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A note on the stratified domination number of generalized planar Petersen like graphs $PP(n, 2)$

Mehmet Şerif Aldemir

ABSTRACT

Let G be a graph with the vertex set $V(G)$. G is called 2-stratified if $V(G)$ is partitioned into red and blue vertices. The stratified domination number of a graph G is the minimum number of red vertices of $V(G)$ in a red-blue coloring of the vertices of $V(G)$ such that every blue vertex v of $V(G)$ lies in a vuw (blue, blue, red) path in G for a blue vertex $u \in V(G)$ $(u \neq v)$ and a red vertex $w \in V(G)$. In this paper we first define the concept of generalized planar Petersen like graphs $PP(n, 2)$ for any positive odd integers and study the stratified domination number of generalized planar Petersen like graphs $PP(n, 2)$. We prove that for $n \geq 5$, $\gamma_F(PP(n, 2)) = 2\left\lceil \frac{n-1}{6} \right\rceil + 1$

1. INTRODUCTION

Let G be a graph with the vertex set $V(G)$. G is called 2-stratified if $V(G)$ is partitioned into red and blue vertices. The stratified domination number of a graph G is the minimum number of red vertices of $V(G)$ in a red-blue coloring of the vertices of $V(G)$ such that every blue vertex v of $V(G)$ lies in a vuw (blue, blue, red) path in G for a blue vertex $u \in V(G)$ $(u \neq v)$ and a red vertex $w \in V(G)$.

Stratified graph theory have been invented by Rashidi [1]. See [2–16] for further studies on stratified domination.

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All the results so far are on upper and lower bounds and exact values for special graphs of stratified domination. For convenience *stratified domination* denoted as *F-domination.*
The domination number of generalized Petersen graphs has been well studied in graph theory. In recent years, there have been many results on generalized Petersen graphs and related to domination parameters. See [17–24] and references therein. We know that for any positive even integer, generalized Petersen graphs $P(n, 2)$ are planar. And for any positive odd integer, generalized Petersen graphs $P(n, 2)$ are not planar. In the next section we give the definition of generalized planar Petersen like graphs $PP(n, 2)$ for any positive odd integer and study the stratified domination number of generalized planar Petersen like graphs $PP(n, 2)$.

2. THE STRATIFIED DOMINATION NUMBER OF GENERALIZED PLANAR PETERSEN LIKE GRAPHS $PP(N, 2)$

The generalized planar Petersen like graphs $PP(n, 2)$ is defined for only positive odd integers.

Definition 1. *The generalized planar Petersen like graph $PP(n, 2)$ is the graph with vertex set $V(P(n, 2)) = U \cup W$, where $U = \{u_i : 1 \leq i \leq n\}$ and $W = \{w_i : 1 \leq i \leq n\}$, and the edge set $E(PP(n, 2)) = \{u_iu_{i+1}, u_iw_i, w_iw_{i+2} : 1 \leq i \leq n-2$, subscripts modulo $n\} \cup \{w_nw_1\}$*

.

And now we begin to compute the stratified domination number in $PP(n, 2)$. For convenience to show planarity we prefer to show $PP(n, 2)$ as the form of three cycles one within the other. The outer cycle denoted by W_o which is consists of the vertices $w_1, w_3, ..., w_n$. The middle cycle consists of the vertices of U. And the inner cycle denoted by W_i which is consists of the vertices $w_2, w_4, ..., w_{n-1}$ See Figures 2,3,4.

Theorem 2. *For any positive odd integer $n \geq 5$,*

$$\gamma_F(PP(n, 2)) = 2\left\lceil \frac{n-1}{6} \right\rceil + 1.$$

Proof. There are three cases.
Case 1: Let $n \equiv 0\,(mod\,3)$. We show first that there exists an F_3-coloring of $PP(n, 2)$ that colors w_1,w_7,...,w_{n-14},w_{n-8},w_{n-2},w_n (the vertices of W_o) and w_2,w_8, ...,w_{n-7},w_{n-1} (the vertices of W_i) red. Let S=$\{w_1$,w_7,...,w_{n-14},w_{n-8},w_{n-2},w_n,w_2,w_8 ,...,w_{n-7},$w_{n-1}\}$. Consider this F_3-coloring of $PP(n, 2)$. Note that all the blue vertices of the cycles U (the middle cycle), W_o (the outer cycle) and W_i (the inner cycle) of $PP(n, 2)$ lie in a blue-blue-red path in $P(n, 2)$ (see Fig.2). Therefore, the vertex set S is an F-dominating set of $PP(n, 2)$. Hence, $\gamma_F(PP(n, 2)) \leq 2\left\lceil \frac{n-1}{6} \right\rceil + 1$ for $n \equiv 0(mod\,3)$.
And now we show that $\gamma_F(PP(n, 2)) \geq 2\left\lceil \frac{n-1}{6} \right\rceil + 1$ for $n \equiv 0(mod\,3)$. We proceed

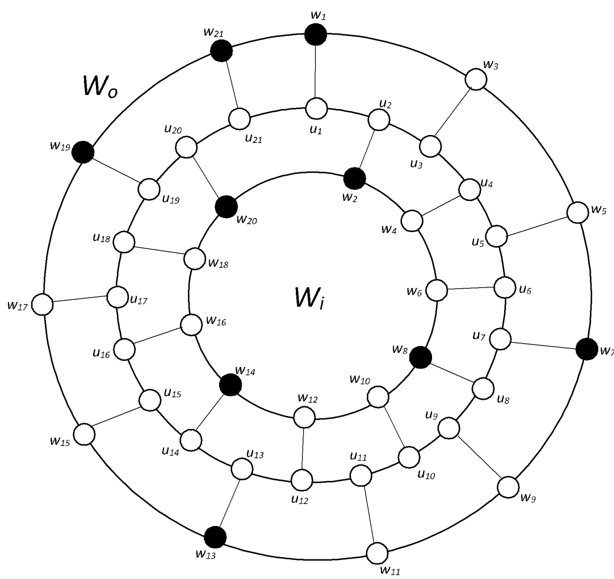

Figure 1: The minimum F domination in $PP(21, 2)$ for the Case 1 of Theorem 1

by induction on the order n. If $n = 9$, the claim is trivial. Suppose that $n \geq 9$. Let accept that the claim is true for $n \geq 15$. We show that the claim is true for $n + 6$. We know that the set S=$\{w_1$,w_7,...,w_{n-14},w_{n-8},w_{n-2},w_n,w_2,w_8,...,w_{n-7},$w_{n-1}\}$ is an F-dominating set of $PP(n, 2)$. We must add 12 extra vertex to $PP(n, 2)$ for to acquire $PP(n + 6, 2)$. For the new added blue vertices must be lied in a blue-blue-red path, we must color red the vertices w_{n+5} and w_{n+6}. Therefore $\gamma_F(PP(n+6, 2)) = \gamma_F(PP(n, 2)) + 2$. Hence $\gamma_F(PP(n+6, 2)) \geq 2\left\lceil \frac{n-1}{6}\right\rceil + 1 + 2 = 2\left\lceil \frac{n+5}{6}\right\rceil + 1 = 2\left\lceil \frac{(n+6)-1}{6}\right\rceil + 1$. So the proof is completed.

Case 2: Let $n \equiv 1\,(mod\,3)$. We show first that there exists an F-coloring of $PP(n, 2)$ that colors w_1,w_7,...,w_{n-12},w_{n-6},w_n (the vertices of W_o) and w_2,w_8,..., w_{n-11},w_{n-5}, (the vertices of W_i) red. Consider this F-coloring of $PP(n, 2)$. Notice that all the blue vertices of the middle cycle U, all the blue vertices of the outer cycle W_o and all the blue vertices of the inner cycle W_i of $PP(n, 2)$ lie in a blue-blue-red path of $PP(n, 2)$(see Fig.3). Therefore $\gamma_F(PP(n, 2)) \leq 2\left\lceil \frac{n-1}{6}\right\rceil + 1$.

And now we show that $\gamma_F(PP(n, 2)) \geq 2\left\lceil \frac{n-1}{6}\right\rceil + 1$ for $n \equiv 1(mod\,3)$. We proceed by induction on the order n. If $n = 7$, the claim is trivial. Let accept that the claim is true for $n \geq 13$. We show that the claim is true for $n + 6$. We know that the set S=$\{w_1$,w_7,...,w_{n-12},w_{n-6},w_n,w_2,w_8,...,w_{n-11},$w_{n-5}\}$ is an F_3-dominating set of $PP(n, 2)$. We must add 12 extra vertex to $PP(n, 2)$ for to acquire

$PP(n+6,2)$. For the new added blue vertices must be lied in a blue-blue-red path, we must color red the vertices w_{n+1} and w_{n+6}. Therefore $\gamma_F(PP(n+6,2)) = \gamma_{F_3}(P(n,2)) + 2$. Hence $\gamma_F(PP(n+6,2)) \geq 2\left\lceil \frac{n-1}{6} \right\rceil + 1 + 2 = 2\left\lceil \frac{n+5}{6} \right\rceil + 1 = 2\left\lceil \frac{(n+6)-1}{6} \right\rceil + 1$. So the proof is completed.

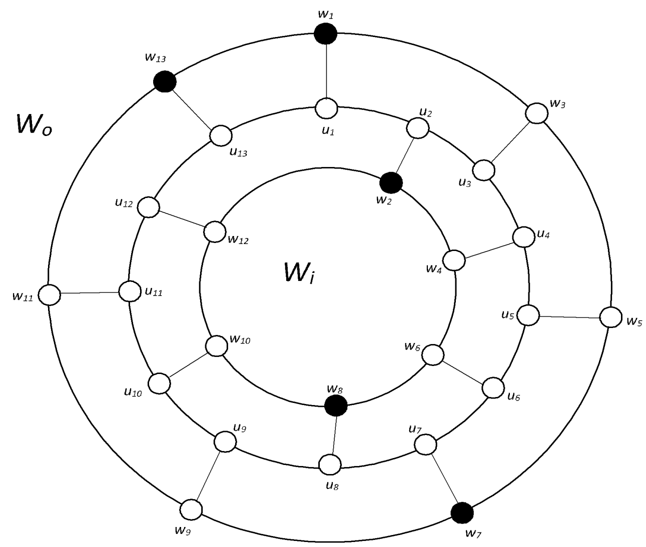

Figure 2: The minimum F domination in $PP(13,2)$ for the Case 2 of Theorem 1

Case 3: Let $n \equiv 2\,(mod\,3)$. We show first that there exists an F-coloring of $PP(n,2)$ that colors w_1,w_7,...,w_{n-16},w_{n-10},w_{n-4} (the vertices of W_o), w_2,w_8,..., w_{n-9},w_{n-3}(the vertices of W_i) and the vertex of u_n of U red. Consider this F-coloring of $PP(n,2)$. Notice that all the blue vertices of the $PP(n,2)$ lie in a blue-blue-red path of $PP(n,2)$ (see Fig.3). Therefore $\gamma_F(PP(n,2)) \leq 2\left\lceil \frac{n-1}{6} \right\rceil + 1$.

And now we show that $\gamma_F(PP(n,2)) \geq 2\left\lceil \frac{n-1}{6} \right\rceil + 1$ for $n \equiv 2(mod\,3)$. We proceed by induction on the order n. If $n = 11$, the claim is trivial. Let accept that the claim is true for $n \geq 11$. We show that the claim is true for $n+6$. We know that the set S={ w_1,w_7,...,w_{n-16},w_{n-10},w_{n-4},w_2,w_8,...,w_{n-9},w_{n-3},u_n } is an F-dominating set of $PP(n,2)$. We must add 12 extra vertex to $P(n,2)$ for to acquire $PP(n+6,2)$. For the new added blue vertices must be lied in a blue-blue-red path, we must color red the vertices u_{n+3} and u_{n+4}. Therefore $\gamma_F(PP(n+6,2)) = \gamma_F(P(n,2)) + 2$. Hence $\gamma_F(PP(n+6,2)) \geq 2\left\lceil \frac{n-1}{6} \right\rceil + 2 + 2 = 2\left\lceil \frac{n+5}{6} \right\rceil + 2 = 2\left\lceil \frac{(n+6)-1}{6} \right\rceil + 2$. So the proof is completed.

□

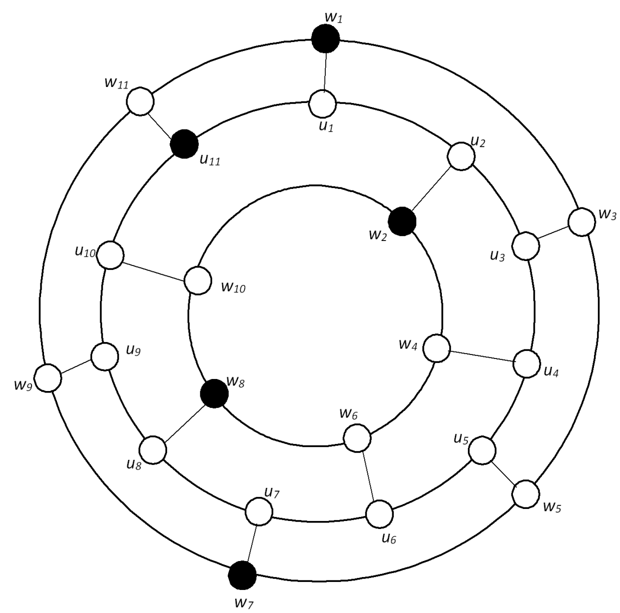

Figure 3: The minimum F domination in $PP(11, 2)$ for the Case 3 of Theorem 1

3. CONCLUSION

In this study we first define the concept of generalized planar Petersen like graphs $PP(n, 2)$ for any positive odd integer and study the stratified domination number of generalized planar Petersen like graphs $PP(n, 2)$. It can be interesting to study the other domination type parameters of generalized planar Petersen like graphs $PP(n, 2)$ for further studies.

REFERENCES

1. R. Rashidi, The Theory and Applications of Stratified Graphs (Ph.D. Dissertation, Western Michigan University, 1994).
2. G. Chartrand, H. Gavlas, M.A. Henning and R. Rashidi, Stratidistance in stratified graphs, Math. Bohem. 122 (1997) 337-347.
3. G. Chartrand, L. Holley, R. Rashidi, N.A. Sherwani, Distance in stratified graphs, Czech. Math. J. 125 (2000) 135-146.
4. G. Chartrand, T.W. Haynes, M.A. Henning, P. Zhang, Stratification and domination in graphs, Discrete Math. 272 (2003) 171-185.
5. G. Chartrand, T.W. Haynes, M.A. Henning, P. Zhang, Stratified claw domination in prisms, J. Combin. Math. Combin. Comput. 33 (2000) 81-96.

6. R. Gera, P. Zhang, Bounds for the F-domination number of a graph, Congr. Numer. 166 (2004) 131-144.
7. R. Gera, P. Zhang, Realizable triples for stratified domination in graphs, Math. Bohem. 130 (2005) 185-202.
8. R. Gera, P. Zhang, On stratified domination in oriented graphs, Congr. Numer. 173 (2005) 175-192.
9. R. Gera, P. Zhang, On stratification and domination in graphs, Discuss. Math. Graph Theory 26 (2006) 249-272.
10. R. Gera, P. Zhang, Stratified domination in oriented graphs, J. Combin. Math. Combin. Comput. 60 (2007) 105-125.
11. M.A. Henning, J.E. Maritz, Stratification and domination in graphs II, Discrete Math. 286 (2004) 203-211.
12. M.A. Henning, J.E. Maritz, Stratification and domination in graphs with minimum degree two, Discrete Math. 301 (2005) 175-194.
13. M.A. Henning, J.E. Maritz, Stratification and domination in graphs with minimum degree two, Discrete Math. 301 (2005) 175-194.
14. M.A. Henning, J.E. Maritz, Stratification and domination in prisms, Ars Combin. 81 (2006) 343-358.
15. M.A. Henning, J.E. Maritz, Simultaneous stratification and domination in graphs with minimum degree two, Quaest. Math. 29 (2006) 1-16.
16. T.W. Haynes, M.A. Henning, P. Zhang, A survey of stratified domination in graphs, Discrete Math. 309 (2009) 5806-5819.
17. G. J. Chang, C. W. Chang, D. Kuo, S.H. Poon, Algorithmic aspect of stratified domination in graphs, Information Processing Letters 113 (2013) 861-865
18. C. W. Chang, D. Kuo, S.C. Liaw and J.H. Yan, F_3-domination problems in graphs, Journal of Combinatorial Optimization 28 (2014) 400-413.
19. J. Liu, X. Zhang, The exact domination number of generalized Petersen graphs, Comp. Appl. Math. (2014) 33:497-506.
20. W. S. Li, H.M. Xing, M. Y. Sohn, On the signed total domination number of generalized Petersen graphs $P(n,2)$, Bull. Korean Math. Soc. 50 (2013) 2021-2026.
21. X. Fu, Y. Yang, B. Jiang, On the domination number of generalized Petersen graphs $P(n,2)$, Discrete Mathematics 309 (2009) 2445-2451.
22. H. Wang, X. Xu, Y. Yang, K. Lü, Liar's domination number of generalized Petersen graphs $P(n,1)$ and $P(n,2)$, Util. Math. 82 (2012) 317-335.
23. R. Barrera, D. Ferrero, Power domination in cylinders, tori, and generalized Petersen graphs, Networks 58 (2011) 43-49.
24. Z. Shao, H. Jiang, P. Wu, S. Wang, J. erovnik, X. Zhang, J.B Liu, On 2-rainbow domination of generalized Petersen graphs. Discrete Appl. Math. 257 (2019), 370384.

Mehmet Şerif Aldemir
Faculty of Science, Van Yuzuncu Yil University, Van, Turkey.
E-mail: msaldemir@yyu.edu.tr