & \mathrm{c}^4 & \mathrm{c}^{11} & -1 & \mathrm{c} & \mathrm{c}^8 & \mathrm{c}^3 & \mathrm{c}^{10} & \mathrm{c}^5 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & \mathrm{c}^5 & \mathrm{c}^{10} & \mathrm{c}^3 & \mathrm{c}^8 & \mathrm{c} & -1 & \mathrm{c}^{11} & \mathrm{c}^4 & \mathrm{c}^9 & \mathrm{c}^2 & \mathrm{c}^7 \\ 1 & \mathrm{c}^4 & \mathrm{c}^8 & 1 & \mathrm{c}^4 & \mathrm{c}^8 & 1 & \mathrm{c}^4 & \mathrm{c}^8 & 1 & \mathrm{c}^4 & \mathrm{c}^8 \\ 1 & \mathrm{c}^3 & -1 & \mathrm{c}^9 & 1 & \mathrm{c}^3 & -1 & \mathrm{c}^9 & 1 & \mathrm{c}^3 & -1 & \mathrm{c}^9 \\ 1 & \mathrm{c}^2 & \mathrm{c}^4 & -1 & \mathrm{c}^8 & \mathrm{c}^{10} & 1 & \mathrm{c}^2 & \mathrm{c}^4 & -1 & \mathrm{c}^8 & \mathrm{c}^{10} \\ 1 & \mathrm{c} & \mathrm{c}^2 & \mathrm{c}^3 & \mathrm{c}^4 & \mathrm{c}^5 & -1 & \mathrm{c}^7 & \mathrm{c}^8 & \mathrm{c}^9 & \mathrm{c}^{10} & \mathrm{c}^{11} \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \\ V_{a3} \\ V_{a4} \\ V_{a5} \\ V_{a6} \\ V_{a7} \\ V_{a8} \\ V_{a9} \\ V_{a10} \\ V_{a11} \end{bmatrix} \tag{10}$$

If unbalanced phase voltages of 12-PPS is known, symmetrical components can be written as in Eq. 11.

$$\begin{bmatrix} \boldsymbol{V_{a0}} \\ \boldsymbol{V_{a1}} \\ \boldsymbol{V_{a2}} \\ \boldsymbol{V_{a3}} \\ \boldsymbol{V_{a4}} \\ \boldsymbol{V_{a5}} \\ \boldsymbol{V_{a6}} \\ \boldsymbol{V_{a7}} \\ \boldsymbol{V_{a8}} \\ \boldsymbol{V_{a9}} \\ \boldsymbol{V_{a10}} \\ \boldsymbol{V_{a11}} \end{bmatrix} = \frac{\mathbf{1}}{\mathbf{12}} [\boldsymbol{T_s}]^{-\mathbf{1}} \begin{bmatrix} \boldsymbol{V_a} \\ \boldsymbol{V_b} \\ \boldsymbol{V_c} \\ \boldsymbol{V_d} \\ \boldsymbol{V_e} \\ \boldsymbol{V_f} \\ \boldsymbol{V_g} \\ \boldsymbol{V_h} \\ \boldsymbol{V_i} \\ \boldsymbol{V_j} \\ \boldsymbol{V_k} \\ \boldsymbol{V_l} \end{bmatrix} \tag{11}$$

$T_S$ is coefficient matrix includes operator "**c**" in Eq. 10.

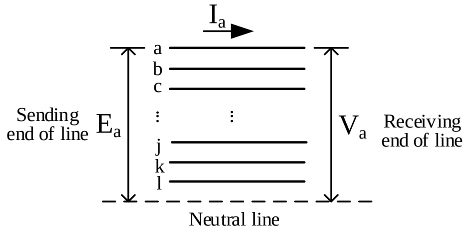


**Figure 5.** 12-PPS

Phase impedance matrix of 12-PPS can be written using Figure 5 in Eq. 12.

$$[Z_p] = \begin{bmatrix} \boldsymbol{Z_{aa}} & \boldsymbol{Z_{ab}} & \cdots & \boldsymbol{Z_{ak}} & \boldsymbol{Z_{al}} \\ \boldsymbol{Z_{ab}} & \boldsymbol{Z_{bb}} & \cdots & \boldsymbol{Z_{bk}} & \boldsymbol{Z_{bl}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{Z_{ak}} & \boldsymbol{Z_{bk}} & \cdots & \boldsymbol{Z_{kk}} & \boldsymbol{Z_{kl}} \\ \boldsymbol{Z_{al}} & \boldsymbol{Z_{bl}} & \cdots & \boldsymbol{Z_{kl}} & \boldsymbol{Z_{ll}} \end{bmatrix} \tag{12}$$

After transmission line transposition impedance matrix changed into in Eq. 13.

$$[Z_p] = \begin{bmatrix} \boldsymbol{Z_{ss}} & \boldsymbol{Z_{sm}} & \cdots & \boldsymbol{Z_{sm}} \\ \boldsymbol{Z_{sm}} & \boldsymbol{Z_{ss}} & & \boldsymbol{Z_{sm}} \\ \vdots & & \ddots & \vdots \\ \boldsymbol{Z_{sm}} & \boldsymbol{Z_{sm}} & \cdots & \boldsymbol{Z_{ss}} \end{bmatrix} \tag{13}$$

Voltage drop of transposed lines can be written as in Eq. 13.

$$\begin{bmatrix} \boldsymbol{E_a - V_a} \\ \boldsymbol{E_b - V_b} \\ \vdots \\ \boldsymbol{E_k - V_k} \\ \boldsymbol{E_l - V_l} \end{bmatrix} = \begin{bmatrix} \Delta \boldsymbol{V_a} \\ \Delta \boldsymbol{V_b} \\ \vdots \\ \Delta \boldsymbol{V_k} \\ \Delta \boldsymbol{V_l} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z_{ss}} & \boldsymbol{Z_{sm}} & \cdots & \boldsymbol{Z_{sm}} \\ \boldsymbol{Z_{sm}} & \boldsymbol{Z_{ss}} & & \boldsymbol{Z_{sm}} \\ \vdots & & \ddots & \vdots \\ \boldsymbol{Z_{sm}} & \boldsymbol{Z_{sm}} & \cdots & \boldsymbol{Z_{ss}} \end{bmatrix} \begin{bmatrix} \boldsymbol{I_a} \\ \boldsymbol{I_b} \\ \vdots \\ \boldsymbol{I_k} \\ \boldsymbol{I_l} \end{bmatrix} \tag{14}$$

Eq. 14 can be written for symmetrical components as Eq. 15.

$$\begin{bmatrix} \Delta \boldsymbol{V_{a0}} \\ \Delta \boldsymbol{V_{a1}} \\ \vdots \\ \Delta \boldsymbol{V_{a10}} \\ \Delta \boldsymbol{V_{a11}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0 - V_{a0}} \\ \boldsymbol{E_{a1} - V_{a1}} \\ \vdots \\ \boldsymbol{0 - V_{a10}} \\ \boldsymbol{0 - V_{a11}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z_0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \ddots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Z_{10}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Z_{11}} \end{bmatrix} \begin{bmatrix} \boldsymbol{I_{a0}} \\ \boldsymbol{I_{a1}} \\ \vdots \\ \boldsymbol{I_{a10}} \\ \boldsymbol{I_{a11}} \end{bmatrix} \tag{15}$$

Considering there is a source in only positive sequence, symmetrical component of voltage at the end of the line can be written as follows in Eq. 16-18.

$$\begin{bmatrix} \boldsymbol{V_{a0}} \\ \boldsymbol{V_{a1}} \\ \vdots \\ \boldsymbol{V_{a10}} \\ \boldsymbol{V_{a11}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{E_{a1}} \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{0} \end{bmatrix} - \begin{bmatrix} \boldsymbol{Z_0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \ddots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Z_{10}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Z_{11}} \end{bmatrix} \begin{bmatrix} \boldsymbol{I_{a0}} \\ \boldsymbol{I_{a1}} \\ \vdots \\ \boldsymbol{I_{a10}} \\ \boldsymbol{I_{a11}} \end{bmatrix} \tag{16}$$

$$Z_0 = Z_{ss} + 11 Z_{sm} \tag{17}$$

$$Z_1 = Z_2 = \cdots = Z_{10} = Z_{11} = Z_{ss} - Z_{sm} \tag{18}$$

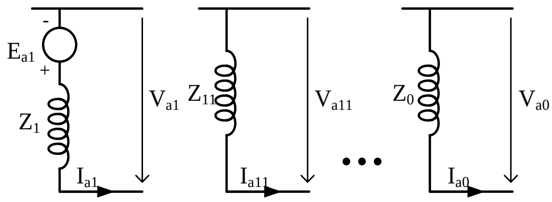


**Figure 6. Positive (first), eleventh (negative) and zero sequence circuits of 12-PPS**

## 3. Fault Analysis in mpps

Fault is contacting phase conductor to neutral or ground. Eliminating load impedance, a high current flows through the line. Symmetric fault occurs when all phase conductor contact all together with/without neutral. Asymmetric fault occurs when one or more conductors, except total the number of phase contacts, comes all together with/without neutral. Therefore, connection of sequence circuits is different for all types of fault [13]. Some certain assumption before fault calculation are as follows.

- Each machine is shown as one voltage source and transient or subtransient reactance.
- All static loads, resistors, magnetizing currents of transformers and capacitive currents of transmission lines are neglected, because they are very small beside fault current.
- All transformers run in nominal voltage level.
- Lines are shown only as reactance when their resistance is small than 1/6 of reactance. [14]
- Bus voltages are at nominal value before fault calculation.

*B. Symmetrical faults in MPPS*

Symmetrical fault in all MPPS is all-phase conductors' contact all together with/without ground. Only positive sequence is included, as in Figure 7.

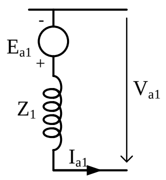


**Figure 7.** Symmetrical fault sequence connection in MPPS

Fault condition can be written as in Eq. 19, 20. Because ground connection, all phase and sequence voltages equals to zero.

$$V_a = V_b = V_c = 0 \tag{19}$$

$$V_{a0} = V_{a1} = V_{a2} = 0 \tag{20}$$

Fault current can be calculated by using Eq. 8.

$$I_a = I_{a1} = \frac{E_{a1}}{Z_1} \tag{21}$$

Connections of sequence circuits and calculation of fault currents are not different in other MPPS. If ground connection is not involved the fault, Eq. 19 turns into Eq. 22.

$$V_a = V_b = V_c \tag{22}$$

Although symmetrical faults have bigger mechanic and dynamic effects than asymmetrical faults, asymmetrical faults is more likely to occur.[13]

*C. Asymmetrical faults in 3-PPS*

All possible asymmetrical faults in 3-PPS are listed below, and necessary information is provided.

*1) One-phase ground fault*

Only faulted phase voltage is zero, and other phase currents is also zero. Fault current is given in Eq. 23.

$$I_{a1} = \frac{E_{a1}}{Z_0 + Z_1 + Z_2} \tag{23}$$

Phase current of faulted phase is equals three times positive sequence fault current as in Eq. 24.

$$I_a = 3. I_{a1} \tag{24}$$

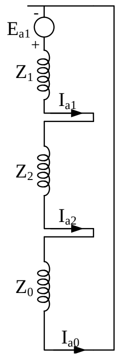


**Figure 8.** One-phase ground fault in 3-PPS.

All relevant equations of one-phase ground fault can be derived from Figure 8.

*2) Two-phase ground fault*

Two phases contact each other with ground. Two-phase fault means two-phase voltages equals to zero, faulted phase current is not zero. Fault current is calculated in Eq. 25.

$$I_{a1} = \frac{E_{a1}}{Z_1 + \frac{Z_2.Z_0}{Z_2 + Z_0}} \tag{25}$$

Phase current of faulted phases are calculated using Eq. 26, 27.

$$V_{a0} = V_{a1} = V_{a2} = E_{a1} - I_{a1}.Z_1 \tag{26}$$

$$\begin{bmatrix} \boldsymbol{V_{a0}} \\ \boldsymbol{V_{a1}} \\ \boldsymbol{V_{a2}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{E_{a1}} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \boldsymbol{Z_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{Z_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{Z_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{I_{a0}} \\ \boldsymbol{I_{a1}} \\ \boldsymbol{I_{a2}} \end{bmatrix} \tag{27}$$

Connection of sequence circuits is given in Figure 9.

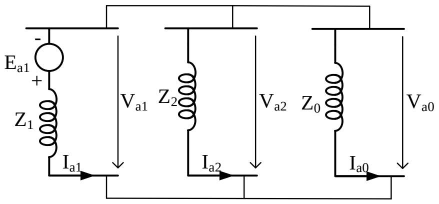


**Figure 9.** Two-phase ground fault in 3-PPS.

*3) Two-phase fault*

Two phases contact each other. Faulted two phase voltages are equal. Faulted phases currents are equal in magnitude, but signs are different. Other phase current is zero. Thus, symmetrical components of faulted phase voltages and currents are given in Eq. 28-32.

$$V_{a0} = \frac{1}{3}V_a + \frac{2}{3}V_b \tag{28}$$

$$V_{a1} = V_{a2} = \frac{1}{3}V_a - \frac{2}{3}V_b \tag{29}$$

$$I_{a0} = 0 \tag{30}$$

$$I_{a1} = \frac{1}{3}I_c(a^2 - a) \tag{31}$$

$$I_{a2} = \frac{1}{3}I_c(a - a^2) \tag{32}$$

Symmetrical component of phase current is Eq. 33.

$$I_{a1} = \frac{E_{a1}}{Z_1 + Z_2} \tag{33}$$

Connection of sequence circuits is given in Figure 10.

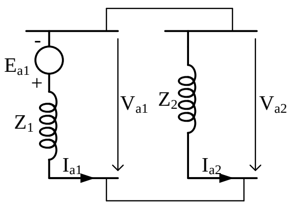


**Figure 10.** Two-phase fault in 3-PPS.

*D.Asymmetrical faults in 12-PPS*

Asymmetrical faults are more frequent faults, although the magnitude of fault current is less than symmetrical fault current. In 12-PPS, asymmetrical faults are twenty one type. For easy of read, only formulations are provided.

*1) One-phase ground fault*

Only faulted phase voltage is zero, and the currents of the rest phases are also zero. Fault current is calculated in Eq. 34, 35.

$$I_{a0} = I_{a1} = \cdots = I_{a10} = I_{a11} = \frac{I_a}{12} \tag{34}$$

$$I_{a1} = \frac{E_{a1}}{Z_0 + Z_1 + \cdots + Z_{10} + Z_{11}} \tag{35}$$

Connection of sequence circuits is given in Figure 11.

*2) Two-phase ground fault*

In two-phase ground fault only faulted two phase voltages equal to zero, and currents of the rest of phases are also zero. Connection of sequence circuits is shown in Figure 12.

*3) Two-phase fault*

Faulted two phase currents are zero. Fault currents and voltages are calculated in Eq. 38, 39.

$$V_{a1} + V_{a3} + V_{a5} + V_{a7} + V_{a9} + V_{a11} = 0 \tag{38}$$

$$I_{a1} = \frac{E_{a1}}{Z_1 + Z_3 + Z_5 + Z_7 + Z_9 + Z_{11}} \tag{39}$$

Connection of sequence circuits is shown in Figure 13.

*4) Three-phase ground fault*

In three-phase ground fault only faulted phase voltages equal to zero, and currents of the rest of phases are also zero. Connection of sequence circuits is shown in Figure 14.

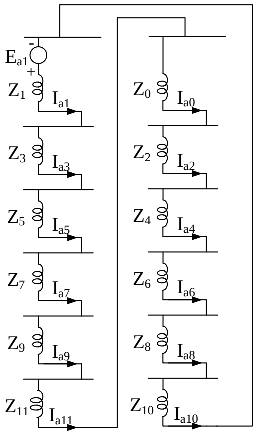


**Figure 11.** One-phase ground fault in 12-PPS

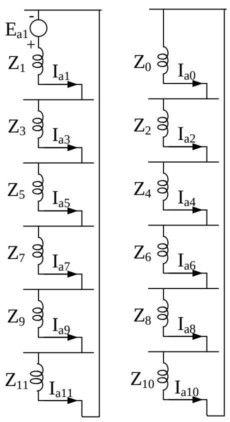


**Figure 12.** Two-phase ground fault in 12-PPS.

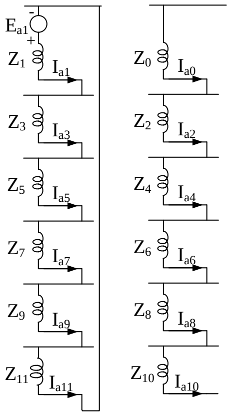


**Figure 13.** Two-phase fault in 12-PPS.

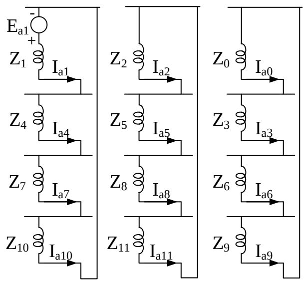


**Figure 14.** Three-phase ground fault in 12-PPS.

*5) Three-phase fault*

Faulted phase voltages are equal, and summation of the faulted phase currents is zero. Currents of the rest of phases equal to zero. Connection of sequence circuits is shown in Figure 15.

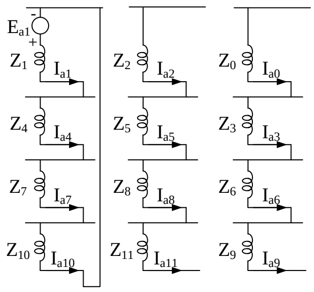


**Figure 15.** Three-phase fault in 12-PPS.

*6) Four-phase ground fault*

Faulted phase voltages, and currents of the rest of phases are zero. Connection of sequence circuits is shown in Figure 16.

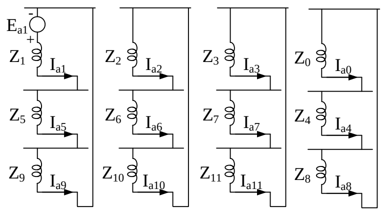


**Figure 16.** Four-phase ground fault in 12-PPS.

*7) Four-phase fault*

Connection of sequence circuits is shown in Figure 17.

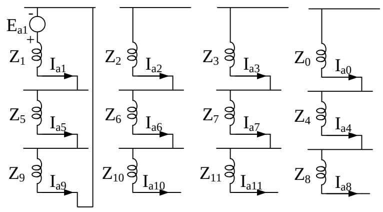


**Figure 17.** Four-phase fault in 12-PPS.

*8) Other asymmetrical faults*

Asymmetrical faults in 12-PPS are very complex. Therefore, only more frequent faults are provided in this study. Further information about fault analysis in 12-PPS can be found in [12].

## 4. Results And Discussion

Fault calculation in MPPS is done by using the example 3-PPS after converting three-phase transmission lines to twelve-phase. The example power system is given in appendix. Before fault calculation all bus voltages were calculated, and load flow details are given in [12], [15], [16].

### *E. 3-PPS faults*

*1) Symmetrical fault currents*

Three-phase ground fault currents are calculated for given for balanced and unbalanced load condition. The faults currents is given in Table 1.

**Table 1.** Three-phase ground fault in 3-PPS.

| **Bus No** | **Phase** | **Balanced loading** | | **Unbalanced loading** | |
|---|---|---|---|---|---|
| | | **Fault current [pu]** | **Angle [°]** | **Fault current [pu]** | **Angle [°]** |
| **1** | a | 8,350 | 178,800 | 7,694 | 178,660 |
| | b | 8,350 | 58,785 | 8,444 | 57,467 |
| | c | 8,351 | -61,160 | 8,468 | -60,050 |
| **2** | a | 8,263 | 178,790 | 6,766 | 178,530 |
| | b | 8,263 | 58,773 | 8,595 | 54,961 |
| | c | 8,263 | -61,170 | 8,612 | -57,540 |
| **3** | a | 8,400 | 178,750 | 7,916 | 178,630 |
| | b | 8,399 | 58,733 | 8,480 | 57,708 |
| | c | 8,400 | -61,210 | 8,485 | -60,320 |
| **4** | a | 8,241 | 178,750 | 8,080 | 178,690 |
| | b | 8,240 | 58,754 | 8,272 | 58,332 |
| | c | 8,241 | -61,190 | 8,271 | -60,880 |
| **5** | a | 8,197 | 178,770 | 8,195 | 178,700 |
| | b | 8,196 | 58,749 | 8,195 | 58,711 |
| | c | 8,197 | -61,200 | 8,196 | -61,270 |

Phase currents for all buses are nearly the same.

*2) Asymmetrical fault currents*

Single-phase ground fault currents are given in Table 2.

**Table 2.** Single-phase ground fault in 3-PPS.

| Bus No | Phase | Balanced loading | | Unbalanced loading | |
|---|---|---|---|---|---|
| | | Fault current [pu] | Angle [°] | Fault current [pu] | Angle [°] |
| **1** | a | 11,203 | 178,500 | 10,613 | 178,270 |
| **2** | a | 10,559 | 178,400 | 8,968 | 178,010 |
| **3** | a | 11,177 | 178,430 | 10,743 | 178,280 |
| **4** | b | 10,610 | 58,360 | 10,602 | 58,350 |
| **5** | a | 11,035 | 178,470 | 11,038 | 178,390 |

The magnitude of single-phase ground fault current is greater than three-phase ground fault current, but effects of fault is less than single-phase ground fault.

*F. 12-PPS faults*

*1) Symmetrical fault currents*

Twelve-phase ground fault currents are calculated for balanced and unbalanced load condition. The faults currents is given in

Table 3.

**Table 3.** Twelve-phase ground faults in 12-PPS.

| Bus No | Phase | Balanced loading | | Unbalanced loading | |
|---|---|---|---|---|---|
| | | Fault current [pu] | Angle [°] | Fault current [pu] | Angle [°] |
| 1 | a | 1,972 | 149,640 | 1,452 | 149,170 |
| | b | 1,972 | 119,640 | 2,080 | 119,320 |
| | c | 1,969 | 89,599 | 2,014 | 90,697 |
| | d | 1,969 | 59,599 | 1,992 | 60,908 |
| | e | 1,972 | 29,547 | 1,967 | 29,039 |
| | f | 1,972 | -0,453 | 1,950 | -1,427 |
| | g | 1,972 | -30,350 | 1,996 | -31,230 |
| | h | 1,972 | -60,350 | 2,001 | -61,870 |
| | i | 1,969 | -90,400 | 1,966 | -91,470 |
| | j | 1,969 | -120,400 | 1,988 | -121,600 |
| | k | 1,972 | -150,400 | 2,011 | -153,200 |
| | l | 1,972 | 179,540 | 2,027 | 177,250 |
| 2 | a | 1,809 | 149,680 | 0,891 | 148,220 |
| | b | 1,809 | 119,680 | 2,023 | 119,940 |
| | c | 1,806 | 89,637 | 1,888 | 92,765 |
| | d | 1,806 | 59,630 | 1,849 | 63,169 |
| | e | 1,809 | 29,580 | 1,806 | 28,910 |
| | f | 1,809 | -0,417 | 1,777 | -2,027 |
| | g | 1,809 | -30,310 | 1,875 | -31,220 |
| | h | 1,809 | -60,310 | 1,886 | -62,420 |
| | i | 1,806 | -90,360 | 1,805 | -91,450 |
| | j | 1,806 | -120,300 | 1,845 | -121,700 |
| | k | 1,809 | -150,400 | 1,891 | -155,200 |
| | l | 1,809 | 179,580 | 1,918 | 175,690 |
| 3 | a | 2,059 | 149,570 | 1,674 | 148,350 |
| | b | 2,059 | 119,570 | 2,140 | 118,990 |
| | c | 2,056 | 89,532 | 2,087 | 89,901 |
| | d | 2,056 | 59,532 | 2,071 | 60,029 |

| Bus No | Phase | Balanced loading | | Unbalanced loading | |
|---|---|---|---|---|---|
| | | Fault current [pu] | Angle [°] | Fault current [pu] | Angle [°] |
| | e | 2,059 | 29,483 | 2,055 | 28,651 |
| | f | 2,059 | -0,517 | 2,043 | -1,699 |
| | g | 2,059 | -30,420 | 2,077 | -31,480 |
| | h | 2,059 | -60,420 | 2,082 | -61,940 |
| | i | 2,056 | -90,460 | 2,054 | -91,710 |
| | j | 2,056 | -120,400 | 2,071 | -121,800 |
| | k | 2,059 | 150,500 | 2,091 | -152,900 |
| | l | 2,059 | 179,480 | 2,103 | 177,420 |
| 4 | a | 1,776 | 149,650 | 1,666 | 148,530 |
| | b | 1,776 | 119,660 | 1,800 | 118,860 |
| | c | 1,773 | 89,611 | 1,783 | 89,120 |
| | d | 1,773 | 59,611 | 1,778 | 59,158 |
| | e | 1,776 | 29,552 | 1,776 | 28,685 |
| | f | 1,776 | -0,448 | 1,772 | -1,437 |
| | g | 1,776 | -30,330 | 1,781 | -31,320 |
| | h | 1,776 | -60,330 | 1,783 | -61,480 |
| | i | 1,773 | -90,380 | 1,774 | -91,440 |
| | j | 1,773 | -120,300 | 1,779 | -121,400 |
| | k | 1,776 | -150,400 | 1,786 | -151,800 |
| | l | 1,776 | 179,550 | 1,790 | 178,270 |
| 5 | a | 1,737 | 149,650 | 1,740 | 148,630 |
| | b | 1,737 | 119,650 | 1,736 | 118,700 |
| | c | 1,733 | 89,592 | 1,735 | 88,635 |
| | d | 1,733 | 59,592 | 1,735 | 58,631 |
| | e | 1,737 | 29,524 | 1,737 | 28,634 |
| | f | 1,737 | -0,476 | 1,738 | -1,364 |
| | g | 1,737 | -30,340 | 1,737 | -31,300 |
| | h | 1,737 | -60,340 | 1,737 | -61,290 |

| Bus No | Phase | Balanced loading | | Unbalanced loading | |
|---|---|---|---|---|---|
| | | Fault current [pu] | Angle [°] | Fault current [pu] | Angle [°] |
| | i | 1,733 | -90,400 | 1,736 | -91,350 |
| | j | 1,733 | -120,400 | 1,735 | -121,300 |
| | k | 1,737 | -150,400 | 1,737 | -151,300 |
| | l | 1,737 | 179,520 | 1,737 | 178,650 |

These fault phase currents are less than symmetrical fault currents in 3-PPS.

*2) Asymmetrical fault currents*

Single-phase ground fault currents are given in Table 4.

**Table 4.** Single-phase ground faults in 12-PPS.

| Bus No | Phase | Balanced loading | | Unbalanced loading | |
|---|---|---|---|---|---|
| | | Fault current [pu] | Angle [°] | Fault current [pu] | Angle [°] |
| 1 | a | 7,902 | 88,620 | 7,338 | 147,850 |
| 2 | a | 3,497 | 148,790 | 1,825 | 147,510 |
| 3 | a | 8,047 | 148,670 | 7,648 | 147,570 |
| 4 | a | 3,577 | 148,610 | 3,445 | 147,630 |
| 5 | a | 7,264 | 148,720 | 7,269 | 147,800 |

The magnitude of single-phase ground fault current in 12-PPS is greater than twelve-phase ground fault current, but effects of fault is less than single-phase ground fault.

## 5. Conclusion

In this study, symmetrical components of 12-PPS are defined and used in fault calculation. All required fault current equations are provided in detail. Using phase coordinate method, load flow analysis is done before and after fault calculation in order to determine bus voltages, and they used in fault calculation. Phase coordinate method is also used for modeling 12-PPS [12], [16]. Maximum fault current found at bus 3 in 3-PPS, and at bus 3 in 12-PPS for symmetrical fault. Maximum fault current found at bus 1 in 3-PPS and at bus 3 in 12-PPS for asymmetrical fault. Unbalanced loading condition was not affected the maximum fault current location. It is seen that fault currents decrease as phase numbers increase.

## Appendix-A

The example 3-PPS is given in Figure A.1.

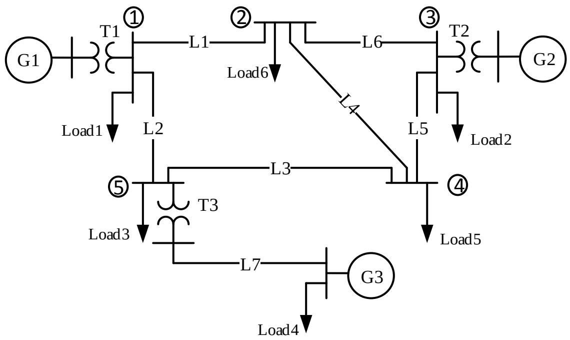


**Figure A.1.** Oneline diagram of 3-PPS.

Transmission line and transformer data used in 3-PPS can be found in Table A.1, and A.2, respectively.

**Table A.1** Transmission line parameters (3-PPS)

| Line | ± sequence components | | | Zero sequence components | | |
|---|---|---|---|---|---|---|
| No. | $R_1$ | $X_1$ | $B_1$ | $R_0$ | $X_0$ | $B_0$ |
| 1 | 0,00011 | 0,0077 | 0,000540 | 0,00036 | 0,0374 | 0,000260 |
| 2 | 0,00023 | 0,0154 | 0,000270 | 0,00073 | 0,0748 | 0,000130 |
| 3 | 0,00014 | 0,0096 | 0,000430 | 0,00044 | 0,0467 | 0,000210 |
| 4 | 0,00058 | 0,0385 | 0,000100 | 0,00180 | 0,1870 | 0,000053 |
| 5 | 0,00017 | 0,0115 | 0,000360 | 0,00053 | 0,0561 | 0,000170 |
| 6 | 0,00029 | 0,0192 | 0,000210 | 0,00090 | 0,0935 | 0,000100 |
| 7 | 0,00950 | 0,6310 | 0,000006 | 0,02900 | 3,0650 | 0,000003 |

**Table A.2.** Transformer parameters (3-PPS)

| Transformer Number | Connection Type | X |
|---|---|---|
| 1 | Delta-Wye | 0,05 |
| 2 | Delta-Wye | 0,05 |
| 3 | Delta-Wye | 0,05 |

Note: Transformer neutral points are solidly earthed.

The example 3-PPS was converted to 12-PPS in order to use for fault calculation in MPPS. Transformers are 3-phase to 12-phase transformer. Detail information about phase shifting transformer is given in [16]. Transmission lines in 12-PPS are modified from 3-PPS, details are in Table A.3. Transformer data used in 12-PPS can be found in Table A.4.

**Table A.3. Transmission line parameters (12-PPS)**

| Line No. | Phase Number | ± sequence components | | | Zero sequence components | | |
|---|---|---|---|---|---|---|---|
| | | $R_1$ | $X_1$ | $B_1$ | $R_0$ | $X_0$ | $B_0$ |
| 1 | 12 | 0,0051 | 0,207 | 0,000020 | 0,0300 | 1,542 | 0,0000060 |
| 2 | 12 | 0,0102 | 0,414 | 0,000010 | 0,0600 | 3,084 | 0,0000033 |
| 3 | 12 | 0,0063 | 0,259 | 0,000016 | 0,0371 | 1,927 | 0,0000053 |
| 4 | 12 | 0,0255 | 1,037 | 0,000004 | 0,1500 | 7,710 | 0,0000013 |
| 5 | 12 | 0,0076 | 0,311 | 0,000013 | 0,0447 | 2,313 | 0,0000044 |
| 6 | 12 | 0,0127 | 0,518 | 0,000008 | 0,0748 | 3,855 | 0,0000026 |
| 7 | 3 | 0,0095 | 0,631 | 0,000006 | 0,0290 | 3,065 | 0,0000030 |

Generators and loads are the same for both 3-PPS and 12-PPS, given in table 4 and 5, respectively.

**Table A.4.** Generator parameters

| Generator No. | ± sequence components | | Zero sequence components | |
|---|---|---|---|---|
| | $R_1$ | $X_1$ | $R_0$ | $X_0$ |
| 1 | 0,0 | 0,268 | 0,0 | 0,129 |
| 2 | 0,0 | 0,212 | 0,0 | 0,106 |
| 3 | 0,0 | 0,212 | 0,0 | 0,106 |

**Table A.5.** Load data

| **Load Number** | **Balanced** (MVA) |
|---|---|
| 1 | 25+j15 |
| 2 | 30+j19 |
| 3 | 35+j22 |
| 4 | 18+j7 |
| 5 | 30+j19 |
| 6 | 40+j25 |

For unbalanced loading condition in 3-PPS, apparent power of Load 5 was changed to $S_a = 10 + j6, S_b = 12 + j8,$
$S_c = 8 + j5$ in MVA. However, another loads are left the same as in Table A.5. For unbalanced loading condition in 12-PPS, apparent power of Load 4 was changed to $S_a = 7 + j1,$ $S_b = 8 + j2, S_c = 3 + j3$ in MVA. However, another loads are left the same as in Table A.5.

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