INTERNATIONAL ENGINEERING, SCIENCE AND EDUCATION GROUP	**Middle East Journal of Science** (2019) 5(1): 41-45 Published online June, 2019 (http://dergipark.org.tr/mejs) doi: 10.23884/mejs.2019.5.1.05 e-ISSN 2618-6136 Received: February 02, 2019 Accepted: March 05, 2019 Submission Type: Research Article

GENERATING NUMERICAL SERIES VIA FUNCTIONS

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Abstract: *Series are the sum of the pattern of numbers or functions. One can produce a series in different ways. This is a study of the construction of series with differentiation of some functions.*

Keywords: Series, functions, numerical

1. Introduction

The basic definition of series is the sum of the pattern of numbers or functions. One can also define a decimal expansion as an infinite series. For example, $\boldsymbol{e}$ can be expressed as an infinite series in the following form

$$e = 2{,}71828247.... = 2 + 7/10 + 1/10^2 + 8/10^3 + ... \quad (1)$$

Series are used in most areas of mathematics. Besides to their inevitability in mathematics, series are also widely used in other areas of sciences such as physics, computer science, statistics, and finance. One can generate series in different ways. Functions are also can be determined with infinite series; Fourier, and Taylor series. On the other hand, one may determine series with function as well.

The use of series in mathematics and physics is well known. Some well-known types of series are arithmetic, geometric, power, and harmonic series. Our aim in this study is not to discuss series, but rather to derive some series with a different method. This method will be discussed in the next section. In this section, we give some basic information about the most used series [1], [2].

It is known that a power series of a variable x can be identified as follows

$$\sum_{n=1}^{\infty} a_n x^n \,. \quad (2)$$

Here a_n are coefficients to be determined. This series specially are used to solve partial differential equations. A geometric series can be produced via the following equation

$$\sum_{n=1}^{\infty} \frac{1}{2^n}. \tag{3}$$

Another well-known type of series, which is known as the harmonic series, can be constructed as follows

$$\sum_{n=1}^{\infty} \frac{1}{n}. \tag{4}$$

Also, basically, an arithmetic series can be defined in the following form

$$\sum_{n=1}^{\infty} n. \tag{5}$$

In the next two sections, we generate some series by a functional operation. Finally, we end the study by writing a short conclusion section.

2. Definition and basic examples

In this section, we see how series can be obtained from functions and their derivatives. For example, a function of a variable x may be expressed as follows

$$f(x) = x^n, \tag{6}$$

where n indicates a constant. After taking the first derivative of this function and multiplying it with x, the result is obtained as

$$f_1(x) = nx\frac{df(x)}{dx} = nf(x). \tag{7}$$

Then, taking the derivative of the equation (7) and multiplying it again with x, we obtain the following result

$$f_2(x) = n^2 f(x). \tag{8}$$

Carrying on these steps to the last equation, we reach

$$f_3(x) = n^3 f(x). \tag{9}$$

Consequently, the mth step gives the mth term as

$$f_m(x) = n^m f(x). \tag{10}$$

Collecting all terms for $f(1)$, the result is obtained in the form of well-known infinite geometric series

$$\sum_{n=1}^{\infty} n^m \quad . \tag{11}$$

For the case $n = 1/2$, it is seen that the result transforms into the convergent geometric series (3)

$$\sum_{m=1}^{\infty} \frac{1}{2^m}. \tag{12}$$

From the equation (6), one can get a finite series with a different decimal approximation. For example, if $f(x) = x$, the value of $f(1)$ is 1, and $\frac{d}{dx}f(1)$ is also equal to 1. Collecting two terms yields

$$f(1) + \frac{1}{1!}\frac{d}{dx}f(1) = 1 + 1 = 2^1 . \tag{13}$$

Also, this approximation gives 2^2 for the function $f(x) = x^2$.

$$f(1) + \frac{1}{1!}\frac{d}{dx}f(1) + \frac{1}{2!}\frac{d^2}{dx^2}f(1) = 1 + 2 + 1 = 2^2 \tag{14}$$

For $f(x) = x^3$, gives $1 + 3 + 3 + 1 = 2^3$, and finally for $f(x) = x^a$, we get

$$f(1) + \frac{1}{1!}\frac{d}{dx}f(1) + \frac{1}{2!}\frac{d^2}{dx^2}f(1) + \cdots .... + \frac{1}{a!}\frac{d^a}{dx^a}f(1) = 2^a . \tag{15}$$

To produce the arithmetic series, one can start with the case

$$f(x) = x^c \tag{16}$$

where c implies a constant. Taking the first derivative and performing the multiplication again with x/c, it leads us to the same equation as given in (16).

$$f(x) = 1x^c . \tag{17}$$

Adding 1 to the coefficient of the former equation and dividing this by that coefficient we get

$$f_1(x) = 1\frac{2}{1}x^c \tag{18}$$

Subsequently, the next term is

$$f_2(x) = 2\frac{3}{2}x^c \tag{19}$$

After the nth steps, the nth term becomes

$$f_n(x) = n\frac{n+1}{n}x^c . \tag{20}$$

Now, collecting all the coefficients of the x^c, the result transforms into an infinite arithmetic series (5) which is written as

$$\sum_{n=1}^{\infty} n . \tag{21}$$

In order to produce the harmonic series, one can also start with equation (6). Taking the first derivative and performing a multiplication again with x/c, it gives us the same conclusion as given in the equation (6).

$$f(x) = x^c \tag{22}$$

Moreover, taking the derivative of the last function and adding 1 to the inverse coefficient of the previous function, one gets the 1st term as

$$f_1(x) = \frac{1}{1+1}x^c = \frac{1}{2}x^c . \tag{23}$$

And, the second one is

$$f_2(x) = \frac{1}{2+1}x^c = \frac{1}{3}x^c\ . \tag{24}$$

After the nth step, the nth term becomes

$$f_n(x) = \frac{1}{n+1}x^c \tag{25}$$

Again, collecting all coefficients of the x^c, the result is an infinite harmonic series (4) which is defined as

$$\sum_{n=1}^{\infty}\frac{1}{n}\quad . \tag{26}$$

3. Logarithmic, exponential, and trigonometric functions

Logarithmic case:

A logarithmic function can be defined as follows

$$f(x) = lnx\ . \tag{27}$$

Next, the corresponding series obtained as follows

$$f(1) + \frac{1}{1!}\frac{d}{dx}f(1) + \frac{1}{2!}\frac{d^2}{dx^2}f(1) + \cdots \ldots . + \frac{1}{n!}\frac{d^n}{dx^n}f(1) + \cdots .. =$$

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots \ldots .. = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} \tag{28}$$

Exponential case:

In the case of $f(x) = e^x$, a series becomes

$$f(0) + \frac{1}{1!}\frac{d}{dx}f(0) + \frac{1}{2!}\frac{d^2}{dx^2}f(0) + \cdots \ldots . + \frac{1}{n!}\frac{d^n}{dx^n}f(0) + \cdots .. =$$

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots \ldots .. = \sum_{n=1}^{\infty}\frac{1}{(n-1)!} \tag{29}$$

Trigonometric case:

A sinusoidal function can be defined as follows

$$f(x) = sinx\ . \tag{30}$$

Therefore, the corresponding series is expanded as

$$f(0)+\frac{1}{1!}\frac{d}{dx}f(0)+\frac{1}{2!}\frac{d^2}{dx^2}f(0)+\cdots\ldots.+\frac{1}{n!}\frac{d^n}{dx^n}f(0)+\cdots..=$$

$$\frac{1}{1!}-\frac{1}{3!}+\frac{1}{5!}-\frac{1}{7!}+\cdots\ldots..=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{(2n-1)!} \tag{31}$$

Correspondingly, the $cosx$ function gives the following expansion

$$f(0)+\frac{1}{1!}\frac{d}{dx}f(0)+\frac{1}{2!}\frac{d^2}{dx^2}f(0)+\cdots\ldots.+\frac{1}{n!}\frac{d^n}{dx^n}f(0)+\cdots..=$$

$$1-\frac{1}{2!}+\frac{1}{4!}-\frac{1}{6!}+\cdots\ldots..=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{(2n-2)!} \tag{32}$$

4. Conclusion

In this study well-known types, some numerical series (geometric, harmonic, and arithmetic) are generated via a new method, which is called as generating function. We believe that this method will give a new aspect to mathematics, physics, and other scientific disciplines for relative purposes. Consequently, it is important to mention here that one can generate most of the series via this method by using a suitable function such as the trigonometric, logarithmic and the exponential functions.

References

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[2] Thomas, G. B., Finney, R. L., "Calculus and Analytic Geometry" **7th Edition**, Addison-Wesley (1988).