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A Novel Approach to Similarity of Soft Sets

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Abstract

The main objective of this paper is to present a new perspective for the similarity between soft sets. Therefore, the concept of similarity coefficient for soft sets is defined. Also, some theoretical investigations for this concept are presented. Finally, it is given examples of how this similarity coefficient can be used to solve the problems in various fields involving the uncertainty.

Keywords: Soft set, Soft set operations, Similarity coefficient.

Esnek Kümelerin Benzerliğine Yeni Bir Yaklaşım

Özet

Bu çalışmanın temel amacı esnek kümeler arasındaki benzerlik için yeni bir bakış açısı sunmaktır. Bu bağlamda, ilk olarak esnek kümeler için benzerlik katsayısı kavramı tanımlanmıştır. Ayrıca, bu kavram için bazı teorik araştırmalar sunulmuştur. Son olarak, bu benzerlik katsayısının belirsizlik içeren çeşitli alanlardaki problemleri çözmede nasıl kullanılabileceğinin örnekleri verilmiştir.

Anahtar Kelimeler: Esnek küme, Esnek küme işlemleri, Benzerlik Katsayısı.

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1. Introduction

In real life, it is extremely difficult or impossible to obtain a solution when the notions, problems or situations involving various uncertainties and unknown data are encountered. In such cases, the classical mathematical methods which can easily bring to a successful conclusion for structures involving certainty and definite data are generally insufficient. Accordingly, many mathematical tools designed according to the nature of uncertainty have been developed. Recently, in handling the structures involving the uncertainties, a novel mathematical tool called soft set [13] has emerged in addition to the existing mathematical tools such as fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set, neutrosophic set and rough set. After this emergence, the soft set which has virtually no limitations that complicate the work, attracted the interest of many researchers. Considering the proliferation of interest, many studies based directly or indirectly on the soft set were published in short time. Maji et al. [10] published a very valuable work which was the first step to the operations of the soft sets. Later on, many studies were presented on the soft operations such as the subset, complement, relative complement of a soft set, the intersection, union, product of two or more soft sets [1-3, 7, 14-16,19]. Maji et al. [11] introduced representations in the form of binary information table of the soft sets. Subsequently, by improving these representations, Çağman and Enginoğlu [6] proposed the concept of soft matrix corresponding to the soft set. In [4, 5, 8], the operations of soft matrices were investigated in detail.

The similarity is fundamentally in almost every scientific field. For many mathematical tools in the literature, this concept has been explored in depth and is still being investigated. Relatedly, for the soft sets, this concept has attracted the attention of many researchers. In [9, 12, 17, 18], the researchers dealt with the similarity between two soft sets. However, there are several conditions for the use of some of the similarity concepts proposed by the authors. For example, the soft sets should not be the null soft sets, the parameter sets of two soft sets should be same and so on. In this study, our goal is to describe the similarity of soft sets without those limitations.

This paper is organized as follows. Section 2 briefly introduces some description and terminology that will be used in this study. Sections 3 contains our main contributes. In this part, we introduce the similarity coefficient for two soft sets. Also, we thoroughly investigate the properties of it. Section 4 presents examples showing that this coefficient can be applied to solve the problems in a variety of areas such as medicine, buying and selling. Section 5 gives some concluding comments.

2. Preliminaries

Throughout this work, $U = \{u_i : i = 1,2, \ldots, n\}$ is an initial universe set, $P(U)$ is the power set of U, $E = \{e_j : j = 1,2, \ldots, m\}$ is a set of parameters and $A \subseteq E$.

Definition 2.1 [13] A soft set over the universe set U is pair (F, A) consisting of a subset A of E and a mapping $F: A \to P(U)$.

Note that the set of all soft sets on U is denoted by $\mathcal{S}(U)$.

Example 2.2 Let $U = \{u_1, u_2, u_3, u_4\}$ be an initial universe set, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ be a set of all parameters, and also $A = \{e_1, e_3, e_4, e_5\}$. If $F: A \to P(U)$ such that $F(e_1) = \{u_1, u_3, u_4\}$, $F(e_3) = \{u_1, u_3\}$, $F(e_4) = \{u_2\}$, $F(e_5) = \emptyset$ then it can be written the soft set

$$(F, A) = \{(e_1, \{u_1, u_3, u_4\}), (e_3, \{u_1, u_3\}), (e_4, \{u_2\}), (e_5, \emptyset)\}.$$

Definition 2.3 [1, 10] Let $(F, A) \in \mathcal{S}(U)$.

a) If $F(e_j) = \emptyset$ for all $e_j \in A$, then (F, A) is called a relative null soft set, denoted by Φ_A. If $A = E$, then the relative null soft set is called a null soft set, denoted by Φ_E.

b) If $F(e_j) = U$ for all $e_j \in A$, then (F, A) is called a relative whole soft set, denoted by $\mathfrak{U}_A$. If $A = E$, then the relative whole soft set is called an absolute soft set, denoted by $\mathfrak{U}_E$.

Definition 2.4 [1] Let $(F, A) \in \mathcal{S}(U)$. Then, the soft set $(F, A)^r = (F^r, A)$ is called a relative complement of (F, A), where $F^r : A \to P(U)$ is a mapping given by $F^r(e_j) = U \setminus F(e_j)$ for all $e_j \in A$.

Example 2.5 Consider the soft set (F, A) in Example 2.2. Then, the relative complement of soft set (F, A) is

$$(F, A)^r = \{(e_1, \{u_2\}), (e_3, \{u_2, u_4\}), (e_4, \{u_1, u_3, u_4\}), (e_5, U)\}.$$

Definition 2.6 [14] Let $(F, A), (G, B) \in \mathcal{S}(U)$. Then

a) (F, A) is a soft subset (G, B), denoted by $(F, A) \tilde{\subseteq} (G, B)$, if $A \subseteq B$ and $F(e_j) \subseteq G(e_j)$ for every $e_j \in A$.

b) (F, A) and (G, B) are equal, denoted by $(F, A) = (G, B)$ if and only if $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\supseteq} (F, A)$.

Definition 2.7 [1] Let $(F, A), (G, B) \in \mathcal{S}(U)$. Then

a) the restricted intersection of (F, A) and (G, B) over U is defined as the soft set $(H, C) = (F, A) \Cap (G, B)$, where $C = A \cap B$ (such that $C \neq \emptyset$) and $H(e_j) = F(e_j) \cap G(e_j)$ for all $e_j \in C$.

b) the restricted union of (F, A) and (G, B) over U is defined as the soft set $(H, C) = (F, A) \Cup (G, B)$, where $C = A \cap B$ (such that $C \neq \emptyset$) and $H(e_j) = F(e_j) \cup G(e_j)$ for all $e_j \in C$.

Lemma 2.8 [1, 2] Let $(F, A) \in \mathcal{S}(U)$.

i) $\Phi_A^r = \mathfrak{U}_A$.

ii) $(F, A) \Cup (F, A)^r = \mathfrak{U}_A$.

iii) $\Phi_A \widetilde{\subseteq} (F,A) \widetilde{\subseteq} \mathfrak{U}_A$.

Proposition 2.9 [2] Let $(F,A),(G,B) \in \mathcal{S}(U)$.

i) $(F,A) \Cap (G,B) = (G,B) \Cap (F,A)$.

ii) $(F,A) \Cup (G,B) = (G,B) \Cup (F,A)$.

Example 2.10 Consider the soft set (F,A) in Example 2.2. Also, let the soft sets $(G,B) = \{(e_2,\{u_1,u_4\}),(e_3,\{u_1,u_3,u_4\}),(e_4,\{u_1,u_3,u_4\}),(e_5,\{u_1,u_4\})\}$ for $B = \{e_2,e_3,e_4,e_5\}$, $(H,C) = \{(e_1,U),(e_3,\{u_1,u_3\}),(e_4,\{u_1,u_2\}),(e_5,\{u_1\}),(e_6,\{u_4\})\}$ for $C = \{e_1,e_3,e_4,e_5,e_6\}$. Then, $(F,A) \widetilde{\nsubseteq} (G,B)$ and $(F,A) \widetilde{\subseteq} (H,C)$. Moreover, the restricted intersection and restricted union of soft sets (F,A) and (G,B) are respectively obtained $(F,A) \Cap (G,B) = \{(e_3,\{u_1,u_3\}),(e_4,\emptyset),(e_5,\emptyset)\}$ and $(F,A) \Cup (G,B) = \{(e_3,\{u_1,u_3,u_4\}),(e_4,U),(e_5,\{u_1,u_4\})\}$.

3. Similarity Coefficient for Soft Sets

In this part, the concept of similarity coefficient between two soft sets is introduced, and also the related theoretical results are presented.

Note: From now on, $|\cdot|$ denotes the cardinality of a set.

Definition 3.1 Let $(F,A),(G,B) \in \mathcal{S}(U)$. Also, let $(M,D) = (K,D) \Cup (L,D)$ where $(K,D) = (F,A) \Cap (G,B)$ and $(L,D) = (F,A)^r \Cap (G,B)^r$ for $D = A \cap B$. Then, similarity coefficient for the soft sets (F,A) and (G,B) is denoted and defined by

$$S_c((F,A),(G,B)) = \frac{\alpha}{n|A \cup B|}$$

where $\alpha = \sum_{j \in J} |M(e_j)|$ for $J = \{j : e_j \in D = A \cap B\}$, and $n = |U|$.

Note that $S_c((F,A),(G,B)) = 0$ if $A \cap B = \emptyset$.

Example 3.2 Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be an initial universe set and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters.

1. If for $A = \{e_1, e_2, e_3\}$ and $B = \{e_1, e_2, e_3\}$,

$$(F, A) = \{(e_1, \{u_1, u_2, u_3\}), (e_2, \{u_2, u_4\}), (e_3, \{u_1, u_3\})\},$$

$$(G, B) = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_4\}), (e_3, \{u_2, u_5\})\}$$

then we obtain the soft sets, for $D = \{e_1, e_2, e_3\}$,

$$(K, D) = (F, A) \Cap (G, B) = \{(e_1, \{u_1, u_2\}), (e_2, \{u_4\}), (e_3, \emptyset)\},$$

$$(L, D) = (F, A)^r \Cap (G, B)^r = \{(e_1, \{u_4, u_5\}), (e_2, \{u_3, u_5\}), (e_3, \{u_4\})\},$$

$$(M, D) = (K, D) \Cup (L, D) = \{(e_1, \{u_1, u_2, u_4, u_5\}), (e_2, \{u_3, u_4, u_5\}), (e_3, \{u_4\})\}.$$

Thus, we find as $S_c((F, A), (G, B)) = \frac{8}{5\times 3} = \frac{8}{15}$.

2. If for $A = \{e_1, e_2, e_3\}$ and $B = \{e_1, e_2, e_3, e_5\}$,

$$(F, A) = \{(e_1, \{u_1, u_2, u_3\}), (e_2, \{u_2, u_4\}), (e_3, \{u_1, u_3\})\},$$

$$(G, B) = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_4\}), (e_3, \{u_2, u_5\}), (e_5, \{u_4, u_5\})\}$$

then we obtain the soft set, for $D = \{e_1, e_2, e_3\}$,

$$(M, D) = (K, D) \Cup (L, D) = \{(e_1, \{u_1, u_2, u_4, u_5\}), (e_2, \{u_3, u_4, u_5\}), (e_3, \{u_4\})\}.$$

Thus, we find as $S_c\big((F, A), (G, B)\big) = \frac{2}{5}$.

3. If for $A = \{e_1, e_2, e_3, e_4\}$ and $B = \{e_1, e_2, e_3, e_5\}$,

$$(F, A) = \{(e_1, \{u_1, u_2, u_3\}), (e_2, \{u_2, u_4\}), (e_3, \{u_1, u_3\}), (e_4, \{u_2, u_3\})\},$$

$$(G, B) = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_4\}), (e_3, \{u_2, u_5\}), (e_5, \{u_4, u_5\})\}$$

then we obtain the soft set, for $D = \{e_1, e_2, e_3\}$,

$$(M, D) = \{(e_1, \{u_1, u_2, u_4, u_5\}), (e_2, \{u_3, u_4, u_5\}), (e_3, \{u_4\})\}.$$

Thus, we find as $S_c((F, A), (G, B)) = \frac{8}{25}$.

4. If for $A = \{e_1, e_2, e_3, e_4\}$ and $B = E$,

$$(F, A) = \{(e_1, \{u_1, u_2, u_3\}), (e_2, \{u_2, u_4\}), (e_3, \{u_1, u_3\}), (e_4, \{u_2, u_3\})\},$$

$$(G, B) = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_4\}), (e_3, \{u_2, u_5\}), (e_4, \emptyset), (e_5, \{u_4, u_5\})\}$$

then we obtain the soft set, for $D = \{e_1, e_2, e_3, e_4\}$,

$$(M, D) = \{(e_1, \{u_1, u_2, u_4, u_5\}), (e_2, \{u_3, u_4, u_5\}), (e_3, \{u_4\}), (e_4, \{u_1, u_4, u_5\})\}.$$

Thus, we find as $S_c((F, A), (G, B)) = \frac{11}{25}$.

5. If for $A = E$ and $B = E$,

$$(F, A) = \{(e_1, \{u_1, u_2, u_3\}), (e_2, \{u_2, u_4\}), (e_3, \{u_1, u_3\}), (e_4, \emptyset), (e_5, \emptyset)\},$$

$$(G, B) = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_4\}), (e_3, \{u_2, u_5\}), (e_4, \emptyset), (e_5, \emptyset)\}$$

then we obtain the soft set, for $D = E$,

$$(M, D) = \{(e_1, \{u_1, u_2, u_4, u_5\}), (e_2, \{u_3, u_4, u_5\}), (e_3, \{u_4\}), (e_4, U), (e_5, U)\}.$$

Thus, we find as $S_c((F, A), (G, B)) = \frac{18}{25}$.

Lemma 3.3 Let $(F, A), (G, B) \in \mathcal{S}(U)$.

i) $S_c((F, A), (F, A)) = 1$.

ii) $S_c((F, A), (F, A)^r) = 0$.

iii) $S_c((F, A), (G, B)) = S_c((G, B), (F, A))$.

iv) $0 \leq S_c((F, A), (G, B)) \leq 1$.

Proof. Let $(F, A), (G, B) \in \mathcal{S}(U)$.

i) Since $(K, D) = (F, A) \Cap (F, A) = (F, A)$ and $(L, D) = (F, A)^r \Cap (F, A)^r = (F, A)^r$, we obtain that $(M, D) = (F, A) \Cup (F, A)^r = \mathfrak{U}_A$, where $D = A$. Then we find as $\alpha = n|A|$. Hence, $S_c((F, A), (F, A)) = 1$.

ii) Since $(K, D) = (F, A) \Cap (F, A)^r = \Phi_A$ and $(K, D) = (F, A)^r \Cap (F, A) = \Phi_A$, we obtain that $(M, D) = \Phi_A$, where $D = A$. Then we find as $\alpha = 0$. Hence, $S_c((F, A), (F, A)) = 0$.

iii) It is trivial from Proposition 2.9, hence omitted.

iv) Since $\Phi_A \tilde{\subseteq} (F, A) \tilde{\subseteq} \mathfrak{U}_A$ and $\Phi_B \tilde{\subseteq} (G, B) \tilde{\subseteq} \mathfrak{U}_B$ by Lemma 2.8, we obtain that $\Phi_D \tilde{\subseteq} (K, D) \tilde{\subseteq} \mathfrak{U}_D$ and $\Phi_D \tilde{\subseteq} (L, D) \tilde{\subseteq} \mathfrak{U}_D$ and so $\Phi_D \tilde{\subseteq} (M, D) \tilde{\subseteq} \mathfrak{U}_D$ where

$D = A \cap B$. Then we find as $0 \le \alpha \le n|A \cap B|$. Since $A \cap B \subseteq A \cup B$, we say that $0 \le S_c((F,A),(G,B)) \le 1$. So, the proof is complete.

Theorem 3.4 Let (F,A), (G,B) and (H,C) be three soft sets over the set U . If $(F,A) \widetilde{\subseteq} (G,B) \widetilde{\subseteq} (H,C)$, then we have the following $S_c((F,A),(H,C)) \le S_c((G,B),(H,C))$ and $S_c((F,A),(H,C)) \le S_c((F,A),(G,B))$.

Proof. Let (F,A), (G,B) and (H,C)be three soft sets over the common universal set U such that $(F,A) \widetilde{\subseteq} (G,B) \widetilde{\subseteq} (H,C)$. Then we can write $F(e_j) \subseteq H(e_j)$and $F^r(e_j) \subseteq H^r(e_j)$ for all $e_j \in A \cap C$. Thus, we have $(K,D_1) = (F,A) \Cap (H,C)$ where $K(e_j) = F(e_j) \cap H(e_j)$=$F(e_j)$ for all $e_j \in D_1$=$A \cap C$, and (L,D_1) =$(F,A)^r \Cap (H,C)^r$ where $L(e_j) = F^r(e_j) \cap H^r(e_j) = H^r(e_j)$ for all $e_j \in D_1 = A \cap C$. Hence, we obtain $(M,D_1) = (K,D_1) \Cup (L,D_1)$ where $M(e_j) = K(e_j) \cup L(e_j) = F(e_j) \cup H^r(e_j)$ for all $e_j \in D_1 = A \cap C$. Similarly, we can write $G(e_j) \subseteq H(e_j)$ and $G^r(e_j) \subseteq H^r(e_j)$ for all $e_j \in B \cap C$. We obtain $(P,D_2) = (G,B) \Cap (H,C)$ where $P(e_j) = G(e_j) \cap H(e_j) = G(e_j)$ for all $e_j \in D_2 = B \cap C$, and $(Q,D_2) = (G,B)^r \Cap (H,C)^r$ where $Q(e_j) = G^r(e_j) \cap H^r(e_j) = H^r(e_j)$ for all $e_j \in D_2 = B \cap C$. So, we have $(R,D_2) = (P,D_2) \Cup (Q,D_2)$ where $R(e_j)$= $P(e_j) \cup Q(e_j)$ $= G(e_j) \cup H^r(e_j)$ for all $e_j \in D_2 = B \cap C$. Then $(M,D_1) \widetilde{\subseteq} (R,D_2)$, that is, $\alpha_1 = \sum_{j \in J_1} |M(e_j)| \le \sum_{j \in J_2} |R(e_j)| = \alpha_2$ for $J_1 = \{ j: e_j \in D_1\} \subseteq \{j: e_j \in D_2\} = J_2$. Hence, we obtain $S_c((F,A),(H,C)) = \frac{\alpha_1}{n|C|} \le \frac{\alpha_2}{n|C|} = S_c((G,B),(H,C))$.

On the other hand, we can write $F(e_j) \subseteq G(e_j)$ and $F^r(e_j) \subseteq G^r(e_j)$ for all $e_j \in A \cap B$. Hence, we have (S,D_3)=$(F,A) \Cap (G,B)$ where $S(e_j) = F(e_j) \cap G(e_j) = F(e_j)$ for all $e_j \in D_3 = A \cap B$, and $(T,D_3) = (F,A)^r \Cap (G,B)^r$ where $T(e_j) = F^r(e_j) \cap G^r(e_j) = G^r(e_j)$ for all $e_j \in D_3 = A \cap B$. Hence, we obtain $(V,D_3) = (S,D_3) \Cup (T,D_3)$ where $V(e_j) = S(e_j) \cup T(e_j) = F(e_j) \cup G^r(e_j)$ for all $e_j \in D_3 = A \cap B$. Then, we calculate as $\alpha_1 = \sum_{j \in J_1} |M(e_j)| \le \sum_{j \in J_3} |V(e_j)| = \alpha_3$ where $J_1 = \{ j: e_j \in A \cap C = A\}$ and $J_3 = \{ j: e_j \in A \cap B = A\}$. Therefore, we obtain obtain $S_c((F,A),(H,C)) = \frac{\alpha_1}{n|C|} \le \frac{\alpha_3}{n|B|} = S_c((F,A),(G,B))$ since $B \subseteq C$.

Example 3.5 Let $U = \{u_1, u_2, u_3, u_4\}$ be an initial universal set, and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ be the parameter set. Also, for the parameter subsets $A = \{e_1, e_2\}$, $B = \{e_1, e_2, e_4\}$, $C = \{e_1, e_2, e_4, e_5, e_7\}$, we consider the following soft sets: $(F, A) = \{(e_1, \{u_1, u_3\}), (e_2, \{u_3\})\}$, $(G, B) = \{(e_1, \{u_1, u_3\}), (e_2, \{u_3, u_4\}), (e_4, \{u_1\})\}$ and $(H, C) = \{(e_1, \{u_1, u_3, u_4\}), (e_2, U), (e_4, \{u_1\}), (e_5, \{u_1, u_4\}), (e_7, \{u_2, u_3, u_4\})\}$. By Definition 2.6, it is clear that $(F, A) \widetilde{\subseteq} (G, B) \widetilde{\subseteq} (H, C)$. Furthermore, by using Definition 3.1, we obtain $S_c((F, A), (H, C)) = \frac{1}{5} < \frac{9}{20} = S_c((G, B), (H, C))$ and $S_c((F, A), (H, C)) = \frac{1}{5} < \frac{7}{12} = S_c((F, A), (G, B))$.

Proposition 3.6 Let (F, A), (G, A) and (H, A) be three soft sets over U. If $S_c((F, A), (G, A)) = S_c((G, A), (H, A)) = 1$, then $S_c((F, A), (H, A)) = 1$.

Proof. Let Let (F, A), (G, A) and (H, A) be the soft sets over U and $S_c((F, A), (G, A)) = S_c((G, A), (H, A)) = 1$. $S_c((F, A), (G, A)) = 1$ implies $\alpha_1 = \sum_{j \in J_1} |M(e_j)| = n|A|$ for $J_1 = \{j : e_j \in A\}$. Thus, $M(e_j) = U$ for every $e_j \in A \subseteq E$. This means that $u_i \in F(e_j) \Rightarrow u_i \in G(e_j)$ or $u_i \notin F(e_j) \Rightarrow u_i \notin G(e_j)$ for every $u_i \in U$ since $(M, A) = ((F, A) \Cap (G, A)) \Cup ((F, A)^r \Cap (G, A)^r)$. Similarly, we show that if $S_c\big((G, A), (H, A)\big) = 1$ then it should be $u_i \in G(e_j) \Rightarrow u_i \in H(e_j)$ or $u_i \notin G(e_j) \Rightarrow u_i \notin H(e_j)$ for every $u_i \in U$. Hence, it can be written as $u_i \in F(e_j) \Rightarrow u_i \in H(e_j)$ or $u_i \notin F(e_j) \Rightarrow u_i \notin H(e_j)$ for every $u_i \in U$. Therefore, it is obtained that $\big((F, A) \Cap (H, A)\big) \Cup ((F, A)^r \Cap (H, A)^r) = \mathfrak{U}_A$, that is, $S_c((F, A), (H, A)) = 1$.

Definition 3.7 Let (F, A) and (G, B) be two soft sets over U. Then (F, A) and (G, B) are said to be ℓ-similar if and only if $S_c((F, A), (G, B)) \geq \ell$ for $\ell \in (0,1)$. If $S_c((F, A), (G, B)) > \frac{1}{2}$ then two soft sets are called significantly similar.

Example 3.8 Let us consider the soft sets (F, A), (G, B) and (H, C) in Example 3.5. Since $S_c((F, A), (H, C)) = \frac{1}{5}$, (F, A) and (H, C) are ℓ-similar where $\ell \in (0, \frac{1}{5}]$. Moreover, (F, A) and (G, B) are significantly similar since $S_c((F, A), (G, B)) = \frac{7}{12} > \frac{1}{2}$.

4. Application

In this part, various practical applications are presented to illustrate the usefulness of similarity coefficient of the soft sets.

Example 4.1 Assume that a company Y operates in four different business areas. The set of these business areas is $U = \{u_1, u_2, u_3, u_4\}$, where u_1= packed products, u_2= electronic devices, u_3= textiles and u_4= paper manufacturing. The company Y wants to make an agreement with any of the companies X_1, X_2 and X_3 which operates in the same business areas. The purpose of this agreement is to make both companies (Y and X_k) more efficient in these business areas. In other words, they are companies that complete each other's shortcomings. Let $E = \{e_1, e_2, e_3, e_4\}$ be a set of all parameters which is specified by the company Y, where $e_j, j = 1,2,3,4$ stand for “profitability”, “customer delight”, “staff efficiency”, and “technology systems”, respectively.
According to this data, suppose that the following soft sets are created.

For the company Y,

$$(G, E) = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_3, u_4\}), (e_3, \{u_1, u_2, u_4\}), (e_4, \{u_2, u_4\})\}.$$

For the company X_1,

$$(F_1, E) = \{(e_1, \{u_2, u_4\}), (e_2, \{u_1, u_3\}), (e_3, \{u_2, u_4\}), (e_4, U)\}.$$

For the company X_2,

$$(F_2, E) = \{(e_1, \{u_1, u_4\}), (e_2, \{u_3\}), (e_3, \{u_3, u_4\}), (e_4, \{u_1, u_3\})\}.$$

For the company X_3,

$$(F_3, E) = \{(e_1, \{u_1, u_2\}), (e_2, \{u_3, u_4\}), (e_3, \{u_1, u_3, u_4\}), (e_4, \{u_2\})\}.$$

Let us try to solve this problem by using the concept of similarity coefficient. The similarity coefficients for soft sets of companies Y and X_k are calculated as follows:

$$S_c((G, E), (F_1, E))=\frac{5}{8},\ S_c((G, E), (F_2, E))=\frac{5}{16} \text{ and } S_c((G, E), (F_3, E))=\frac{3}{4}.$$

Since the similarity coefficient between the soft sets of company Y and company X_2 is the lowest value, they are the most suitable companies for this agreement on completing each other's shortcomings in the determined business areas. Consequently, according to the above evaluations, we say that the company Y can make an agreement with the company X_2.

Example 4.2 The similarity coefficient of soft sets can be used to detect whether a patient is suffering from a certain disease or not. An attempt is made to estimate the possibility that a psychological patient having certain visible symptoms is suffering from depression. For this purpose, a model soft set for depression and the soft set for the patient are created. Later on, the similarity coefficient of two soft sets is computed. It is determined that the patient is possibly suffering from depression if the soft sets are significantly similar.

Assume that the universe set consists of only two elements which are yes and no, that is, $U = \{y, n\}$. $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ is the set of parameters which are certain visible symptoms. These parameters are "pessimism", "panic attack", "anorexia", "heart throbbing", "loss of concentration", "sleep disturbance", "future anxiety", respectively. With the help of a psychologist, a model soft set for depres- sion are created as follows:

$$(F, E) = \{(e_1, \{y\}), (e_2, \{n\}), (e_3, \{y\}), (e_4, \{n\}), (e_5, \{y\}), (e_6, \{y\}), (e_7, \{n\})\}.$$

After talking to two patients having psychological disorders, the following soft sets are constructed.

The soft set of first patient is as follows:

$$(G, E) = \{(e_1, \{n\}), (e_2, \{y\}), (e_3, \{n\}), (e_4, \{n\}), (e_5, \{y\}), (e_6, \{n\}), (e_7, \{y\})\}.$$

The soft set of second patient is as follows:

$$(H, E) = \{(e_1, \{y\}), (e_2, \{n\}), (e_3, \{n\}), (e_4, \{n\}), (e_5, \{y\}), (e_6, \{y\}), (e_7, \{y\})\}.$$

Then $S_c((F, E), (G, E))=\frac{2}{7}$, that is, the soft sets (F, E) and (G, E) are not significantly similar. Hence, it is determined that the first patient who is psychological disorder is not possibly suffering from depression. However, $S_c((F, E), (H, E))=\frac{5}{7}$, that is, the soft sets (F, E) and (H, E) are significantly similar. Hence, it is determined that the second patient who is psychological disorder is possibly suffering from depression.

5. Conclusion

In this paper, we proposed a novel approach for the similarity of soft sets. Also, we put forward that this approach makes easier to handle some problems containing the uncertainty or unknown data. This convenience manifests itself in applications, as well. Therefore, we gave various applications aiming to solve the problems in the fields such as medicine, buying and selling. Consequently, due to the wide application area of soft set, we expect that this proposed approach for the similarity between two soft sets will have a wide range in the literature in the coming years.

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