A note on the stratified domination number of generalized planar Petersen like graphs $PP(n, 2)$

Mehmet Serif Aldemir

ABSTRACT

Let $G$ be a graph with the vertex set $V(G)$. $G$ is called 2-stratified if $V(G)$ is partitioned into red and blue vertices. The stratified domination number of a graph $G$ is the minimum number of red vertices of $V(G)$ in a red-blue coloring of the vertices of $V(G)$ such that every blue vertex $v$ of $V(G)$ lies in a $uvw$ (blue, blue, red) path in $G$ for a blue vertex $u \in V(G) \ (u \neq v)$ and a red vertex $w \in V(G)$. In this paper we first define the concept of generalized planar Petersen like graphs $PP(n, 2)$ for any positive odd integers and study the stratified domination number of generalized planar Petersen like graphs $PP(n, 2)$. We prove that for $n \geq 5$, $\gamma_F(PP(n, 2)) = 2 \left\lceil \frac{n-1}{6} \right\rceil + 1$

1. INTRODUCTION

Let $G$ be a graph with the vertex set $V(G)$. $G$ is called 2-stratified if $V(G)$ is partitioned into red and blue vertices. The stratified domination number of a graph $G$ is the minimum number of red vertices of $V(G)$ in a red-blue coloring of the vertices of $V(G)$ such that every blue vertex $v$ of $V(G)$ lies in a $uvw$ (blue, blue, red) path in $G$ for a blue vertex $u \in V(G) \ (u \neq v)$ and a red vertex $w \in V(G)$.

Stratified graph theory have been invented by Rashidi [1]. See [2–16] for further studies on stratified domination.
All the results so far are on upper and lower bounds and exact values for special graphs of stratified domination. For convenience stratified domination denoted as $F$-domination. The domination number of generalized Petersen graphs has been well studied in graph theory. In recent years, there have been many results on generalized Petersen graphs and related to domination parameters. See [17–24] and references therein. We know that for any positive even integer, generalized Petersen graphs $P(n, 2)$ are planar. And for any positive odd integer, generalized Petersen graphs $P(n, 2)$ are not planar. In the next section we give the definition of generalized planar Petersen like graphs $PP(n, 2)$ for any positive odd integer and study the stratified domination number of generalized planar Petersen like graphs $PP(n, 2)$.

2. THE STRATIFIED DOMINATION NUMBER OF GENERALIZED PLANAR PETERSEN LIKE GRAPHS $PP(N, 2)$

The generalized planar Petersen like graphs $PP(n, 2)$ is defined for only positive odd integers.

**Definition 1.** The generalized planar Petersen like graph $PP(n, 2)$ is the graph with vertex set $V(PP(n, 2)) = U \cup W$, where $U = \{u_i : 1 \leq i \leq n\}$ and $W = \{w_i : 1 \leq i \leq n\}$, and the edge set $E(PP(n, 2)) = \{u_iu_{i+1}, u_iw_i, w_iw_{i+2} : 1 \leq i \leq n - 2, \text{subscripts modulo } n\} \cup \{w_nw_1\}$.

And now we begin to compute the stratified domination number in $PP(n, 2)$. For convenience to show planarity we prefer to show $PP(n, 2)$ as the form of three cycles one within the other. The outer cycle denoted by $W_o$ which is consists of the vertices $w_1, w_3, ..., w_n$. The middle cycle consists of the vertices of $U$. And the inner cycle denoted by $W_i$ which is consists of the vertices $w_2, w_4, ..., w_{n-1}$ See Figures 2,3,4.

**Theorem 2.** For any positive odd integer $n \geq 5$,

$$\gamma_F(PP(n, 2)) = 2 \left\lceil \frac{n-1}{6} \right\rceil + 1.$$

**Proof.** There are three cases.

**Case 1:** Let $n \equiv 0 \pmod{3}$. We show first that there exists an $F_3$-coloring of $PP(n, 2)$ that colors $w_1, w_7, w_{n-14}, w_{n-8}, w_{n-2}, w_n$ (the vertices of $W_o$) and $w_3, w_8, ..., w_{n-7}, w_{n-1}$ (the vertices of $W_i$) red. Let $S = \{w_1, w_7, w_{n-14}, w_{n-8}, w_{n-2}, w_n, w_3, w_8, ..., w_{n-7}, w_{n-1}\}$. Consider this $F_3$-coloring of $PP(n, 2)$. Note that all the blue vertices of the cycles $U$ (the middle cycle), $W_o$ (the outer cycle) and $W_i$ (the inner cycle) of $PP(n, 2)$ lie in a blue-blue-red path in $P(n, 2)$ (see Fig.2). Therefore, the vertex set $S$ is an $F$-dominating set of $PP(n, 2)$. Hence, $\gamma_F(PP(n, 2)) \leq 2 \left\lceil \frac{n-1}{6} \right\rceil + 1$ for $n \equiv 0 \pmod{3}$.

And now we show that $\gamma_F(PP(n, 2)) \geq 2 \left\lceil \frac{n-1}{6} \right\rceil + 1$ for $n \equiv 0 \pmod{3}$. We proceed
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Figure 1: The minimum $F$ domination in $PP(21, 2)$ for the Case 1 of Theorem 1

by induction on the order $n$. If $n = 9$, the claim is trivial. Suppose that $n \geq 9$. Let accept that the claim is true for $n \geq 15$. We show that the claim is true for $n + 6$. We know that the set $S = \{w_1, w_7, ..., w_{n-14}, w_{n-8}, w_{n-2}, w_n, w_2, w_8, ..., w_{n-7}, w_{n-1}\}$ is an $F$-dominating set of $PP(n, 2)$. We must add 12 extra vertex to $PP(n, 2)$ for to acquire $PP(n + 6, 2)$. For the new added blue vertices must be lied in a blue-blue-red path, we must color red the vertices $w_{n+5}$ and $w_{n+6}$. Therefore $\gamma_F(PP(n + 6, 2)) = \gamma_F(PP(n, 2)) + 2$. Hence $\gamma_F(PP(n + 6, 2)) \geq 2 \left\lceil \frac{n+1}{6} \right\rceil + 1 + 2 = 2 \left\lceil \frac{n+7}{6} \right\rceil + 1 = 2 \left\lceil \frac{n+6}{6} \right\rceil + 1$. So the proof is completed.

Case 2: Let $n \equiv 1 \pmod{3}$. We show first that there exists an $F$-coloring of $PP(n, 2)$ that colors $w_1, w_7, ..., w_{n-12}, w_{n-6}, w_n$ (the vertices of $W_o$) and $w_2, w_8, ..., w_{n-11}, w_{n-5}$, (the vertices of $W_i$) red. Consider this $F$-coloring of $PP(n, 2)$. Notice that all the blue vertices of the middle cycle $U$, all the blue vertices of the outer cycle $W_o$ and all the blue vertices of the inner cycle $W_i$ of $PP(n, 2)$ lie in a blue-blue-red path of $PP(n, 2)$ (see Fig.3). Therefore $\gamma_F(PP(n, 2)) \leq 2 \left\lceil \frac{n+1}{6} \right\rceil + 1$.

And now we show that $\gamma_F(PP(n, 2)) \geq 2 \left\lceil \frac{n-1}{6} \right\rceil + 1$ for $n \equiv 1 \pmod{3}$. We proceed by induction on the order $n$. If $n = 7$, the claim is trivial. Let accept that the claim is true for $n \geq 13$. We show that the claim is true for $n + 6$. We know that the set $S = \{w_1, w_7, ..., w_{n-12}, w_{n-6}, w_n, w_2, w_8, ..., w_{n-11}, w_{n-5}\}$ is an $F_3$-dominating set of $PP(n, 2)$. We must add 12 extra vertex to $PP(n, 2)$ for to acquire
For the new added blue vertices must be lied in a blue-blue-red path, we must color red the vertices $w_{n+1}$ and $w_{n+6}$. Therefore $\gamma_F(PP(n+6,2)) = \gamma_F(P(n,2)) + 2$. Hence $\gamma_F(PP(n+6,2)) \geq 2 \left\lceil \frac{n-1}{6} \right\rceil + 1 + 2 = 2 \left\lceil \frac{n+5}{6} \right\rceil + 1 = 2 \left\lceil \frac{(n+6)-1}{6} \right\rceil + 1$. So the proof is completed.

Figure 2: The minimum $F$ domination in $PP(13,2)$ for the Case 2 of Theorem 1

**Case 3:** Let $n \equiv 2 \pmod{3}$. We show first that there exists an $F$-coloring of $PP(n,2)$ that colors $w_1,w_7,...,w_{n-16},w_{n-10},w_{n-4}$ (the vertices of $W_o$), $w_2,w_8,...,w_{n-9},w_{n-3}$ (the vertices of $W_i$) and the vertex of $u_n$ of $U$ red. Consider this $F$-coloring of $PP(n,2)$. Notice that all the blue vertices of the $PP(n,2)$ lie in a blue-blue-red path of $PP(n,2)$ (see Fig.3). Therefore $\gamma_F(PP(n,2)) \leq 2 \left\lceil \frac{n+5}{6} \right\rceil + 1$.

And now we show that $\gamma_F(PP(n,2)) \geq 2 \left\lceil \frac{n-1}{6} \right\rceil + 1$ for $n \equiv 2\pmod{3}$. We proceed by induction on the order $n$. If $n = 11$, the claim is trivial. Let accept that the claim is true for $n \geq 11$. We show that the claim is true for $n+6$. We know that the set $S=\{w_1,w_7,...,w_{n-16},w_{n-10},w_{n-4},w_2,w_8,...,w_{n-9},w_{n-3},u_n\}$ is an $F$-dominating set of $PP(n,2)$. We must add 12 extra vertex to $P(n,2)$ for to acquire $PP(n+6,2)$. For the new added blue vertices must be lied in a blue-blue-red path, we must color red the vertices $u_{n+3}$ and $u_{n+4}$. Therefore $\gamma_F(PP(n+6,2)) = \gamma_F(P(n,2)) + 2$. Hence $\gamma_F(PP(n+6,2)) \geq 2 \left\lceil \frac{n-1}{6} \right\rceil + 2 + 2 = 2 \left\lceil \frac{n+5}{6} \right\rceil + 2$. So the proof is completed.
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Figure 3: The minimum $F$ domination in $PP(11, 2)$ for the Case 3 of Theorem 1

3. CONCLUSION

In this study we first define the concept of generalized planar Petersen like graphs $PP(n, 2)$ for any positive odd integer and study the stratified domination number of generalized planar Petersen like graphs $PP(n, 2)$. It can be interesting to study the other domination type parameters of generalized planar Petersen like graphs $PP(n, 2)$ for further studies.

REFERENCES


Mehmet Şerif Aldemir
Faculty of Science, Van Yuzuncu Yil University, Van, Turkey.
E-mail: msaldemir@yuu.edu.tr