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Research Article

On topological properties of hexagonal and silicate networks

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Abstract

Topological indices are the numerical quantities associated to a simple and finite graphs which represent its structure. There are certain types of topological indices such as degree based topological indices, distance based topological indices and counting related topological indices etc. The degree based topological indices are exploited to estimate the bioactivity of chemical compounds. In this paper, we compute first general Zagreb index, general Randić connectivity index, general sum-connectivity index, atom-bond connectivity index, geometric-arithmetic index, ABC_4 index and GA_5 index of the line graphs of silicate networks, chain silicate networks and hexagonal networks by using the subdivision method.

Mathematics Subject Classification (2010). 05C90

Keywords. Topological index, Zagreb index, hexagonal network, silicate network

1. Introduction and preliminary results

Topological indices are the molecular descriptors that describe the structures of chemical compounds and they help us to predict certain physico-chemical properties such as boiling point, strain energy, stability and so forth. They are same if and they are defined under the isomorphism of graphs. Among all the molecular descriptors topological indices play a vital role in QSPR/QSAR study. A *topological index* is a numeric quantity associated with a graph which characterize the molecular topology of graph. The distance based topological indices, degree based topological indices and counting related polynomials and indices are the some classes of graphs. These topological indices play an important role in chemical graph theory and particularly in theoretical chemistry. Some degree based

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topological indices are Randić index, Zagreb indices, sum-connectivity index and atom-bond connectivity index.

Let G be a simple, connected and undirected graph having order n and size m. d_u and s_u denotes the degree of a vertex $u \in V(G)$ and the sum of vertices lying at unit distance from vertex u. Two vertices u and v in a graph G are adjacent if and only if $e = uv \in E(G)$ and two edges are incident to each other if and only if they share a common vertex. The subdivision graph $S(G)$ is the graph that is obtain from G by replacing each of its edges by a path of length 2. The line graph $L(G)$ of a graph G is the graph whose vertices are the edges of existing graph G, two vertices f and g are adjacent if and only if they are incident in G. Following [20], we can construct the line graph of subdivision graph $L(S(G))$ of a graph G, as follows:

(1) The vertices of $L(S(G))$ are the edges of $S(G)$,
(2) Two vertices f and g are adjacent in $L(S(G))$ if and only if they are incident in $S(G)$.

The *Wiener index* introduced in 1947 by the chemist Harold Wiener [24], he was also working on boiling point of paraffin. The *Wiener index* $W(G)$ is defined as follows:

$$W(G) = \frac{1}{2} \sum_{u \in V(G)} \sum_{v \in V(G)} d_G(u, v),$$

where $d_G(u, v)$ is the shortest distance between u and v. The first degree based topological index is *Randić connectivity index* [17], it was defined as follows:

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}. \tag{1.1}$$

Latter on, the generalization of Randić index was introduced and known as *general Randić connectivity index* or *general product-connectivity index*, is defined as follows:

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha, \tag{1.2}$$

where α is a real number. Then $R_{-1/2}$ is the *classical Randić connectivity index*. The concept of sum-connectivity index [25] was extended to the *general sum-connectivity index* [26] and is defined as follows:

$$\chi_\alpha(G) = \sum_{uv \in E(G)} (d_u + d_v)^\alpha, \tag{1.3}$$

where α is a real number. If $\alpha = -1/2$, then this is the classical sum-connectivity index. Li and Zhao [12] introduced the first general Zagreb index as follows:

$$M_\alpha(G) = \sum_{u \in V(G)} (d_u)^\alpha, \tag{1.4}$$

where α is a real number. Estrada et al. [5] defined the atom-bond connectivity ABC index as follows:

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}. \tag{1.5}$$

Vukičević [23] introduced the geometric-arithmetic (GA) index defined as follows:

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}. \tag{1.6}$$

In 2010, the fourth version of ABC index (ABC_4) was introduced by Ghorbani and Hosseinzadeh [6] and defined as follows:

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{s_u + s_v - 2}{s_u s_v}}. \tag{1.7}$$

In 2011, the fifth version of GA index (GA_5) is proposed by Graovac [7] and defined as follows:

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{s_u s_v}}{s_u + s_v}. \tag{1.8}$$

For further study of these topological indices consult [1–4, 8–11, 13, 14, 22].

2. Topological indices of $L(S(G))$

In 2015, Nadeem [15] computed ABC_4 and GA_5 indices of the line graphs of the tadpole, wheel and ladder graphs by using the notion of subdivision. Nadeem [16] calculated topological properties of the line graphs of subdivision graphs of certain nanostructures in 2016. In 2015, Su et al. [21] calculated the general sum-connectivity indices and co-indices of the line graphs of the tadpole, wheel and ladder graphs with subdivision. Ranjini [19] in 2011 was calculated the explicit expressions for the Shultz indices of the subdivision graphs of the tadpole, wheel, helm and ladder graphs. They also computed the Zagreb indices of the line graphs of the tadpole, wheel and ladder graphs with subdivision in [18].

2.1. Silicate networks SL_n

Silicates are obtained by fusing metal oxides or metal carbonates with sand. The tetrahedron (SiO_4) is basic unit of silicates. Almost all silicates contain (SiO_4) tetrahedra. A silicate sheet is a ring of tetrahedrons which are linked by shared oxygen vertices to other rings in a two dimensional plane that produces a sheet-like structure. Cyclic silicates are structures which give cycles of different length after being linked by shared oxygen vertices. From chemical point of view, the corner vertices of tetrahedron (SiO_4) are actually the oxygen atoms and central vertex represents silicon atom. These corner atoms called as oxygen vertices, central atom as silicon vertex and bonds between them as edges from graphical point of view. SL_n denotes a silicate network of dimension n, where n is the number of hexagons between the center and boundary of SL_n. The number of vertices in SL_n are $15n^2 + 3n$ and number of edges are $36n^2$.

The number of vertices and the number of edges in the line graph of the subdivision graph of SL_n are $72n^2$ and $27n(7n - 1)$, respectively. In this section, we compute first general Zagreb index, general Randić connectivity index, general sum connectivity index, ABC index, GA index, ABC_4 index and GA_5 index of the line graph of the subdivision graph of SL_n. The subdivision graph of SL_n and the line graph of subdivision graph $L(S(SL_n))$ of SL_n are shown in Figure 1.

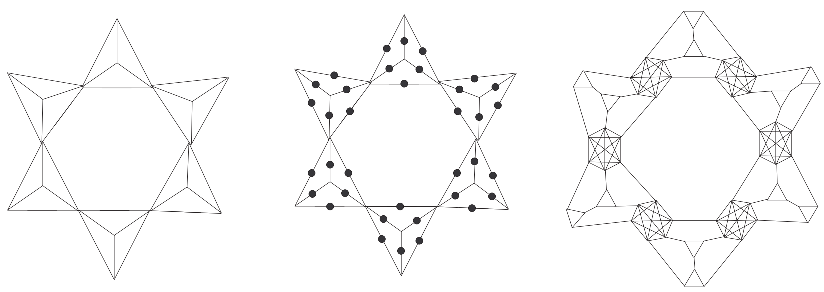

Figure 1. (a) SL_1 (b) A Subdivision of SL_1 (c) A line graph of subdivision of SL_1.

(d_u, d_v) where $uv \in E(G)$	$(3,3)$	$(3,6)$	$(6,6)$
Number of edges	$6n(3n+4)$	$6n(3n+1)$	$3n(51n-19)$

Table 1. The edge partition of SL_n based on degree of end vertices of each edge.

Theorem 2.1. *Consider the line graph of the subdivision graph G of SL_n. Then*
(1) $M_\alpha(G) = 2n(n+1).3^{\alpha+2} + 3n(3n-1)6^{\alpha+1}$,
(2) $R_\alpha(G) = 2n(3n+4).3^{2\alpha+1} + 6n(3n+1).18^\alpha + 3n(51n-19).6^{2\alpha}$,
(3) $\chi_\alpha(G) = n(3n+4)6^{\alpha+1} + 2n(3n+1)3^{2\alpha+1} + 3n(51n-19)12^\alpha$,
where α is a real number.

Proof. The graph G has total $72n^2$ vertices. The number of vertices of degree 3 and 6 are $18n(n+1)$ and $18n(3n-1)$, respectively. Use these partition of vertices in the formulae (1.4) and compute the required result. The graph G has $27n(7n-1)$ total number of edges. Using values of Table 1 in the formulae of general Randić index and general sum-connectivity index and obtain the required results. ☐

Theorem 2.2. *Consider the line graph of subdivision graph G of SL_n. Then*
(1) $ABC(G) = (12 + 3\sqrt{14} + \frac{51\sqrt{10}}{2})n^2 + (16 + \sqrt{14} - \frac{19\sqrt{10}}{2})n$,
(2) $GA(G) = (171 + 12\sqrt{2})n^2 + (4\sqrt{2} - 33)n$.

Proof. The graph G contains only 3 types of edges i.e, $(3,3)$, $(3,6)$ and $(6,6)$. From using the values of Table 1 in the formulae (1.5), we get

$$\begin{aligned} ABC(G) &= (3,3)\sqrt{\frac{3+3-2}{3\times 3}} + (3,6)\sqrt{\frac{3+6-2}{3\times 6}} + (6,6)\sqrt{\frac{6+6-2}{6\times 6}} \\ &= 6n(3n+4)\frac{2}{3} + 6n(3n+1)\frac{\sqrt{7}}{3\sqrt{2}} + 3n(51n-19)\frac{\sqrt{10}}{6} \end{aligned}$$

After simplification, we get

$$ABC(G) = \left(12 + 3\sqrt{14} + \frac{51\sqrt{10}}{2}\right)n^2 + \left(16 + \sqrt{14} - \frac{19\sqrt{10}}{2}\right)n.$$

Similarly, by using the values of Table 1 in the formula (1.6), we can easily calculate the geometric-arithmetic index of G. ☐

(s_u, s_v) where $uv \in E(G)$	Number of edges
$(9,9)$	$6n$
$(9,12)$	$24n$
$(12,12)$	$6n(3n-1)$
$(12,33)$	$6n(3n+1)$
$(33,33)$	$3n(51n-19)$

Table 2. The edge partition of the line graph of the subdivision graph G of SL_n based on degree sum of neighbor vertices of end vertices of each edge.

Theorem 2.3. *Consider the line graph of the subdivision graph G of SL_n, then its ABC_4 and GA_5 indices are as follows:*
(1) $ABC_4(G) = (\frac{3\sqrt{22}}{2} + \frac{3\sqrt{43}}{\sqrt{11}} + \frac{51\sqrt{43}}{11})n^2 + (\frac{8}{3} + \frac{4\sqrt{19}}{\sqrt{3}} - \frac{\sqrt{22}}{2} + \frac{\sqrt{43}}{\sqrt{11}} - \frac{19\sqrt{34}}{11})n$,

(2) $GA_5(G) = (\frac{24\sqrt{11}}{5} + 171)n^2 + (\frac{96\sqrt{2}}{7} + \frac{8\sqrt{11}}{5} - 57)n.$

Proof. By using values of Table 2 in (1.7), we obtain

$$ABC_4(G) = 6n\sqrt{\frac{9+9-2}{9\times 9}} + 24n\sqrt{\frac{9+12-2}{9\times 12}} + 6n(3n-1)\sqrt{\frac{12+12-2}{12\times 12}} + 6n(3n+1)\sqrt{\frac{12+33-2}{12\times 33}} + 3n(51n-19)\sqrt{\frac{33+33-2}{33\times 33}}$$

After a bit calculation, we obtain

$$ABC_4(G) = \left(\frac{3\sqrt{22}}{2} + \frac{3\sqrt{43}}{\sqrt{11}} + \frac{51\sqrt{43}}{11}\right)n^2 + \left(\frac{8}{3} + \frac{4\sqrt{19}}{\sqrt{3}} - \frac{\sqrt{22}}{2} + \frac{\sqrt{43}}{\sqrt{11}} - \frac{19\sqrt{34}}{11}\right)n.$$

Similarly, we can compute the fifth geometric-arithmetic index of G by using the values of Table 2 in the formula (1.8). □

2.2. Chain silicate networks CS_n

A chain silicate network CS_n of dimension n is obtained by linearly arranging n tetrahedra. The number of vertices and the number of edges in CS_n with $n > 1$ are $3n + 1$ and $6n$, respectively. The number of vertices and the number of edges in the line graph of the subdivision graph $L(S(CS_n)) = G$ of CS_n are $12n$ and $27n - 9$, respectively. In this section, we compute first general Zagreb index, general Randić connectivity index, general sum connectivity index, ABC index, GA index, the fourth version of ABC index and the fifth version of GA index of the line graph of the subdivision graph of CS_n. The subdivision graph of CS_n and its line graph $L(S(G))$ are depicted in Figure 2.

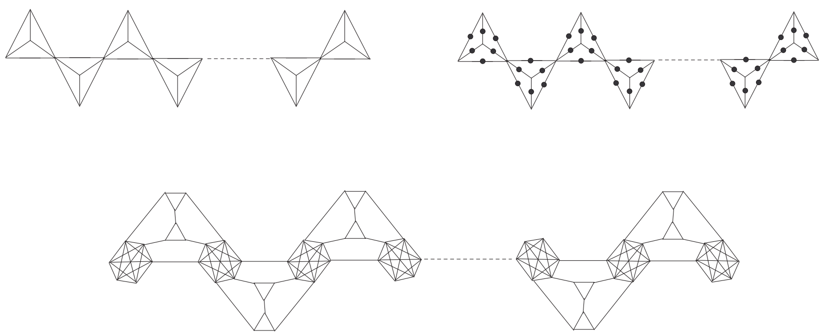

Figure 2. (a) CS_n (b) A subdivision graph of CS_n (c) A line graph of subdivision graph of CS_n.

m_{d_u,d_v} where $uv \in E(G)$	$m_{3,3}$	$m_{3,6}$	$m_{6,6}$
Number of edges	$7n + 10$	$2(2n - 1)$	$16n - 17$

Table 3. The edge partition of the line graph of the subdivision graph of CS_n based on degree of end vertices of each edge.

Theorem 2.4. *Consider the line graph of the subdivision graph G of CS_n. Then*

(1) $M_\alpha(G) = 2(n + 1).3^{\alpha+1} + (n - 1)6^{\alpha+1},$

(2) $R_\alpha(G) = (7n+10).3^{2\alpha} + 2(2n-1).18^\alpha + (16n-17).6^{2\alpha}$,
(3) $\chi_\alpha(G) = (7n+10).6^\alpha + 2(2n-1).3^{2\alpha} + (16n-17).12^\alpha$,
where α is a real number.

Proof. The line graph of subdivision graph G of CS_n has total $12n$ vertices. The vertices of degree 3 and degree 6 are $6(n+1)$ and $6(n-1)$, respectively. Using these values in the formula of first general Zagreb index and get the required result.

The line graph of the subdivision graph G of CS_n has $(27n-9)$ total number of edges. Using Table 3 in the formulae of general Randić index and general sum-connectivity index and obtain the required results. The graph G has only 18 edges of $m_{3,3}$. By using this in the formulae of general Randić index and general sum-connectivity index, and obtain the required result. □

Theorem 2.5. *Consider the line graph of subdivision graph G of CS_n. Then its ABC and GA indices are as follows:*

(1) $ABC(G) = \begin{cases} 12, & n=1; \\ (\frac{14}{3} + \frac{2\sqrt{14}}{3} + \frac{8\sqrt{10}}{3})n + (\frac{20}{3} - \frac{\sqrt{14}}{3} - \frac{17\sqrt{10}}{6}), & n>1. \end{cases}$

(2) $GA(G) = \begin{cases} 18, & n=1; \\ (23 + \frac{8\sqrt{2}}{3})n - (\frac{4\sqrt{2}}{3} + 7), & n>1. \end{cases}$

Proof. The graph G contains only 3 types of edges i.e, $(3,3)$, $(3,6)$ and $(6,6)$.
Case I: For $n>1$, From the definition of ABC index and by using the Table 3, we obtain

$$ABC(G) = (7n+10)\sqrt{\frac{3+3-2}{3\times 3}} + 2(2n-1)\sqrt{\frac{3+6-2}{3\times 6}} + (16n-17)\sqrt{\frac{6+6-2}{6\times 6}}$$

Simplify and then we get

$$ABC(G) = \left(\frac{14}{3} + \frac{2\sqrt{14}}{3}\frac{8\sqrt{10}}{3}\right)n + \left(\frac{20}{3} - \frac{\sqrt{14}}{3} - \frac{17\sqrt{10}}{6}\right).$$

Case II: For $n=1$, from the definition of ABC index

$$\begin{aligned} ABC(G) &= \sum_{uv\in E(G)} \sqrt{\frac{d_u+d_v-2}{d_u d_v}} \\ &= 18\sqrt{\frac{3+3-2}{3\times 3}} \\ &= 12. \end{aligned}$$

Similarly, we compute GA index by the use of the edge partition of the line graph of the subdivision graph of CS_n that shown in Table 3. □

(s_u, s_v) where $uv \in E(G)$	Number of edges
$(9,9)$	n+10
$(9,12)$	$4(n+1)$
$(12,12)$	$2(n-2)$
$(12,33)$	$2(2n-1)$
$(33,33)$	$16n-17$

Table 4. The edge partition of the line graph of the subdivision graph of CS_n based on degree sum of neighbor vertices of end vertices of each edge.

Theorem 2.6. *Consider the line graph of the subdivision graph G of CS_n, then its ABC_4 and GA_5 indices are given by*

$$(1)\ ABC_4(G) = \begin{cases} 8, & n = 1; \\ (\frac{4}{9} + \frac{2\sqrt{19}}{3\sqrt{3}} + \frac{\sqrt{22}}{2} + \frac{2\sqrt{43}}{3\sqrt{11}} + \frac{128}{33})n & \\ +(\frac{40}{9} + \frac{2\sqrt{19}}{3\sqrt{3}} - \sqrt{22} - \frac{\sqrt{43}}{3\sqrt{11}} - \frac{136}{33}), & n > 1. \end{cases}$$

$$(2)\ GA_5(G) = \begin{cases} 18, & n = 1; \\ (19 + \frac{16\sqrt{3}}{7} + \frac{16\sqrt{11}}{15})n + (\frac{16\sqrt{3}}{7} - 11 - \frac{8\sqrt{11}}{15}), & n > 1. \end{cases}$$

Proof. There are two cases to discuss while proving this result.
Case I: For $n > 1$. From the definition of fourth atom-bond connectivity index and by Table 4, we get

$$ABC_4(G) = (n+10)\sqrt{\frac{9+9-2}{9\times 9}} + 4(n+1)\sqrt{\frac{9+12-2}{9\times 12}} + 2(n-2)\sqrt{\frac{12+12-2}{12\times 12}} + 2(2n-1)\sqrt{\frac{12+33-2}{12\times 33}} + (16n-17)\sqrt{\frac{33+33-2}{33\times 33}}$$

After a bit simplification, we obtain

$$ABC_4(G) = \left(\frac{4}{9} + \frac{2\sqrt{19}}{3\sqrt{3}} + \frac{\sqrt{22}}{2} + \frac{2\sqrt{43}}{3\sqrt{11}} + \frac{128}{33}\right)n + \left(\frac{40}{9} + \frac{2\sqrt{19}}{3\sqrt{3}} - \sqrt{22} - \frac{\sqrt{43}}{3\sqrt{11}} - \frac{136}{33}\right).$$

Case II: For $n = 1$. From the definition of fourth atom-bond connectivity index

$$\begin{aligned} ABC_4(G) &= \sum_{uv\in E(G)} \sqrt{\frac{s_u + s_v - 2}{s_u s_v}} \\ &= 18\sqrt{\frac{9+9-2}{9\times 9}} \\ &= 8. \end{aligned}$$

From the definition of fifth geometric-arithmetic index and by the edge partition of Table 4, we calculate the GA_5 index of G. □

2.3. Hexagonal networks HX_n

There exist three regular plane tiling with composition of same kind of regular polygons such as triangular, hexagonal and square. In the construction of hexagonal networks, triangular tiling is being used. HX_n denotes a hexagonal network of dimension n, where n is the number of vertices on each side of hexagon. The number of vertices and number of edges in HX_n with $n > 1$ are $3n^2 - 3n + 1$ and $9n^2 - 15n + 6$, respectively. The number of vertices and the number of edges in the line graph of the subdivision graph $L(S(HX_n)) = G$ of HX_n are $2(9n^2 - 15n + 6)$ and $3(18n^2 - 38n + 19)$, respectively. In this section, we compute first general Zagreb index, general Randić connectivity index, general sum connectivity index, ABC index, GA index, ABC_4 index and GA_5 index of the line graph of the subdivision graph G of HX_n. The subdivision graph of HX_n and its line graph $L(S(G))$ are shown in Figure 3.

Theorem 2.7. *Let G be the line graph of the subdivision graph of HX_n. Then*

(1) $M_\alpha(G) = 2.3^{\alpha+2} + 3(n-2).2^{2\alpha+3} + (3n^2 - 9n + 7).6^{\alpha+1}$,

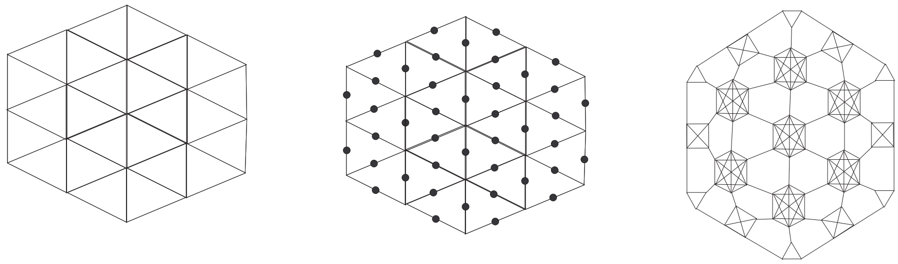

Figure 3. (a) HX_3 (b) A subdivision graph of HX_3 (c) A line graph of subdivision graph of HX_3.

m_{d_u,d_v} where $uv \in E(G)$	$m_{3,3}$	$m_{3,6}$	$m_{6,6}$
Number of edges	24	6	15

Table 5. The edge partition of the line graph of the subdivision graph G of Hexagonal network HX_2 based on degree of end vertices of each edge.

m_{d_u,d_v} where $uv \in E(G)$	Number of edges
$m_{3,3}$	18
$m_{3,4}$	12
$m_{4,4}$	$6(7n-15)$
$m_{3,6}$	6
$m_{4,6}$	$12(n-2)$
$m_{6,6}$	$3(18n^2-56n+45)$

Table 6. The edge partition of the line graph of the subdivision graph G of Hexagonal network HX_n, $n>2$, based on degree of end vertices of each edge

$$(2)\ \ R_\alpha(G) = \begin{cases} 24.9^\alpha + 6.18^\alpha + 15.6^{2\alpha}, & n=2; \\ 2.9^{\alpha+1} + 12^{\alpha+1} + 6(7n-15)4^{2\alpha} & \\ +6.18^\alpha + 12(n-2)24^\alpha & \\ +3(18n^2-56n+45)6^{2\alpha}, & n>2, \end{cases}$$

$$(3)\ \ \chi_\alpha(G) = \begin{cases} 4.6^{\alpha+1} + 2.3^{2\alpha+1} + 15.12^\alpha, & n=2; \\ 3.6^{\alpha+1} + 12.7^{\alpha+1} + 3(7n-15)2^{2\alpha+1} + 2.3^{2\alpha+1} & \\ +12(n-2)10^\alpha + 3(18n^2-56n+45)12^\alpha, & n>2, \end{cases}$$

where α is a real number.

Proof. The graph G has total $2(9n^2-15n+6)$ vertices among which vertices of degree 3, degree 4 and degree 6 are 18, $24(n-2)$ and $6(3n^2-9n+7)$, respectively. Using these values in the formula of first general Zagreb index and compute the required result. There are two cases to discuss while proving the general Randić index of G.

Case I: For $n>2$. From the definition of the general Randić index and by using the values of Table 6, we obtain

$$\begin{aligned} R_\alpha(G) &= m_{3,3}(3\times 3)^\alpha + m_{3,4}(3\times 4)^\alpha + m_{4,4}(4\times 4)^\alpha + m_{3,6}(3\times 6) \\ &\quad + m_{4,6}(4\times 6) + m_{6,6}(6\times 6) \\ &= 2.9^{\alpha+1} + 12^{\alpha+1} + 6(7n-15).4^{2\alpha} + 6.18^\alpha + 12(n-2).24^\alpha \\ &\quad + 3(18n^2+56n+45).6^{2\alpha}. \end{aligned}$$

Case II: For $n = 2$. From the definition of the general sum-connectivity index and by Table 5, we obtain

$$\begin{aligned} R_\alpha(G) &= m_{3,3}(3\times 3)^\alpha + m_{3,6}(3\times 6)^\alpha + m_{6,6}(6\times 6)^\alpha \\ &= 24.9^\alpha + 6.18^\alpha + 15.6^{2\alpha}. \end{aligned}$$

On the similar way, by using the values of Table 5 and 6 in (1.3), we calculate the general sum-connectivity index of G. □

Theorem 2.8. *Let G be the line graph of the subdivision graph of HX_n. Then its atom-bond connectivity index is equal to*

$$ABC(G) = \begin{cases} 16 + \sqrt{14} + \frac{5\sqrt{10}}{2}, & n = 2; \\ 9\sqrt{10}n^2 + (\frac{21\sqrt{6}}{2} + 4\sqrt{3} - 28\sqrt{10})n + \\ (12 + 2\sqrt{15} - 8\sqrt{3} - \frac{45\sqrt{6}}{2} + \frac{45\sqrt{10}}{2}), & n > 2. \end{cases}$$

Proof. **Case I:** For $n = 2$. From the definition of ABC index and by using Table 5, we get

$$\begin{aligned} ABC(G) &= 24\sqrt{\frac{3+3-2}{3\times 3}} + 6\sqrt{\frac{3+6-2}{3\times 6}} + 15\sqrt{\frac{6+6-2}{6\times 6}} \\ &= 16 + \sqrt{14} + \frac{5\sqrt{10}}{2}. \end{aligned}$$

Case II: For $n > 2$. From the definition of ABC index and by using Table 6, we get

$$\begin{aligned} ABC(G) = {} & 18\sqrt{\frac{3+3-2}{3\times 3}} + 12\sqrt{\frac{3+4-2}{3\times 4}} + 6(7n-15)\sqrt{\frac{4+4-2}{4\times 4}} \\ & + 6\sqrt{\frac{3+6-2}{3\times 6}} + 12(n-2)\sqrt{\frac{4+6-2}{4\times 6}} \\ & + 3(18n^2 - 56n + 45)\sqrt{\frac{6+6-2}{6\times 6}} \\ & = 9\sqrt{10}n^2 + \left(\frac{21\sqrt{6}}{2} + 4\sqrt{3} - 28\sqrt{10}\right)n + \Big(12 + 2\sqrt{15} - 8\sqrt{3} \\ & - \frac{45\sqrt{6}}{2} + \frac{45\sqrt{10}}{2}\Big). \end{aligned}$$

□

In the following theorem, we compute geometric-arithmetic GA index of the line graph of the subdivision graph G of SL_n. The proof is similar to the proof of Theorem 2.8, hence omitted.

Theorem 2.9. *Consider the line graph of the subdivision graph G of SL_n. Then its geometric-arithmetic GA index is equal to*

$$GA(G) = \begin{cases} 39 + 4\sqrt{2}, & n = 2; \\ (54n^2 + (\frac{24\sqrt{6}}{5} - 126)n + (\frac{48\sqrt{3}}{7} - \frac{48\sqrt{6}}{5} + 4\sqrt{2} + 63), & n > 2. \end{cases}$$

m_{s_u,s_v} where $uv \in E(G)$	$m_{9,9}$	$m_{9,12}$	$m_{12,33}$	$m_{33,33}$
Number of edges	12	12	6	15

Table 7. The edge partition of the line graph of the subdivision graph of HX_2 based on degree sum of vertices lying at unit distance from end vertices of each edge.

m_{s_u,s_v}	Number of edges	m_{s_u,s_v}	Number of edges
$m_{10,10}$	6	$m_{18,34}$	12
$m_{10,12}$	12	$m_{33,34}$	12
$m_{10,15}$	12	$m_{34,34}$	6
$m_{15,15}$	6	$m_{33,36}$	18
$m_{15,18}$	24	$m_{34,36}$	36
$m_{18,18}$	6	$m_{36,36}$	45
$m_{12,33}$	6		

Table 8. The edge partition of the line graph of the subdivision graph of HX_3 based on degree sum of vertices lying at unit distance from end vertices of each edge.

m_{s_u,s_v}	Number of edges	m_{s_u,s_v}	Number of edges
$m_{10,10}$	6	$m_{12,33}$	6
$m_{10,12}$	12	$m_{18,33}$	$12(n-2)$
$m_{10,15}$	12	$m_{33,34}$	12
$m_{15,16}$	12	$m_{34,34}$	$6(n-2)$
$m_{16,16}$	$6(2n-7)$	$m_{33,36}$	18
$m_{15,18}$	24	$m_{34,36}$	$12(4n-9)$
$m_{16,18}$	$24(n-3)$	$m_{36,36}$	$3(18n^2-74n+75)$
$m_{18,18}$	$6(n-2)$		

Table 9. The edge partition of the line graph of the subdivision graph of HX_n, $n > 3$, based on degree sum of vertices lying at unit distance from end vertices of each edge.

Theorem 2.10. *Consider the line graph of the subdivision graph G of HX_n. Then the ABC_4 index is*

$$ABC_4(G) = \begin{cases} \frac{296\sqrt{33}+66\sqrt{209}+33\sqrt{129}}{33\sqrt{33}}, & n=2; \\ 2\sqrt{6}+6\sqrt{2}+\frac{9\sqrt{2}}{5}+\frac{12\sqrt{23}}{5\sqrt{6}}+\frac{4\sqrt{7}}{5}+\frac{4\sqrt{186}}{5}+\frac{\sqrt{34}}{3} & \\ +\frac{\sqrt{43}}{\sqrt{11}}+\frac{10\sqrt{2}}{\sqrt{17}}+\frac{12\sqrt{65}}{\sqrt{1122}}+\frac{3\sqrt{66}}{17}+\frac{\sqrt{201}}{\sqrt{11}}+\frac{5\sqrt{70}}{4}, & n=3; \\ \frac{3\sqrt{70}}{2}n^2+(\frac{3\sqrt{30}}{4}+8+\frac{\sqrt{34}}{3}+\frac{10\sqrt{2}}{\sqrt{17}}+\frac{3\sqrt{66}}{17}+8\sqrt{2} & \\ -\frac{37\sqrt{70}}{6})n+(\frac{9\sqrt{2}}{5}+2\sqrt{6}+\frac{2\sqrt{138}}{5}+\frac{3\sqrt{29}}{\sqrt{15}}-\frac{21\sqrt{30}}{8} & \\ +\frac{8\sqrt{31}}{\sqrt{30}}-24-\frac{2\sqrt{34}}{3}+\frac{\sqrt{43}}{\sqrt{11}}-\frac{20\sqrt{2}}{\sqrt{17}}+\frac{12\sqrt{65}}{\sqrt{1122}}-\frac{6\sqrt{66}}{17} & \\ +\frac{\sqrt{201}}{\sqrt{11}}-18\sqrt{2}+\frac{25\sqrt{70}}{4}), & n>3. \end{cases}$$

Proof. We find the edge partition of the line graph of the subdivision graph of HX_n based on degree sum of vertices lying at unit distance from end vertices of each edge. Now we can compute ABC_4 index for values of n.

Case I: For $n = 2$. From the definition of ABC index and by Table 7, then we have

$$\begin{aligned}ABC_4(G) &= 12\sqrt{\frac{9+9-2}{9\times 9}} + 12\sqrt{\frac{9+12-2}{9\times 12}} + 6\sqrt{\frac{12+33-2}{12\times 33}} + 15\sqrt{\frac{33+33-2}{33\times 33}}\\ &= \frac{296\sqrt{33}+66\sqrt{209}+33\sqrt{129}}{33\sqrt{33}}.\end{aligned}$$

Case II. For $n = 3$. From the definition of ABC index and Table 8, we obtain

$$\begin{aligned}ABC_4(G)6&\sqrt{\frac{10+10-2}{10\times 10}} + 12\sqrt{\frac{10+12-2}{10\times 12}} + 12\sqrt{\frac{10+15-2}{10\times 15}} + 6\sqrt{\frac{15+15-2}{15\times 15}}\\ &+ 24\sqrt{\frac{15+18-2}{15\times 18}} + 6\sqrt{\frac{18+18-2}{18\times 18}} + 6\sqrt{\frac{12+33-2}{12\times 33}}\\ &+ 12\sqrt{\frac{18+34-2}{18\times 34}} + 12\sqrt{\frac{33+34-2}{33\times 34}} + 6\sqrt{\frac{34+34-2}{34\times 34}}\\ &+ 18\sqrt{\frac{33+36-2}{33\times 36}} + 36\sqrt{\frac{34+36-2}{34\times 36}} + 45\sqrt{\frac{36+36-2}{36\times 36}}\\ &= 2\sqrt{6} + 6\sqrt{2} + \frac{9\sqrt{2}}{5} + \frac{12\sqrt{23}}{5\sqrt{6}} + \frac{4\sqrt{7}}{5} + \frac{4\sqrt{186}}{5} + \frac{\sqrt{34}}{3} + \frac{\sqrt{43}}{\sqrt{11}}\\ &+ \frac{10\sqrt{2}}{\sqrt{17}} + \frac{12\sqrt{65}}{\sqrt{1122}} + \frac{3\sqrt{66}}{17} + \frac{\sqrt{201}}{\sqrt{11}} + \frac{5\sqrt{70}}{4}.\end{aligned}$$

Case III: For $n > 3$. From the definition of ABC index and Table 9, we obtain

$$\begin{aligned}ABC_4(G) = 6&\sqrt{\frac{10+10-2}{10\times 10}} + 12\sqrt{\frac{10+12-2}{10\times 12}} + 12\sqrt{\frac{10+15-2}{10\times 15}}\\ &+ 12\sqrt{\frac{15+16-2}{15\times 16}} + 6(2n-7)\sqrt{\frac{16+16-2}{16\times 16}}\\ &+ 24\sqrt{\frac{15+18-2}{15\times 18}} + 24(n-3)\sqrt{\frac{16+18-2}{16\times 18}} + 6(n-2)\sqrt{\frac{18+18-2}{18\times 18}}\\ &+ 6\sqrt{\frac{12+33-2}{12\times 33}} + 12(n-2)\sqrt{\frac{18+34-2}{18\times 34}} + 12\sqrt{\frac{33+34-2}{33\times 34}}\\ &+ 6(n-2)\sqrt{\frac{34+34-2}{34\times 34}} + 18\sqrt{\frac{33+36-2}{33\times 36}} + 12(4n-9)\sqrt{\frac{34+36-2}{34\times 36}}\\ &+ 3(18n^2-74n+75)\sqrt{\frac{36+36-2}{36\times 36}}\\ &= \frac{3\sqrt{70}}{2}n^2 + \left(\frac{3\sqrt{30}}{4} + 8 + \frac{\sqrt{34}}{3} + \frac{10\sqrt{2}}{\sqrt{17}} + \frac{3\sqrt{66}}{17} + 8\sqrt{2} - \frac{37\sqrt{70}}{6}\right)n\\ &+ \left(\frac{9\sqrt{2}}{5} + 2\sqrt{6} + \frac{2\sqrt{138}}{5} + \frac{3\sqrt{29}}{\sqrt{15}} - \frac{21\sqrt{30}}{8}\frac{8\sqrt{31}}{\sqrt{30}} - 24 - \frac{2\sqrt{34}}{3}\right.\\ &\left. + \frac{\sqrt{43}}{\sqrt{11}} - \frac{20\sqrt{2}}{\sqrt{17}} + \frac{12\sqrt{65}}{\sqrt{1122}} - \frac{6\sqrt{66}}{17} + \frac{\sqrt{201}}{\sqrt{11}} - 18\sqrt{2} + \frac{25\sqrt{70}}{4}\right).\end{aligned}$$

□

In the following theorem, we compute geometric-arithmetic GA index of the line graph of the subdivision graph G of SL_n. The proof is similar to the proof of Theorem 2.10, hence omitted.

Theorem 2.11. *Consider the line graph of the subdivision graph G of HX_n. Then its GA_5 index is equal to*

$$GA_5(G) = \begin{cases} \frac{48\sqrt{3}}{7} + \frac{8\sqrt{11}}{5} + 27, & n = 2; \\ 69 + \frac{72\sqrt{30}}{11} + \frac{24\sqrt{6}}{5} + \frac{8\sqrt{11}}{5} + \frac{36\sqrt{17}}{13} & \\ +\frac{24\sqrt{1122}}{67} + \frac{72\sqrt{33}}{23} + \frac{216\sqrt{34}}{35}, & n = 3; \\ 54n^2 + (\frac{288\sqrt{2}}{17} + \frac{36\sqrt{17}}{13} - \frac{288\sqrt{34}}{35} - 198)n & \\ +\frac{72\sqrt{30}}{11} + \frac{24\sqrt{6}}{5} + \frac{96\sqrt{15}}{31} - \frac{864\sqrt{2}}{17} + \frac{8\sqrt{11}}{5} & \\ -\frac{72\sqrt{17}}{13} + \frac{24\sqrt{1122}}{67} + \frac{72\sqrt{33}}{23} - \frac{648\sqrt{34}}{35} + 165, & n > 3. \end{cases}$$

3. Conclusion

In this paper, some important degree based indices, namely first general Zagreb index M_α, general Rndić connectivity index R_α, general sum-connectivity index χ_α, atom-bond connectivity index ABC, geometric-arithmetic index GA, ABC_4 index and GA_5 index are studied for the line graphs of subdivision graphs of silicate networks SL_n, chain silicate networks CS_n and hexagonal networks HX_n.

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