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Difference Sequence Spaces Derived by using Pascal Transform

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Abstract

The essential goal of this manuscript is to investigate some novel sequence spaces of $p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$ which are comprised by all sequence spaces whose differences are in Pascal sequence spaces p_∞, p_c and p_0, respectively. Furthermore, we determine both γ-, β-, α- duals of newly defined difference sequence spaces of $p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$. We also obtain bases of the newly defined difference sequence spaces of $p_c(\Delta)$ and $p_0(\Delta)$. Finally, necessary and sufficient conditions on an infinite matrix belonging to the classes $(p_c(\Delta) : l_\infty)$ and $(p_c(\Delta) : c)$ are characterized.

1. Introduction

Real or complex valued sequences spaces are represented by w along with the manuscript. Each sub-classes of real or complex valued sequences spaces is known as a sequence space. A sequence space of null, convergent, and bounded sequences are respectively demonstrated by c_0 , c, and l_∞. Moreover cs, l_1, bs depict convergent, absolutely convergent, and bounded series respectively.
K space is defined by any sequence space λ with a linear topology satisfying following transformation for a continuous term of $p_s(m) = m_s$ $s \in N$ such that $p_s : \lambda a \to C$, where $N = \{0,1,2,\ldots\}$ and C represents the set of complex number. If λ is a complete linear metric space then K-space is named by FK- space. BK-space is defined as normable topological space of FK-space [1].
Infinite matrix of complex or real numbers $A = (a_{nk})$ is defined for $n, k \in N$. Let X and Y be any two sequence spaces. Then, A is defined as a transformation between X to Y such that following equality holds.

$$(Ax)_n = \sum_k a_{nk} x_k \tag{1.1}$$

for each $n \in N$. $(X : Y)$, shows the family of matrices where $A : X \to Y$. Hence series given by the (1.1) converges for every $x \in X$ and each $n \in N$ iff $A \in (X : Y)$. One also has $Ax = \{(Ax)_n\} \in Y$. Here collection of entire finite subsets on K and N is denoted by F, where $N \subset F$. Studies on the sequence space have been mainly focused on some elementary concepts which are inclusions of sequence spaces, matrix mapping, determination of topologies, [2]. Let X be a sequence space and A be an infinite matrix in X then the domain of matrix is determined by

$$X_A = \{x = (x_k) \in w : Ax \in X\}$$

In general limitation matrix A produces novel sequence space X_A and it is either contraction or the expansion of the original space. Indeed, it is obviously clear that inclusion relations of $X \subset X_\Delta$ and $X_S \subset X$ are decidedly satisfied for $X \in \{c, l_\infty, c_0\}$ [3]. In particular, the the difference operator and sequence spaces which are fundamental samples for the matrix A and they have been investigated comprehensively through the mentioned methods.
Let P represeents the means of Pascal which is described by the matrix of Pascal [4] then it is defined by

$$P = [p_{nk}] = \begin{cases} \binom{n}{n-k}, & (0 \le k \le n) \\ 0, & (k > n) \end{cases} \quad , (n,k \in N)$$

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and the inverse of matrix of Pascal $P_n = (p_{nk})$ is defined by

$$P^{-1} = [p_{nk}]^{-1} = \begin{cases} (-1)^{n-k}\binom{n}{n-k}, & (0 \le k \le n) \\ 0 & ,(k > n) \end{cases}, (n,k \in N).$$

Pascal matrix contains some fascinating features. For instance; we can form three types of matrices: symmetric, lower triangular, and upper triangular, for any integer $n > 0$. The n-th order symmetric Pascal matrix n is given by

$$S_n = (s_{ij}) = \binom{i+j-2}{j-1}, \text{ for } i, j = 1, 2, ..., n, \tag{1.2}$$

$n-$th order lower triangular Pascal matrix is presented by

$$L_n = (l_{ij}) = \begin{cases} \binom{i-1}{j-1}, (0 \le j \le i) \\ 0, \quad (j > i) \end{cases}, \tag{1.3}$$

and the n-th order upper triangular Pascal matrix of order is presented by

$$U_n = (u_{ij}) = \begin{cases} \binom{j-1}{i-1}, (0 \le i \le j) \\ 0, \quad (j > i) \end{cases}. \tag{1.4}$$

We notice that $U_n = (L_n)^T$, n is any natural number.
i. Let S_n be the n-th order symmetric Pascal matrix given by (1.2), L_n be the n-th order lower triangular Pascal matrix given by (1.3), and U_n be the n-th order upper triangular Pascal matrix given by (1.4), then $S_n = L_n U_n$ and $det(S_n) = 1$ [5].
ii. Let S_n be the n-th order symmetric Pascal matrix given by (1.2), then S_n is similar to its inverse S_n^{-1} [5].
iii. Let A and B be $n \times n$ matrices. It is already known obviously that A is similar to B if one can define $n \times n$ invertible matrix Pi which satisfies following
$P^{-1}AP = B$ [6].
iv. Let L_n be the n-th order Pascal matrix. It is also assumed that it is a lower triangular matrix which is given by (1.3), then $L_n^{-1} = ((-1)^{i-j} l_{ij})$ [7].
Recently, Pascal sequence spaces was investigated by Polat [8] p_∞, p_c and p_0 like as follows:

$$p_\infty = \left\{ x = (x_k) \in w : \sup_n \left| \sum_{k=0}^{n} \binom{n}{n-k} x_k \right| < \infty \right\},$$

$$p_c = \left\{ x = (x_k) \in w : \lim_{n\to\infty} \sum_{k=0}^{n} \binom{n}{n-k} x_k \text{ exists} \right\},$$

and

$$p_0 = \left\{ x = (x_k) \in w : \lim_{n\to\infty} \sum_{k=0}^{n} \binom{n}{n-k} x_k = 0 \right\}.$$

$l_\infty(\Delta) = \{x \in w : (x_k - x_{k+1}) \in l_\infty\}$, $c(\Delta) = \{x \in w : (x_k - x_{k+1}) \in c\}$ and $c_0(\Delta) = \{x \in w : (x_k - x_{k+1}) \in c_0\}$ are known as difference sequence space and they are firstly defined by Kızmaz [9]. Further, various authors have defined and studied the difference sequence spaces, which can be seen in the following papers [10]-[15].
In this manuscript, Pascal difference sequence spaces of $p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$ are defined. They contain entire sequences whose differences are in Pascal sequence spaces p_∞, p_c and p_0, respectively. What is more, we determine the bases of the novel difference sequence spaces $p_c(\Delta)$ and $p_0(\Delta)$, and the α-, β- of the difference sequence spaces $p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$. Finally, we give the characterization of the necessary and sufficient conditions on an infinite matrix belonging to families of $(p_c(\Delta) : l_\infty)$ and $(p_c(\Delta) : c)$.

2. Inverse formula of the Pascal matrix and Pascal sequence spaces

We define the operators $\Delta : w \to w$ here and after it may be written for the sequence $(x_k - x_{k-1})$ that $(\Delta x)_k = \Delta x$. The well known difference matrix and the inverse of the difference matrix are defined as follows:

$$\left(\Delta^{(1)}\right)_{nk} = \begin{cases} (-1)^{n-k}, (n-1 \le k \le n) \\ 0, (0 \le k < n-1 \text{ or } k > n) \end{cases}, (n,k \in N)$$

and

$$\left(\left(\Delta^{(1)}\right)^{-1}\right)_{nk} = \begin{cases} 1, (0 \le k \le n) \\ 0, \quad (k > n) \end{cases}, (n,k \in N).$$

Pascal difference sequence spaces are defined by $p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$ by

$$p_\infty(\Delta) = \{x = (x_k) \in w : (x_k - x_{k-1}) \in p_\infty\},$$

$$p_c(\Delta) = \{x = (x_k) \in w : (x_k - x_{k-1}) \in p_c\},$$

and

$$p_0(\Delta) = \{x = (x_k) \in w : (x_k - x_{k-1}) \in p_0\}.$$

Let be a sequence $y = \{y_n\}$, which is generally utilized as H- mapping or H- transformation of a sequence $x = (x_k)$ and $H = P\Delta^{(1)}$ i.e.,

$$y_n = (Hx)_n = \sum_{k=0}^{n} \binom{n}{n-k} (x_k - x_{k-1}) \tag{2.1}$$

$$= \sum_{k=0}^{n} \left[\binom{n}{k} - \binom{n}{k+1} \right] x_k$$

for each $n \in N$. It can be easily shown that $p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$ are linear and normed spaces by the following norm:

$$\|x\|_\Delta = \|y\|_\infty = \sup_n |y_n|. \tag{2.2}$$

Theorem 2.1. *$p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$ sequence spaces are Banach spaces provided with the norm function given by (2.2).*

Proof. In the space of $p_\infty(\Delta)$, let we define following sequence and suppose that it is a Cauchy sequence $\{x^i\}$ such that $\{x^i\} = \{x_k^i\} = \{x_0^{(i)}, x_1^{(i)}, x_2^{(i)}, \ldots\} \in p_\infty(\Delta)$ for every $i \in N$. For a given $\varepsilon > 0$ it may be found a positive integer $N_0(\varepsilon)$ such that $\|x_i^k - x_i^n\|_\Delta < \varepsilon$ for all k, $n > N_0(\varepsilon)$. Hence

$$\left| H(x_i^k - x_i^n) \right| < \varepsilon$$

for all k, $n > N_0(\varepsilon)$ and for each $i \in N$. Therefore, following sequence is a reeal Cauchy sequence $\{(Hx^k)_i\} = \{(Hx^0)_i, (Hx^1)_i, (Hx^2)_i, \ldots\}$ for every fixed $i \in N$. Since real number of set R is complete, it converges, say

$$\lim_{i\to\infty} (Hx^i)_k \to (Hx)_k$$

for each $k \in N$. So, we have

$$\lim_{n\to\infty} \left| H(x_i^k - x_i^n) \right| = \left| H(x_i^k - x_i) \right| \le \varepsilon$$

for each $k \ge N_0(\varepsilon)$. This implies that $\|x^k - x\|_\Delta < \varepsilon$ for $k \ge N_0(\varepsilon)$, that is, $x^i \to x$ as $i \to \infty$.
Now, we must show that $x \in p_\infty(\Delta)$. We have

$$\|x\|_\Delta = \|Hx\|_\infty = \sup_n \left| \sum_{k=0}^{n} \binom{n}{n-k} \Delta x_k \right|$$

$$= \sup_n \left| \sum_{k=0}^{n} \binom{n}{n-k} (x_k - x_{k-1}) \right|$$

$$= \sup_n \left| \sum_{k=0}^{n} \left[\binom{n}{k} - \binom{n}{k+1} \right] x_k \right|$$

$$\le \sup_n \left| H(x_k^i - x_k) \right| + \sup_n \left| Hx_k^i \right|$$

$$\le \left\| x^i - x \right\|_\Delta + \left| Hx_k^i \right| < \infty$$

for all $i \in N$. This implies that $x = (x_i) \in p_\infty(\Delta)$. Therefore $p_\infty(\Delta)$ is a Banach space.
It can be shown that $p_c(\Delta)$ and $p_0(\Delta)$ are closed subspaces of $p_\infty(\Delta)$ which implies that $p_c(\Delta)$ and $p_0(\Delta)$ are also Banach spaces. Moreover, $p_\infty(\Delta)$ is a BK- space due to the fact that it is a Banach space with continuous coordinates □

3. The Bases of the sequence spaces $p_c(\Delta)$ and $p_0(\Delta)$

In this part, it is firstly gien the Schauder basis for the spaces $p_0(\Delta)$ and$p_c(\Delta)$. In normed sequence space X, Schauder basis (or briefly bases) is a sequnce of $\left\{b^{(k)}\right\}_{k\in\mathbb{N}}$ such that $x\in\lambda$ and (λ_k) of scalars such that

$$\lim_{n\to\infty}\|x-(\lambda_0x_0+\lambda_1x_1+...+\lambda_nx_n)\|=0.$$

Theorem 3.1. *Let $b^{(k)}=\left\{b_n^{(k)}\right\}_{n\in\mathbb{N}}$ be the sequence of elements of the space $p_0(\Delta)$ for each $k\in\mathbb{N}$ by*

$$b_n^{(k)}=\begin{cases}0, & (0\leq n<k)\\ \sum\limits_{i=k}^{n}(-1)^{i-k}\binom{i}{i-k}, & (n\geq k\end{cases}$$

Then the following assertions are true:
i. *The sequence $\left\{b^{(k)}\right\}_{k\in\mathbb{N}}$ is a basis for the space $p_0(\Delta)$, and for any $x\in p_0(\Delta)$ there exists a unique representation of the given form*

$$x=\sum_k\lambda_k(\Delta)b^{(k)}.$$

ii. *The set $\left\{t,b^{(1)},b^{(2)},b^{(3)},...\right\}$ is a basis for the space $p_c(\Delta)$, and for any $x\in p_0(\Delta)$ there exists a unique representation of the given form*

$$x=lt+\sum_k(\lambda_k(\Delta)-l)b^{(k)},$$

where $t=\{t_n\}$ with $t_n=\sum\limits_{k=0}^{n}\left[\sum\limits_{i=k}^{n}(-1)^{i-k}\binom{i}{i-k}\right]$, $\lambda_k(\Delta)=(Hx)_k$, $k\in\mathbb{N}$ and $l=\lim\limits_{k\to\infty}(Hx)_k$.

Theorem 3.2. *The sequence spaces $p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$ are linearly isomorphic to given spaces l_∞, c and c_0 respectively, i.e., $p_\infty(\Delta)\cong l_\infty$, $p_c(\Delta)\cong c$ and $p_0(\Delta)\cong c_0$.*

Proof. To begin the proof of $p_0(\Delta)\cong c_0$, it is firstly needed to indicate the presence of a linear bijection among spaces $p_0(\Delta)$ and c_0. Let we also take the map T described by the (2.1), from $p_0(\Delta)$ to c_0 by $x\to y=Tx$. T is trivially linear. It is also evident that $x=0$ since $Tx=0$ and thus T is an injective.
Let $y\in c_0$ and define the sequence $x=\{x_n\}$ by

$$x_n=\sum_{k=0}^{n}\left[\sum_{i=k}^{n}(-1)^{i-k}\binom{i}{i-k}\right]y_k.$$

Then,

$$\lim_{n\to\infty}(Hx)_k=\lim_{n\to\infty}\sum_{k=0}^{n}\binom{n}{n-k}\Delta x_k=\sum_{k=0}^{n}\binom{n}{n-k}(x_k-x_{k-1})$$

$$=\sum_{k=0}^{n}\left[\binom{n}{k}-\binom{n}{k+1}\right]x_k=\lim_{n\to\infty}y_n=0.$$

Thus, we have $x\in p_0(\Delta)$. Finally, T is is norm preserving and surjective. Thus, T is a linearly bijective. Therfore $p_0(\Delta)$ and c_0 spaces are linearly isomorphic. Similarly, it might be demonstated that $p_\infty(\Delta)$ and $p_c(\Delta)$ are respectively linearly isomorphic to l_∞ and c. □

4. The α-, β- and γ- duals of the sequence spaces $p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$

Here we present some facts together with their proofs to determine α-, β- and γ- duals of Pascal difference sequence spaces $p_\infty(\Delta)$**,** $p_c(\Delta)$ and $p_0(\Delta)$. Let λ and μ be two sequence space and let we determine the set $S(\lambda,\mu)$ where

$$S(\lambda,\mu)=\{z=(z_k)\in w: xz=(x_kz_k)\in\mu \text{ for all } x\in\lambda\}. \tag{4.1}$$

From the (4.1), duals of α-, β- and γ- of the sequence space λ that are denoted severally by λ^α, λ^β and λ^γ formed by Garling [17] as the following manner,

$$\lambda^\alpha=S(\lambda,l_1),\ \lambda^\beta=S(\lambda,cs) \text{ and } \lambda^\gamma=S(\lambda,bs).$$

Following facts presented by Tietz and Stieglitz [18] are useful to prove following theorems.

Lemma 4.1. $A\in(c_0:l_1)$ *if and only if*

$$\sup_{K\in F}\sum_n\left|\sum_{k\in K}a_{nk}\right|<\infty.$$

Lemma 4.2. *$A \in (c_0 : c)$ if and only if*

$$\sup_n \sum_k |a_{nk}| < \infty,$$

$$\lim_{n\to\infty} a_{nk} - \alpha_k = 0.$$

Lemma 4.3. *$A \in (c_0 : l_\infty)$ if and only if*

$$\sup_n \sum_k |a_{nk}| < \infty.$$

Theorem 4.4. *Let $a = (a_k) \in w$ and the matrix $B = (b_{nk})$ by*

$$b_{nk} = \left[\sum_{i=k}^{n} (-1)^{i-k} \binom{i}{i-k} a_n \right].$$

Then the α- dual of the spaces $p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$ is the set

$$b = \left\{ a = (a_n) \in w : \sup_{K\in F} \sum_n \left| \sum_{k\in K} \sum_{i=k}^{n} (-1)^{i-k} \binom{i}{i-k} a_n \right| < \infty \right\}.$$

Proof. Let us assume to have $a = (a_n) \in w$ and specially defined matrix B such that rows of the given matrix are the products of the rows of the given matrix $\left(\Delta^{(1)}\right)^{-1} P^{-1}$. From the (2.1), it is derived immediately that

$$a_n x_n = \sum_{k=0}^{n} \left[\sum_{i=k}^{n} (-1)^{i-k} \binom{i}{i-k} a_n \right] y_k = \sum_{k=0}^{n} b_{nk} y_k = (By)_n \tag{4.2}$$

$i, n \in \mathbb{N}$. We therefore see from the (4.2) that $ax = (a_n x_n) \in l_1$ when $x \in p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$ iff $By \in l_1$ whenever $y \in l_\infty$, c and c_0. Consequently, it is obtained from the first lemma that

$$\sup_{K\in F} \sum_n \left| \sum_{k\in K} \sum_{i=k}^{n} (-1)^{i-k} \binom{i}{i-k} a_n \right| < \infty$$

which yields the consequence that $[p_\infty(\Delta)]^\alpha = [p_c(\Delta)]^\alpha = [p_0(\Delta)]^\alpha = b$. □

Theorem 4.5. *Let $a = (a_k) \in w$ and the matrix $C = (c_{nk})$ by*

$$c_{nk} = \begin{cases} \sum\limits_{i=k}^{n} \sum\limits_{j=k}^{i} (-1)^{j-k} \binom{j}{j-k} a_i \ if\ 0 \le k \le n, \\ 0\ if\ k > n, \end{cases}$$

and define sets c_1, c_2, c_3 and c_4 by

$$c_1 = \left\{ a = (a_k) \in w : \sup_n \sum_k |c_{nk}| < \infty \right\},$$

$$c_2 = \left\{ a = (a_k) \in w : \lim_{n\to\infty} c_{nk} \text{ exists for each } k \in N \right\},$$

$$c_3 = \left\{ a = (a_k) \in w : \lim_{n\to\infty} \sum_k |c_{nk}| = \sum_k \left| \lim_{n\to\infty} c_{nk} \right| \right\},$$

and

$$c_4 = \left\{ a = (a_k) \in w : \lim_{n\to\infty} \sum_k c_{nk} \text{ exists} \right\}.$$

Then $[p_0(\Delta)]^\beta$, $[p_c(\Delta)]^\beta$ and $[p_\infty(\Delta)]^\beta$ is $c_1 \cap c_2$, $c_1 \cap c_2 \cap c_4$ and $c_2 \cap c_3$, respectively.

Proof. We solely present the proof for $p_0(\Delta)$ space. Since the rest of proof is accomplished by using the similar argument for $p_c(\Delta)$ and $p_\infty(\Delta)$. Let we take the following equation

$$\sum_{k=0}^{n} a_k x_k = \sum_{k=0}^{n} \left[\sum_{i=k}^{n} \sum_{j=k}^{i} (-1)^{j-k} \binom{j}{j-k} y_j \right] a_k$$

$$= \sum_{k=0}^{n} \left[\sum_{i=k}^{n} \sum_{j=k}^{i} (-1)^{j-k} \binom{j}{j-k} a_i \right] y_k$$

$$= (Cy)_n .$$

Hence, it is deduced by the second lemma and aforementioned equality that $ax = (a_n x_n) \in cs$ when $x \in p_0(\Delta)$ iff $Cy \in c$ whenever $y \in c_0$. Consequently, it may be shown due to the second lemma that $\{p_0(\Delta)\}^\beta = c_1 \cap c_2$. □

Theorem 4.6. *The γ-dual of the spaces $p_\infty(\Delta)$, $p_c(\Delta)$ and $p_0(\Delta)$ is the set c_1*

Proof. Proof is accomplished by utilizing the similar method as in the above case. □

5. Some matrix transformations on the sequence spaces $p_c(\Delta)$

We shall for brevity that

$$\tilde{a}_{nk} = \sum_{i=k}^{\infty} \sum_{j=k}^{i} (-1)^{j-k} \binom{j}{j-k} a_{ni}$$

and

$$\hat{g}_{nk} = \sum_{i=k}^{n} \sum_{j=k}^{i} (-1)^{j-k} \binom{j}{j-k} a_{ni}$$

In this part, some classes $(p_c(\Delta) : l_\infty)$ and $(p_c(\Delta) : c)$ are characterized. Following proofs of theorems is finalized by considering familiar approaches. Detais left to the reader.

Theorem 5.1. $A \in (p_c(\Delta) : l_\infty)$ *if and only if*

$$\sup_n \sum_k |\hat{g}_{nk}| < \infty, \tag{5.1}$$

$$\lim_{n\to\infty} \sum_k \hat{g}_{nk} \text{ exists for all } m \in N, \tag{5.2}$$

$$\sup_{n\in N} \sum_k |\tilde{a}_{nk}| < \infty, \ (n \in N) \tag{5.3}$$

and

$$\lim_{n\to\infty} \tilde{a}_{nk} \text{ exists for all } n \in N. \tag{5.4}$$

Theorem 5.2. $A \in (p_c(\Delta) : c)$ *iff* (5.1)-(5.4) *hold, and*

$$\lim_{n\to\infty} \sum_k \tilde{a}_{nk} = \alpha,$$

$$\lim_{n\to\infty} (\tilde{a}_{nk}) = \alpha_k , \ (k \in N).$$

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