Investigation of the effect of stator and rotor pole ratios on torque and efficiency in Inverted Switched Reluctance Motor

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In this study, the effect of rotor pole ratio coefficient and stator pole ratio coefficient on torque and efficiency in 18/12 pole 3 phase Inverted Switched Reluctance Motor have been investigated. For this purpose, dimension, electrical and magnetic parameters of 18/12 pole 3-phase Inverted Switched Reluctance Motor have been determined. These parameters were obtained in our previous study by the heuristic optimization method according to the determined motor characteristics (efficiency, torque, etc.). The other dimensions of motor have been kept constant and the effect of rotor pole ratio coefficient and stator pole ratio coefficient on the torque and efficiency have been determined according to the fixed and calculated total ampere-turns values. As a result of the analyzes performed in MATLAB, it has been observed that there have certain relationships between these coefficients and torque-efficiency. However, since the effects of rotor pole ratio coefficient and stator pole ratio coefficient on torque and efficiency are not exactly linear, it can seen that these coefficients will be considered as optimization problems in future studies and optimization of these coefficients will be beneficial in terms of motor performance.

1. Introduction

The Switched Reluctance Motor (SRM) is a simple-structured electric machine consisting of stator and rotor salient poles and without winding in the rotor. SRMs are converts mechanical energy to electrical energy with reluctance force. The difference of Inverted Switched Reluctance Motors (ISRM) from SRMs is the change of rotor and stator places. All the operating features of ISRMs are exactly the same as a normal SRM. The ability to produce a continuous torque depends on the position of the rotor pole relative to the stator pole and the suitable phase windings being fed respectively. A power electronics circuit and control mechanism are also required to supply this. Although SRMs were introduced as a principle in the 1800s, it was possible to drive the motor in the early 1960s due to the developments in power electronics. Especially the studies on this motor have been concentrated after 1980. In recent years, significant developments have been achieved in SRMs. In parallel with the developments in software, design and technology, the reason for tending to SRM is that the structure of these motors is simple, reliable and stable, almost independent from the working environment, can be operated at low and high speeds [1-3]. In addition, the cost of production and maintenance-repair costs are low [4-5].
SRMs are widely used in automobile, railway, aviation and marine industries, electrical household appliances, fans, pumps and elevators [6-8].

In order to rotate the SRM, the number of stator and rotor poles are different. If high speed is desired from the SRMs, the number of rotor poles must be smaller than number of stator poles, and if the high torque is desired, the number of rotor poles is selected to be close to the number of stator poles. Stator and rotor pole combinations used in SRMs are shown in Table 1 [9-10].

**Table 1.** Pole combinations and the corresponding number of phases

<table>
<thead>
<tr>
<th>Ns/Nr</th>
<th>6/4</th>
<th>6/8</th>
<th>12/8</th>
<th>18/12</th>
<th>8/6</th>
<th>8/10</th>
<th>16/12</th>
<th>24/18</th>
<th>10/6</th>
<th>10/12</th>
<th>12/10</th>
<th>14/12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of phases</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

SRMs can be produced in different phase numbers starting from a phase. However, 3 and 4 phase SRMs are widely produced for commercial. In Figure 1, one, two, three and four phase SRM types are shown. The structure of one-phase SRMs is very simple, the machine with the least connection between the drive circuit and the motor. Although they are attractive for very high speed applications, these motors cannot run by themselves, and they have more vortex losses. This type of motors 2/2 and 4/4 poles can be made but in this case, the produced torque is very ripple [11]. The problem of running in two-phase SRMs was overcome by an asymmetric structure in the rotor poles. But the ripple in the torque is still high. Three-phase SRMs are generally made of 6/4 and 12/8 poles. They have a high starting torque. In these motors, the ripple in the torque is decreasing but continues. Four-phase SRMs are made of 8/6-pole and five-phase SRMs are made of 10/8 poles, and are recommended to reduce the torque ripple. However, the number of connections between the driver circuit and the motor increases as more power electronics are used [12-13].

![Figure 1. SRM types according to stator and rotor poles number](image)

a) One-phase 4/4 pole SRM, b) Two-phase 4/2 pole SRM, c) Three-phase 6/4-pole SRM, d) Four-phase 8/6-pole SRM
SRMs has basically two problems. The first is the acoustic noise caused by the torque ripple and radial forces when SRM is operating. The other is the determination of the motor position. The forces in the direction of the radius and acting on the rotor poles cause vibrations in the bearings. These vibrations are perceived as acoustic noise in the stator. These vibrations to the bearings can cause the bearings to malfunction after a period of time. Vibration and acoustic noise can be reduced thanks to innovations in control systems and motor design.

SRMs are very advantageous compared to other AC and DC motors because of their simple and robust structure, high torque to weight ratio and high fault tolerance [14-16]. In SRMs, there are only windings in the stator and no windings in the rotor and the absence of magnets in the structures make these motors advantageous in terms of cost and production. In addition, the high power output of the motors in SRMs is a powerful alternative to AC and DC motors, especially due to the high efficiency and cost advantages of the system [17]. Table 2 shows the comparison of SRM with other motors.

Table 2. The comparison of SRM with other motors

<table>
<thead>
<tr>
<th></th>
<th>Induction Motor</th>
<th>Synchronous Motor</th>
<th>DC Motor</th>
<th>Step Motor</th>
<th>SRM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage</td>
<td>Alternating voltage</td>
<td>Alternating voltage</td>
<td>Direct voltage</td>
<td>Direct voltage</td>
<td>Direct voltage</td>
</tr>
<tr>
<td>Excitation voltage</td>
<td>Not required</td>
<td>Direct voltage</td>
<td>Direct voltage</td>
<td>Not required</td>
<td>Not required</td>
</tr>
<tr>
<td>Driver</td>
<td>Necessary in variable speed applications</td>
<td>Necessary in variable speed applications</td>
<td>Necessary in variable speed applications</td>
<td>Always necessary</td>
<td>Always necessary</td>
</tr>
<tr>
<td>Operation and</td>
<td>Low</td>
<td>Low</td>
<td>Average</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>maintenance costs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>Low</td>
<td>High</td>
<td>Average</td>
<td>Average</td>
<td>Average</td>
</tr>
<tr>
<td>Driver cost</td>
<td>High</td>
<td>High</td>
<td>Average</td>
<td>Average</td>
<td>High</td>
</tr>
<tr>
<td>Efficiency</td>
<td>High</td>
<td>High</td>
<td>Average</td>
<td>Average</td>
<td>High</td>
</tr>
</tbody>
</table>

In SRM design, rotor and stator pole numbers, pole width, pole arcs are directly effective on motor performance. There are many studies on these parameters in literature. Choi et al. proposed a new stator pole face with an irregular air gap profile and a pole shoe attached to the lateral face of the rotor pole to reduce unwanted torque ripple of a Switched Reluctance Motor. The effects of the design parameters were simulated using the finite element method and optimized by applying the response surface methodology (RSM). By optimizing the pole shoe and the air gap profile, the torque wave was reduced to 23% of its value before optimization [18]. Arumugam et al. investigated the effect of the pole arc/pole pitch ratio of the rotor and stator on the performance of a Switched Reluctance Motor. They used an analytical method based on two-dimensional finite element analysis and magnetic flux path. They indicated that the range of values that could be used for the pole arc/pole pitch ratio of the rotor could be between 0.3 to 0.45 and the stator could be between 0.35 and 0.5 [19]. Gungor and Uygun have tried to find out an easy way to determine the suitable and acceptable ratio between the stator and rotor tooth width in the Switched Reluctance Motors for torque and output power, smoother magnetic field energy. In order to find the most appropriate ratio, they did 8 different SRM designs and used the finite element method analysis [20].

In this study, the effects of rotor and stator pole ratio coefficients on torque and efficiency in ISRM have been investigated. For this purpose, the electrical and dimensional parameters of ISRM have been calculated in MATLAB considering the non-linearity of the motor. Then, the results for fixed and variable cases of total amperes have been examined.
2. Material And Method

In this study, firstly, dimension, electrical and magnetic parameters of 18/12 pole 3-phase ISRM have been determined. These parameters were obtained in [21] by the heuristic optimization method according to the determined motor characteristics (efficiency, torque, etc.). In the next step, the other dimension parameters of the ISRM were kept constant and the effect of $C_{spr}$ and $C_{rpr}$ coefficients on the torque and efficiency was examined according to the fixed and calculated TAT values. The equations that symbolize the coefficients are given in Equation (1-2). The parameters in the equations are shown in Figure 2. In order for this study to be realistic, it has been checked whether the number of windings per phase obtained will be fitted into in stator slots. Winding results that could not be fit in the slots were eliminated.

Rotor ratio coefficient ($C_{rpr}$) is

$$C_{rpr} = \frac{C}{C + h_r} \tag{1}$$

Stator ratio coefficient ($C_{spr}$) is

$$C_{spr} = \frac{C_{in}}{C_{in} + h_s} \tag{2}$$

![Figure 2. Dimension parameters of ISRM](image)

2.1 Torque Generation in Switched Reluctance Motor

In SRM, the torque generation is based on the conversion of magnetic energy to mechanical energy by taking advantage of the reluctance change between the unaligned position and the aligned position. As shown in Figure 3, when a phase in the SRM is excitation, positive torque is generated positive where the inductance slope is positive, zero in the region where it is fixed, and finally negative torque generated where the slope is negative.

![Figure 3. Variation of the torque generated in SRM due to inductance of a phase](image)
Instantaneous torque in any phase $i$ [22]:

$$T_i = \frac{1}{2} i^2 \frac{\partial L_i(\theta)}{d\theta}$$

(3)

can be calculated by relation. By using the inductance curve in Figure 3 with the expression in Equation (3), the mathematical torque relations in Equation (4) for phase $A$ can be derivation [9]:

$$T_a = \begin{cases} \frac{1}{2} K_i a^2 & 0 \leq \theta \leq \beta_s \\ 0 & \beta_s \leq \theta \leq \beta_r \\ -\frac{1}{2} K_i a^2 & \beta_s \leq \theta \leq \beta_s + \beta_r \\ 0 & \beta_s + \beta_r \leq \theta \leq \alpha_r \end{cases}$$

(4)

The total torque developed by the motor is equal to the sum of the instantaneous torques generated by the individual phases.

$$T_e(\theta, i_a, ..., i_d) = \sum_{i=1}^{q} T_i = \frac{1}{2} \sum_{i=1}^{q} i^2 \frac{\partial L_i(\theta)}{d\theta}$$

(5)

In order to ensure that the instantaneous motor torque is not zero at any rotor position, the $\beta_s$ must be greater than $\alpha_r/q$.

2.2 Deriving Equations of ISRM Dimension and Electrical Parameters

The electrical and dimensional parameters of the ISRM are calculated using Equations (6-25) via MATLAB. The dimension parameters in the equations are shown in Figure 4. In addition, the B-H characterization of the silicone sheet should be taken into account when the nonlinear equations of the ISRM are derived. The B-H characteristic of the silicon sheet is shown in Figure 5. In the ISRM design, the magnetic induction density must not exceed the knee point in nominal operating conditions.

![Figure 4. Dimension parameters of ISRM](image-url)
Figure 5. B-H Characteristic of silicone sheet

The flux density of stator pole ($B_s$) is assumed as equal to $B_{\text{max}}$. Omitting leakage flux and packaging factor, stator pole area ($A_s$) is estimated as [21];

$$A_s = \left(\frac{D}{2} - g\right) \cdot L \cdot \beta_s$$

(6)

where $L$ refers to package length of motor.

Flux in stator pole is

$$\varphi = B_s \cdot A_s$$

(7)

Flux in yoke is

$$\varphi_y = \frac{\varphi}{2} = \frac{B_s \cdot A_s}{2}$$

(8)

Area of yoke is

$$A_y = C_{\text{in}} \cdot L$$

(9)

Flux density of yoke is

$$B_y = \frac{Q_y}{A_y}$$

(10)

Length of stator pole is

$$h_s = \left(\frac{D}{2} - g - \frac{D_{\text{sh}}}{2}\right) - \frac{A_y}{L}$$

(11)

Length of rotor pole is

$$h_r = \frac{D_0}{2} - c - \frac{D}{2}$$

(12)

Magnetic field intensity in air gap is calculated by

$$H_g = \frac{B_g}{4 \cdot \pi \cdot 10^{-7}}$$

(13)

B-H curve is determined as a function then magnetic field density values corresponding to $B_s$, $B_g$, $B_r$, and $B_{ro}$ are obtained by this function. Consequently, the flux path equations are estimated. Thus, path length of rotor region is

$$l_r = h_r + \frac{C}{2}$$  \hspace{1cm} (14)

Path length of rotor yoke is

$$l_{ry} = \frac{(2 \cdot \pi \cdot NP \cdot 0.5 \cdot (D_0 - C))}{N_s}$$  \hspace{1cm} (15)

Path length of air gap region is

$$l_g = g$$  \hspace{1cm} (16)

Path length of stator region is

$$A_s = \left(\frac{D}{2} - g\right) \cdot L \cdot \beta_s$$  \hspace{1cm} (17)

Path length of stator core is

$$l_s = (0.25 \cdot D - 0.5 \cdot g + 0.5 \cdot h_s - 0.25 \cdot D_{sh})$$  \hspace{1cm} (18)

In this situation, total ampere-turns (TAT) can be calculated as

$$TAT = 2 \cdot (H_s \cdot l_s + H_r \cdot l_r) + \left(\frac{B_g \cdot A_g}{P_a}\right) + (H_{ry} \cdot l_{ry} + H_{sy} \cdot l_{sy})$$  \hspace{1cm} (19)

where $P_a$ refers to permeability.

Turn per phase is

$$T_{ph} = \frac{TAT}{I_p}$$  \hspace{1cm} (20)

where $I_p$ refers to rated current.

Current rate is

$$I_p = \frac{TAT}{T_{ph}}$$  \hspace{1cm} (21)
Accurate estimation of inductances, $L_a$ and $L_u$ is crucial for reliable design. High estimation error of inductances causes the torque to be calculated wrong. Therefore, to calculate the inductances ($L_a$ and $L_u$) at aligned and unaligned positions, respectively 2 base and 7 base flux-paths are selected. The rest of the fluxes can cause raising the estimation complexity and variation of the result will be trivial, as shown in Figure 6 and Figure 7.

**Figure 6.** Flux paths for inductance calculation of ISRM at aligned position (La) [21].

**Figure 7.** Flux paths for inductance calculation of ISRM at unaligned position (Lu) [21].

The inductance values obtained by using the flux paths are transferred to the energy calculation. Energy calculation at aligned and unaligned position, average torque of pole pair, and average torque are all respectively given as:

\[
W_a = \frac{1}{2} \cdot L_a \cdot i_p^2
\]

(22)

\[
W_u = \frac{1}{2} \cdot L_u \cdot i_p^2
\]

(23)

\[
T_{pp} = \frac{((W_a - W_u) \cdot N_s \cdot N_r)}{N_{tpp} \cdot \pi}
\]

(24)

\[
T_{average} = T_{pp} \cdot N_{tpp} \cdot 0.5
\]

(25)
3. Results and Discussion

In the first case, TAT value calculated from Equation (19) has not been calculated and it has been taken constant at different intervals from 4000 AT to 6000 AT and torque and efficiency changes were examined according to $C_{spr}$ and $C_{rpr}$ values. Figures 8a and 8b show the effect of $C_{spr}$ and $C_{rpr}$ ratios on the generated torque. As it is seen in the figure as the $C_{spr}$ ratio increased, the torque increased but the torque decreased as the $C_{rpr}$ ratio increased. In addition, when Figure 8a is carefully examined, the torque is not calculated at 5000 AT and above values because the windings do not fit into the slots with these $C_{spr}$ values.

![Figure 8](image)

**Figure 8.** Torque vs pole ratio coefficients for different ampere-turns

Figures 9a and 9b show the effects of $C_{spr}$ and $C_{rpr}$ values on efficiency of ISRM. It has been observed that the increase $C_{spr}$ ratio had a negative effect on efficiency, but $C_{rpr}$ had no effect on efficiency because the total ampere-turns did not change and only copper losses were examined in this study.

![Figure 9](image)

**Figure 9.** Efficiency vs pole ratio coefficients for different ampere-turns

Figures 10 and 11 show the three-dimensional figures of the effects of $C_{spr}$ and $C_{rpr}$ in the nominal current (75A) on the torque and efficiency, respectively. When Figure 10 is examined, it is observed that high torque can be obtained at high values of $C_{spr}$ and low values of $C_{rpr}$. Figure 11 shows that lower $C_{spr}$ values are required for higher efficiency. The higher values of $C_{rpr}$ increase the torque while the lower values increase the efficiency. It is necessary to pay attention to this case and the most appropriate value should be selected according to the desired case.

![Figure 10](image)

**Figure 10.** Torque vs pole ratio coefficients
In the second case, the effect of $C_{spr}$ and $C_{rpr}$ ratios on the torque and efficiency was investigated by taking into account the Equation (19) in different current values (60-100 A). In Figures 12a and 12b, the effect of $C_{spr}$ and $C_{rpr}$ ratios on the torque is shown. Since the torque values in both figures follow the same curve, the torque in each current value was plotted by multiplying 0.01 times according to the previous figure ($\alpha=1, Tort=Tort*\alpha, \alpha=\alpha+0.01$). In this way, the figures can be made clearer and interpreted. When Figure 8a is examined, it is observed that there is no complete linearity between $C_{spr}$ ratio and torque. As the $C_{spr}$ coefficient increased until about 0.55, it was observed that the torque value increased as in Figure 4a. However, a negative effect was observed in the torque after 0.55 value of $C_{spr}$. It should also be considered that the number of windings calculated in some current values cannot be fitted in the stator pole slots. For example, torque generation started from 0.45 values of $C_{spr}$ for $Ip=60$ A and started at 0.3 for $Ip = 80$ A. In Figure 12b, it has been observed that the ISRM starts to generate torque after 0.44 value of $C_{rpr}$ and the torque decreased as the ratio increased as in Figure 8b.

Figures 13a and 13b show the effect of $C_{spr}$ and $C_{rpr}$ on efficiency of ISRM. Both naturally have the same values as $C_{spr}$ and $C_{rpr}$ in Figures 12a and 12b (horizontal axis is the same). As is also observed from the figures, the efficiency increased as the ratio increased in both. These results were observed to be completely opposite to the fixed TATs. However, it should not be forgotten that only copper losses are investigated in this study.
Figures 14 and 15 show the three-dimensional figures of the effects of $C_{spr}$ and $C_{rpr}$ with respect to the torque and efficiency, respectively, according to the nominal current. Considering the limitations described in figures previous to $C_{spr}$ and $C_{rpr}$ when Figure 14 is examined, the high value of $C_{spr}$ and the low value of $C_{rpr}$ should be selected for high torque. This result is the same as the fixed TAT except for the limitations. When Figure 15 is examined, high values of both $C_{spr}$ and $C_{rpr}$ should be selected for high efficiency by taking into account the limitations that appear. However, these ratios should be selected by taking into consideration torque-efficiency relationship.

![Figure 14. Torque versus pole ratio coefficients](image)

![Figure 15. Efficiency versus pole ratio coefficients](image)

4. Conclusions

In this study, the effect of stator and rotor pole ratio coefficients on torque and efficiency have been investigated for two case which constant total ampere-turns and variable total ampere-turns. The results obtained for the case which TAT is kept constant are listed as follows.

It has been observed that torque increased as $C_{spr}$ value increased and the torque decreased as $C_{rpr}$ value increased.

As the $C_{spr}$ value increased, the efficiency of ISRM decreased by approximately between 0.3% and 0.95% for fixed TATs. It has been observed that there is a partly change between $C_{rpr}$ values and efficiency efficiency. Because the total ampere-turns did not change and only copper losses have been examined in this study.

The results obtained for the case in which TAT is calculated and for different current variations are listed as follows.
It has been observed that there is no exactly linear relationship between \( C_{spr} \) values and torque. As the \( C_{spr} \) coefficient increased until about 0.55, it has been observed that the torque value increased. However, a negative effect has been observed in the torque after 0.55 value of \( C_{spr} \). It has been observed that the ISRM starts to generate torque after the 0.44 value of \( C_{spr} \) and the torque of ISRM decreased as the ratio increased.

As the \( C_{spr} \) value increased, the efficiency increased by approximately between 0.5% and 0.8% according to different currents. As the \( C_{spr} \) value increased, the efficiency of ISRM increased by approximately 1.68% for different currents.

In this paper, it is observed that stator and rotor pole ratio coefficients had an effect on the output parameters of the ISRM. Stator and rotor pole ratio coefficients must be taken into account before starting the ISRM design. The effect of stator and rotor pole ratio coefficients on the desired motor performance is presented. As a result, if these coefficients are determined as an optimization problem, it can be stated that appropriate results can be obtained in the torque and efficiency of the ISRM.

5. References


