



Implementation of the Network-Based Moving Sliding Mode Control Algorithm to the Rotary Inverted Pendulum System

Muhammet AYDIN^{1*}, Oğuz YAKUT², Hakan TUTUMLU³

^{1,2} Firat University, Faculty of Engineering, Mechatronics Engineering, ELAZIG/TURKEY muhammeta@firat.edu.tr, oyakut@firat.edu.tr

³ Firat University, Faculty of Engineering, Mechanical Engineering, ELAZIG/TURKEY htutumlu@firat.edu.tr

ARTICLE INFO

Article history:

Received 29 Nov 2018

Received in revised form 2 Jan 2019

Accepted 6 Jan 2019

Available online 23 May 2019

Keywords:

Rotary inverted pendulum,
Moving sliding mode control,
Artificial neural networks

* Corresponding author.

E-mail address:

muhammeta@firat.edu.tr

ABSTRACT

In this study, the most preferred in control applications rotary inverted pendulum system (rip) is dealt. The coordinates of the center of gravity of the rip elements were found and the total kinetic and potential energies of the system were obtained. Lagrange function has been formed by using kinetic and potential energy expressions. Expressions giving the equations of motion of the system have been found by taking into consideration the Lagrange method. Using the state variables, the pendulum angle of the system has been controlled by the moving sliding mode control method via the program written in Matlab. The slope of the sliding surface is calculated by artificial neural networks. Optimum values of weight and bias coefficients of artificial neural networks are found by using the genetic algorithm. From the results, it has been seen that the pendulum angle reaches to about 25 Nm motor torque and the reference value reaches about 3 seconds and the error is about zero.

2019 Batman University. All rights reserved

1. Introduction

The controllability of the inverted pendulum system has an important place in the application of control techniques. The inverted pendulum problem is one of the most appropriate systems to be controlled to provide training in control engineering. There are inverted pendulum systems in different structures that have been developed and controlled up to now. These are single inverted pendulum systems on a cart [1,2], double inverted pendulum systems on a cart [3], single and double rotary inverted pendulum systems [4,5].

One of the most preferred inverted pendulum systems is the rip system. The rip system is an excellent test system for working on the control of indirectly driven nonlinear unstable systems. The production of rip is more preferred in recent times because it is easier and less expensive than the inverted pendulum cart type. The rip system consists of two movable rigid rods. One of this limbs, a horizontal cylindrical arm, is moved by a rotational drive element and the other one is a vertically movable shaft (pendulum). In this system, the aim is to stabilize the pendulum [6,7].

The rotary inverted pendulum system shown in Figure 1 is available in the literature works such as adaptive pid with sliding mode controller, fuzzy control, sliding mode control, pid control based on particle swarm optimization and the sliding mode control with the artificial neural network [8-12].

In this study, the pendulum control was carried out by the moving sliding mode control method by obtaining the nonlinear model of the rotary inverted pendulum system. The slope of the sliding surface has been found by artificial neural network (ann) method. The values of the ann constants are optimized using the genetic algorithm. In the structure of genetic algorithm in matlab, fitness limit, generations and population size were taken as 1e-10, 100 and 50, respectively.

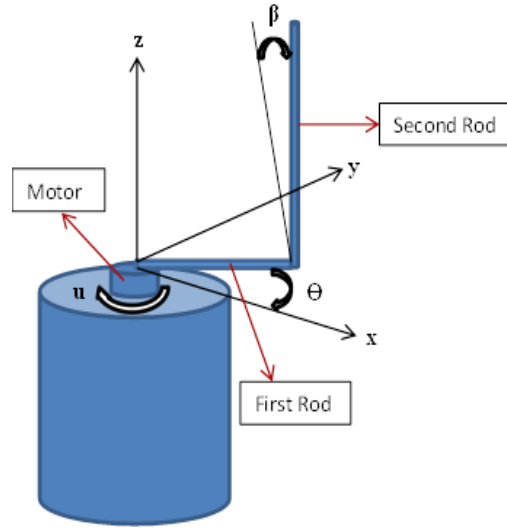


Fig. 1. Rotary inverted pendulum system (Rip system)

2. The Modeling Of The Rotary Inverted Pendulum System

The system in Figure 1 is two degrees of freedom system driven by a single motor. Θ and β are the variable parameters of the system. The coordinate axis of the system is shown in figure 1. If the total kinetic energy of the system is calculated according to this coordinate axis set;

$$T = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}I_1\dot{\theta}^2 + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2) + \frac{1}{2}I_2\dot{\beta}^2 \quad (1)$$

In the equations x_1 and y_1 are coordinates of the center of gravity of the first rod. x_2 , y_2 , and z_2 represent the coordinates of the center of gravity of the second rod. m_1 and m_2 are the masses of each arm. I_1 and I_2 represent the inertia of the rods. The limb sizes according to the centers of gravity are L_1 and L_2 , respectively. Frictional coefficients at joints are b_1 and b_2 . U indicates the control signal that the motor applies.

The equations of the unknown expressions given in Eq. (1) are obtained as follows.

$$x_1 = L_1 \cos\theta \quad (2)$$

$$y_1 = L_1 \sin\theta \quad (3)$$

$$x_2 = x_1 - L_2 \sin\beta \sin\theta \quad (4)$$

$$y_2 = y_1 + L_2 \sin\beta \cos\theta \quad (5)$$

$$z_2 = L_2 \cos\beta \quad (6)$$

If the above expressions are replaced to find the total kinetic energy of the system;

$$T = \frac{1}{2}(m_1 L_1^2 + m_2 L_1 + m_2 L_2^2 \sin^2\beta + I_1)\dot{\theta}^2 + \frac{1}{2}m_2(L_2^2 + I_2)\dot{\beta}^2 + m_2 L_1 L_2 \cos\beta \dot{\theta} \dot{\beta} \quad (7)$$

The potential energy of the system is calculated by using equation (8) and (9) respectively.

$$V = m_2 g z_2 \quad (8)$$

$$V = m_2 g L_2 \cos \beta \quad (9)$$

Here the Lagrange function is constructed as follows.

$$L = T - V \quad (10)$$

$$L = \frac{1}{2} m_2 [(L_1 + L_2^2 \sin^2 \beta) \dot{\theta}^2 + 2L_1 L_2 \cos \beta \dot{\theta} \dot{\beta} + L_2^2 \dot{\beta}^2] + \frac{1}{2} (m_1 L_1^2 + I_1) \dot{\theta}^2 + \frac{1}{2} I_2 \dot{\beta}^2 - m_2 g L_2 \cos \beta \quad (11)$$

The equation of motion for θ is:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = Q_\theta \quad (12)$$

If the expressions in this equation are calculated and substituted, the equation of motion for θ is obtained as follows.

$$(m_1 L_1^2 + I_1 + m_2 L_1 + m_2 L_2^2 \sin^2 \beta) \ddot{\theta} - m_2 L_1 L_2 \cos \beta \ddot{\beta} + m_2 L_1 L_2 \sin \beta \dot{\beta}^2 + 2m_2 L_2^2 \sin \beta \cos \beta \dot{\beta} \dot{\theta} = u - b_1 \dot{\theta} \quad (13)$$

The equation of motion for β is:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\beta}} \right) - \frac{\partial L}{\partial \beta} = Q_\beta \quad (14)$$

After performing the necessary operations in the above equation, the equation of motion for β is obtained as follows.

$$-m_2 L_1 L_2 \cos \beta \ddot{\theta} + (m_2 L_2^2 + I_2) \ddot{\beta} - m_2 L_2^2 \sin \beta \cos \beta \dot{\theta}^2 - m_2 g L_2 \sin \beta = -b_2 \dot{\beta} \quad (15)$$

If the expressions $\ddot{\theta}$ and $\ddot{\beta}$ in the equations of motion are extracted, the following equations are found.

$$\ddot{\theta} = \frac{(m_2 L_2^2 + I_2)(b_1 \dot{\theta} - u + m_2 L_1 L_2 \sin \beta \dot{\beta}^2 + 2m_2 L_2^2 \sin \beta \cos \beta \dot{\beta} \dot{\theta})}{(m_2 L_1 L_2 \cos \beta)^2 - (m_2 L_2^2 + I_2)(m_1 L_1^2 + I_1 + m_2 L_1 + m_2 L_2^2 \sin^2 \beta)} + \frac{m_2 L_1 L_2 \cos \beta (b_2 \dot{\beta} - m_2 L_2^2 \sin \beta \cos \beta \dot{\theta}^2 - m_2 g L_2 \sin \beta)}{(m_2 L_1 L_2 \cos \beta)^2 - (m_2 L_2^2 + I_2)(m_1 L_1^2 + I_1 + m_2 L_1 + m_2 L_2^2 \sin^2 \beta)} \quad (16)$$

$$\ddot{\beta} = \frac{(m_1 L_1^2 + I_1 + m_2 L_1 + m_2 L_2^2 \sin^2 \beta)(b_2 \dot{\beta} - m_2 L_2^2 \sin \beta \cos \beta \dot{\theta}^2 - m_2 g L_2 \sin \beta)}{(m_2 L_1 L_2 \cos \beta)^2 - (m_2 L_2^2 + I_2)(m_1 L_1^2 + I_1 + m_2 L_1 + m_2 L_2^2 \sin^2 \beta)} + \frac{m_2 L_1 L_2 \cos \beta (b_1 \dot{\theta} - u + m_2 L_1 L_2 \sin \beta \dot{\beta}^2 + 2m_2 L_2^2 \sin \beta \cos \beta \dot{\beta} \dot{\theta})}{(m_2 L_1 L_2 \cos \beta)^2 - (m_2 L_2^2 + I_2)(m_1 L_1^2 + I_1 + m_2 L_1 + m_2 L_2^2 \sin^2 \beta)} \quad (17)$$

If expressions in equations are transformed into state variables, following equations are obtained.

$$\theta = x(1) \quad (18)$$

$$\dot{\theta} = x(2) \quad (19)$$

$$\beta = x(3) \quad (20)$$

$$\dot{\beta} = x(4) \quad (21)$$

The moving sliding mode control method is implemented through the program written in Matlab using these state variables for the rip system. The pendulum angle β will be set to the desired zero reference point with this control method.

3. Moving Sliding Mode Control Design

Sliding mode control is a prominent, nonlinear, robust control method that is achieved by switching over time on a predetermined sliding surface with high speed, nonlinear feedback and switching discontinuously [13]. If parameters of a system are variable or cannot be precisely measured because they cannot be modeled and if the system is affected by disturbances, the sliding mode control provides robust control as long as their limit values are known.

The sliding mode controller design process can be considered as a two-step procedure. These steps are to determine the sliding surface and to obtain a rule that allows the sliding surface to be determined [14]. When the sliding surface is reached, the sliding mode which has been insensitivity to external disturbances and parameter uncertainties of the system trajectory begins to slip. The chattering in the sliding mode control applications results from oscillations around the equilibrium point that the system wants to achieve. Which reveals the unmodified high-frequency dynamics of the system.

A sliding mode control statement that has a sign function;

$$u = -k \operatorname{sign}(S) \quad (22)$$

can be written as. Here S is the sliding surface function and is expressed as follows, depending on the error (e) taken from the system response and the time-dependent variation of the error (de).

$$S = Ce + de \quad (23)$$

The saturation function can be used at a sliding surface to reduce because the sign function causes chattering problem in the system. A sliding mode control statement that has a saturation function;

$$u = -k \operatorname{sat}(S) \quad (24)$$

The sliding surface has a certain slope as seen in Figure 2. This slope is specified by the C coefficient in the equation (23).

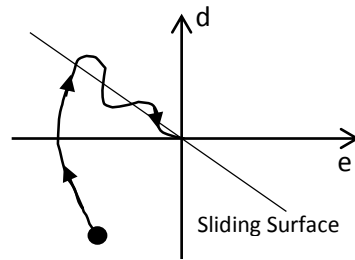


Fig. 2. Sliding surface

The success of the controller is provided by determining the most appropriate value of this specified slope. In this study, the slope coefficients of the sliding mode controller have been taken as moving. Ann has been used to calculate the slope coefficient C . As inputs to the Ann structure, the pendulum angle error and the derivative of the pendulum angle error. The slope coefficient of the sliding surface of the sliding mode control is obtained as the output. The optimum values of the weight and bias coefficients of Ann structure are calculated by using the genetic algorithm. The model for the Ann structure used is shown below.

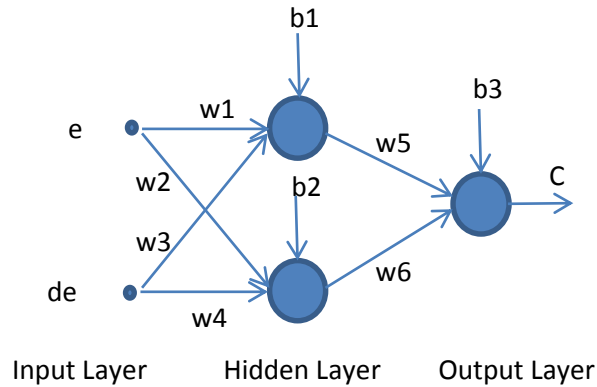


Fig. 3. ANN Structure

Since the obtained coefficient C is variable every time, our control method has a moving sliding surface. The figure below shows the motion of the C coefficient.

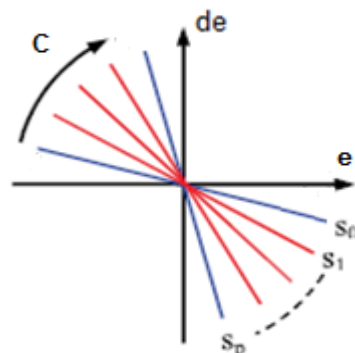


Fig. 4. Variation of the C coefficient

Figure 5 shows the block diagram of the operations performed in the program written in Matlab environment.

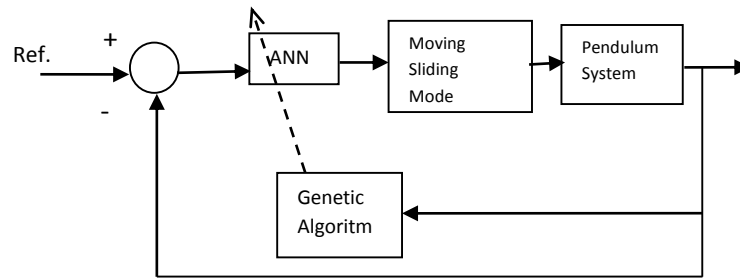


Fig. 5. Block Diagram of Controller Matlab Program

4. Results And Discussion

In Figure 6, the angular position of the first arm connected to the motor changes with time. Initially, it is starting at zero points and changing direction in the first seconds. This situation is an expected result to be able to lift the pendulum up.

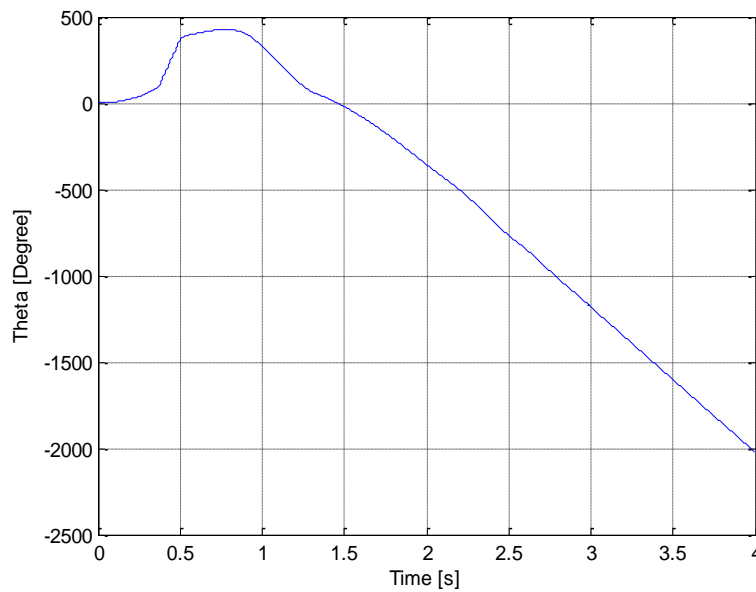


Fig. 6. θ angle change with time

In the graph shown in Fig. 7, the angular velocity of the first arm according to time is shown. In the beginning, it is seen that the angular velocity value is reaching 40 rad / s and fixed at -15 rad / s.

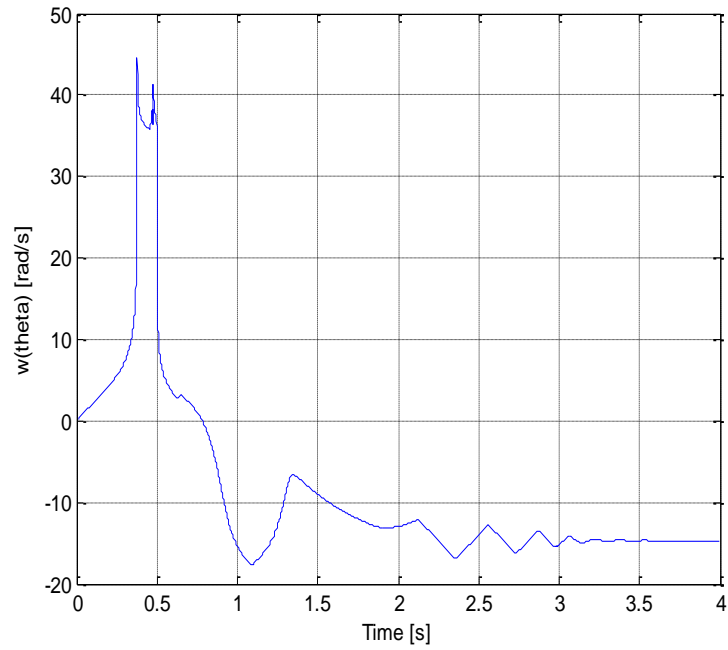


Fig. 7. First rod angular velocity change with time

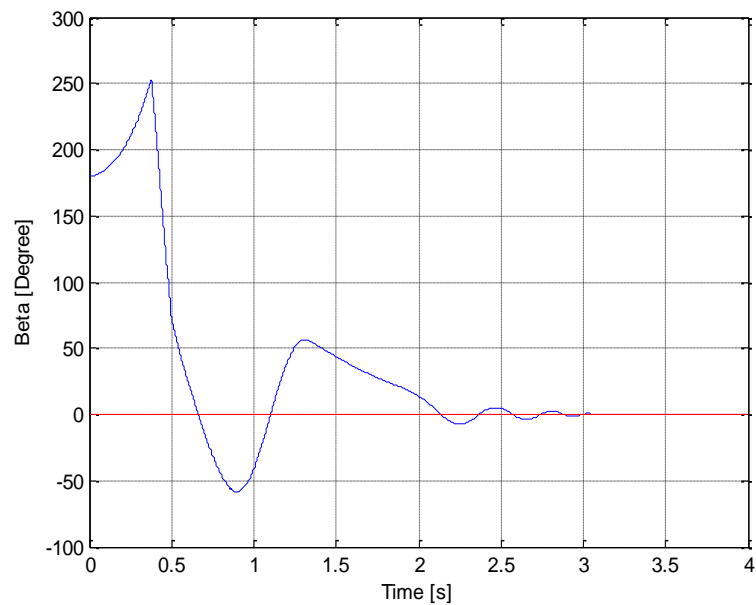


Fig. 8. β angle (pendulum angle) change with time

Figure 8 shows the variation of the pendulum angle according to time. It is desired that the pendulum is able to stop at the unstable equilibrium point. For this reason, the pendulum angle needs to reach the desired zero reference point. As can be seen from the figure, the pendulum has reached the desired reference value in about 3 seconds.

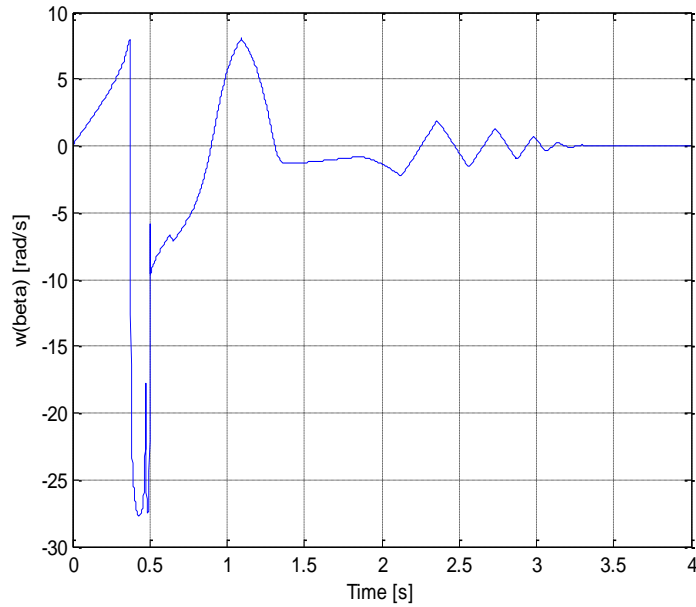


Fig. 9. Angular velocity change of the pendulum with time

Figure 9 shows the angular velocity of the pendulum with respect to time. The angular velocity of the pendulum reaches zero after the third second.

Respectively, figures 10 and 11 show the control signal values, which should be applied to the motor, and error graph. When the control signal graph is examined, it is clear that 25 Nm of engine torque will be sufficient to bring the pendulum to the desired reference value. It can be said that this torque value is reasonable for real applications. After a time like 3.5 seconds, the control signal reaches zero. When looking at the error graph in Figure 11, the error is zero because the pendulum angle captures the desired reference value after the 3rd second.

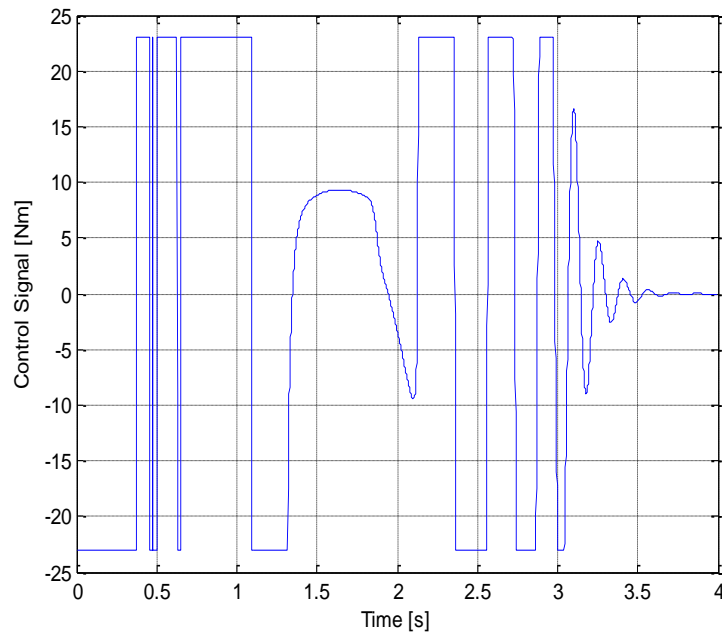


Fig. 10. Control signal change with time

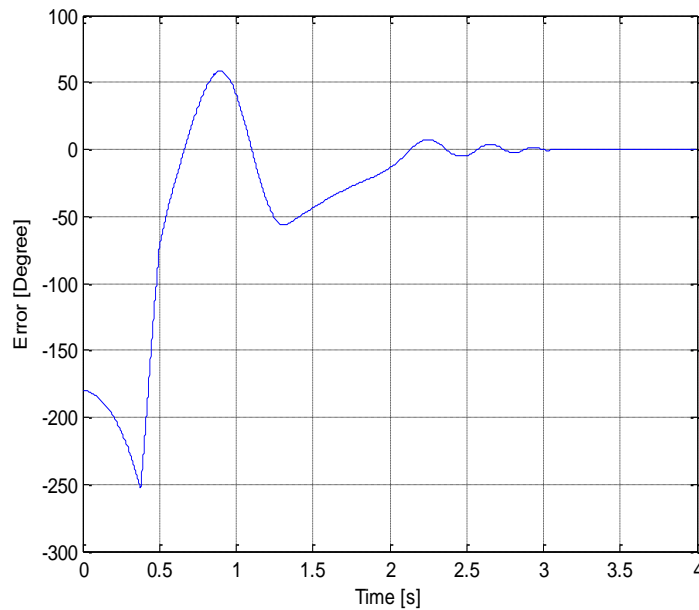


Fig. 11. Error graph of the pendulum angle

5. Conclusion

In this study, firstly the nonlinear model of the rotary inverted pendulum system with two degrees of freedom was obtained by the Lagrange method. Through the obtained model, the control of the pendulum angle was carried out with the program created in Matlab using the state variables. The moving sliding mode control method has been applied to the system for this purpose. Variation of the slope of the sliding surface is provided by the artificial neural network. Constant coefficients in the network structure are calculated by using the genetic algorithm. At the end of the work, it was observed that the pendulum reached the desired reference value, the error was about zero, and the control signal reached zero after 3 seconds. There is a 0.001 % error. In future studies, it is planned to control both the pendulum angle and the first pendulum angle by using the moving sliding mode control method.

6. References

- [1] Bugeja, M., "Non-Linear Swing-Up and Stabilizing Control of an Inverted Pendulum System", EUROCON, Ljubljana, Slovenia (2003).
- [2] Stimac, A.K., "Standup and Stabilization of the Inverted Pendulum", Massachusetts Institute of Technology, 1999.
- [3] Zhong, W., Röck, H., "Energy and passivity Based control of the double inverted pendulum on a cart", IEEE Conference on Control Applications, 2001.
- [4] Krishen, J., Becerra, V.M., "Efficient Fuzzy Control of a Rotary Inverted Pendulum Based on LQR Mapping", IEEE International Symposium on Intelligent Control, 2701-2706, Germany (2006).
- [5] Craig, K., Awtar, S., "Inverted Pendulum Systems: Rotary And Arm-Driven A Mechatronic System Design Case Study", Mechatronics, 12, 357-370, 2001.
- [6] Awtar S.. et al., "Inverted pendulum Systems: rotary and arm driven – a mechatronicssystem design case", Pergamon Pres, Mechatronics, 357-370, 2002.
- [7] Yan, Q., "Output Tracking of Underactuated Rotary Inverted Pendulum by Nonlinear Controller", Proceedings of the 42nd IEEE Conference on Decision and Control, Maui, Hawaii, USA, December, 2003.
- [8] T. C. Kuo, Y. J. Huang, B. W. Hong, "Adaptive PID with Sliding Mode Control for the Rotary Inverted Pendulum System", 2009 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, Singapore, July 14-17, 2009.

- [9] Krishen, J., Becerra, V. M., Efficient Fuzzy Control of a Rotary Inverted Pendulum Based on LQR Mapping, Proceedings of the 2006 IEEE International Symposium on Intelligent Control Munich, Germany, October 4-6, 2006.
- [10] Khanesar, M. A., Teshnehlab, M., Shoorehdeli, M. A., Sliding Mode Control of Rotary Inverted Pendulum, 2007 Mediterranean Conference on Control and Automation, July 27-29, 2007. Athens, Greece.
- [11] Hassanzadeh, I., Mobayen, S., PSO-Based Controller Design for Rotary Inverted Pendulum System, Journal of Applied Science 8 (16), 2907-2912, 2008.
- [12] Aydın, M., Yakut, O., ALLİ, H., Yapay Sinir Ağlı Kayan Kipli Kontrolün Dönel Ters Sarkaç Sistemine Uygulanması, Sigma Mühendislik ve Fen Bilimleri Dergisi, Sigma 5, 39-50, 2013.
- [13] Young, K.D., Utkin, V.I., Ozguner, U. A Control Engineer's Guide to Sliding Mode Control. IEEE Transactions on Control Systems Technology 1999; 7 (3): 328-342.
- [14] Edwards C, Spurgeon S K. Sliding Mode Control: Theory and Applications. London: Taylor and Francis; 1998.