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DEBATE BETWEEN WİLLİNG AND MC KENZİE ON WELFARE MEASURE

Yard.Doç.Dr. Recep SAYMAN
University of Istanbul,
Faculty of Economics

At the Comment on the R. Willing's article of "Consumer Surplus Without Apology," published by AER, 6/79, G.W. McKenzie basically tries to prove that the welfare indicator presented by him and F. Pearce, is superior to Willing's indicator (actually it might be). To do that he picks certain sections (or subsections) and point from Willig's paper (probably without considering Willing's article as a whole) and comes to following conclusions:

a- Willing's indicator can not be easily generalized to cases where prices of several commodities vary. According to him, Willing's welfare measure is based on constant income elasticity of demand (or consumer preferences are homotetic) and as a result, his consumer surplus measure is not generalizable to the case of more than one good, if the matrix of uncompensated price effects is symmetric (condition of homoteticity) therefore, his indicator is valid only for rare cases and he failed to establish general conditions under which consumer surplus theoretically sound or empirically accurate.

b- McKenzie's second point is loosely approximation. "Approximations of compensating and equivalent variations by using consumer surplus neglects the fact that the remainder term involved may be quite substantial if consumer surplus (A) is not very small." To prove it, McKenzie uses utility function of $U = \varepsilon\ b_i \ln(x_i - c_i)$.b = marginal propensity to consume, c = price elasticity. He takes two commodities and both of their prices and incomes vary. He compares the initial situation with three cases, and tries to show that even though three alternative cases on the same indifference surface (have the same Equivalent Variation), consumer surplus indicator is incapable to identify this result and show cases significantly different (Table 1, pg. 466).

As a result, Willig's approach is incapable of generalization to several prices and income changes.

c- At third point, McKenzie comes to case which he has been trying to. He summarizes the characteristics of good welfare measure (being capable of correctly ranking all relevant, alternative price/quantity situations, expressible in terms of monetary units, it is capable of expression in terms of parameters of ordinary, observable demand functions). He concludes that having all desirable properties of consumer surplus and being equal to equivalent variation, McKenzie-Pears' welfare indicator is better and exact, not approximate welfare indicator since they might write any utility function $U = U(x^1, \ldots x^n)$ as an indirect cost of utility function $e(U) = g(m, p, p_1^1, \ldots p_n)$ because e and m are both measured in terms of the same monetary units, the marginal utility of money (λ) equals one, and all its higher derivatives with respect to income $\delta^i\lambda/\delta Y^i$ equal zero, given initial prices. This result regarding λ enables to express any utility function in terms of a Taylor-series expansion involving only the parameters of ordinary demand functions, and eliminating all traces of marginal utility of money and its derivatives with respect to price and income. He gives an example where a second order approximation is valid, their welfare indicator is written as

$$(\Delta e = -\sum X_i \, \Delta p_i - \frac{1}{2} \sum_i \sum_j \left(\frac{\delta X}{\delta p_j} - X_i \frac{\delta X_i}{\delta m}\right) \Delta p_j \, \Delta p_i + \Delta y - \sum \frac{\delta X_i}{\delta Y} \Delta p_i \, \Delta m$$

Higher order terms can be added to achieve an approximation to any desired degree of accuracy. As additional terms are added, the change indicated by Δe will approach the true equivalent variation in the limit.

McKenzie concludes that even though theirs and Willing's approaches require the same information, observable consumer demand function, integration and Taylor-series approximation, their indicator can generate more accurate and exact results, it is the one which should be used in practice.

WILLING'S REPLY:

Also answer to first point of McKennzie, Willing states that he did not base his argument for consumer surplus on the case of constant income elasticities. Instead he presented it as an instructive special case that introduced ,some of the techniques utilized to analyze in Section IV the general case of nonconstant income elasticity of demand. He also makes reference to his paper published in 1973.

As an answer to the second point of McKenzie (loosely applying the second order approximations to the derived exact expressions for the compensation and equivalent variations) Willing explains that the entire section of IV of the article was devoted to calculating, analyzing, and bounding the remainder term, and the principle contribution of his paper was the derivation of precise upper and lower bounds on the percentage errors of approximating the compensating and equivalent variations with consumer surplus. Also, Willing shows that McKenzie's calculations of consumer surplus (by using the utility model of $U = \varepsilon b_i \ln(x_i - c_i)$) are wrong because McKenzie calculates his version of multi-product consumer's surplus by integrating along the directed segments running from (P_1^0,P_2^0) to (P_1^1,P_2^0) and running from (P_1^0,P_2^0) to (P_1^0,P_2^1) and these segments do not form a path from (P_1^0,P_2^0) to (P_1^1,P_2^1). This procedure is incorrect and it would yield wrong answers even if income elasticity is negligible and also Willing shows how to calculate it.

After that, Willing tries to analyze different cases. By using what he calls Theorem 1, he shows that if all price movements are in the same direction, then multi-Product consumer's surplus bears the same relationship to the compensating and equivalent variations as does consumer's surplus over a single price change. In these cases, the rules of thumb derived in CSWA and following inequalities are held

$$(1) \quad \frac{n|A|}{2M_0} \leq \frac{C - A}{|A|} \leq \frac{\tilde{n}|A|}{2M^0}$$

$$(2) \quad \frac{n|A|}{2M_0} \leq \frac{A - E}{|A|} \leq \frac{\tilde{n}|A|}{2M^0}$$

However, the cases of price increases with price decreases is far more complex since A/m^0 may be small in absolute value while its components of opposite sings, have large absolute values. In such a situation, compensated demands can stray significantly far from Marshallian demands along the path of integration, and consequently A can fail to closely approximate C and E. He shows how to handle such situations (by Lemma, Theorem 2). However, if each of the price changes causes only a moderate change in new income, then relationship of (1), and (2) above will hold.

As an answer to McKenzie's third point (there is better indicator) Willing asks how many terms of infinite Taylor series would McKenzie and Pears recommend including in their approximation. He says they would continue adding terms until the terms become small, or until the partial sums seem to reveal their limit, and he points out that these are dangerous procedures that can result in arbitrarily large errors.

Kaynakça:

- Willing, R. "Consumer's Surplus Without Apology", American Economic Review, 9/76.
- McKenzie, G. "Consumer's Surplus Without Apology: Comment", AER, 6/79.
- Willing, R. "Consumer's Surplus Without Apology: Reply", AER, 6/79.