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Construction of Exact Solutions to Partial Differential Equations with CRE Method

Filiz Taşcan[1]*, Arzu Akbulut[2]

Abstract
In this article, the consistent Riccati expansion (CRE) method is presented for constructing new exact solutions of (1+1) dimensional nonlinear dispersive modified Benjamin Bona Mahony (DMBBM) and mKdV-Burgers equations. The exact solutions obtained are composed of hyperbolic and exponential functions. The outcomes obtained confirm that the proposed method is an efficient technique for analytic treatment of a wide variety of nonlinear partial differential equations.

Keywords: Partial differential equations, Exact solution, The consistent Riccati expansion.
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[1] *Eskisehir Osmangazi University, Art-Science Faculty, Department of Mathematics-Computer, Eskisehir-TURKEY, ORCID: 0000-0003-2697-5271*
[2] *Eskisehir Osmangazi University, Art-Science Faculty, Department of Mathematics-Computer, Eskisehir-TURKEY, ORCID: 0000-0003-2448-2481*
***Corresponding author**: ftascan@ogu.edu.tr
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1. Introduction

Nonlinear evolution equations (NLEEs) in mathematical physics play a vital role in different fields, such as fluid mechanics, plasma physics, optical fibers, solid state physics, chemical kinematic, chemical physics and geochemistry. Since obtaining exact solutions of NLEEs come into prominence, there become significant improvements in this domain[1]. Many effective and powerful methods have been established and improved, such as modified simple equation method [2], symmetry reduction method[3], trial equation method [4], the (G'/G)-expansion method [5], sub equation method [6], $\exp(-\Phi(\xi))$ method[7], functional variable method[8], first integral method[9], modified exp-function method [10] and so on.

The aim of this paper is search new solutions of (1+1) dimensional nonlinear dispersive modified Benjamin Bona Mahony (DMBBM) equation and modified Korteweg-de Vries (mKdV)-Burgers equation with consistent Riccati expansion (CRE) method. In section 2, we give the definition of the method. In section 3, there found solutions of the given equations. In section 4, conslusions are given.

2. Consistent Riccati expansion (CRE) method

Lets assume that we have a nonlinear differential equation, remark in the independent variables x and t and dependent variable u, given by

$$F(u,u_x,u_t,u_{xx},u_{tt},...)=0, \tag{2.1}$$

where F is a polynomial of $u(x,t)$ and its various partial derivatives including the highest order derivatives and nonlinear terms.

According to the algoritm, we can seek for the solutions of Eq. (2.1) in the form

$$u=\sum_{i=0}^{n}u_i\left(x,t\right)R^i\left(w\right), \tag{2.2}$$

where u_i $(i=0,...,n)$ are functions to be detected later and the positive integer n can be detected by using homogeneous balance method. Here $R(w)$ is a solution of the Riccati equation

$$R_w=a_0+a_1R+a_2R^2 \tag{2.3}$$

where a_0,a_1,a_2 are parameters to be determined and w is an undetermined function of x and t.

The positive integer n can be detected by considering the homogeneous balance between the highest order derivative term with the highest order nonlinear term appearing in Eq. (2.1). Then by sett ing Eq. (2.2) along with Eq. (2.3) into Eq. (2.1) and equating the coefficients of all powers of $R\left(w\right)$ to zero yields a set of algebraic equations for unknowns u_i,a_0,a_1 and a_2[11, 12].

3. Exercises

In this part, we have dealed with two partial differential equations as an application of the CRE method.

3.1 (1+1) dimensional nonlinear dispersive modified Benjamin Bona Mahony (DMBBM) equation

Firstly, we look at the (1+1) dimensional nonlinear dispersive modified Benjamin Bona Mahony (DMBBM) equation [13]

$$u_t+u_x-\alpha u^2u_x+u_{xxx}=0, \tag{3.1}$$

where where α is a nonzero constant. This equation was first derived to describe an approximation for surface long waves in nonlinear dispersive media. It can also characterize the hydro magnetic waves in cold plasma, acoustic waves in inharmonic crystals and acoustic gravity waves in compressible fluids [14].

Here, it is clear from the homogoneous balance principle that the balancing number is 1. From here, we infer from that the exact solution of Eq. (3.1) is

$$u(x,t)=u_0\left(x,t\right)+u_1\left(x,t\right)R\left(w\left(x,t\right)\right) \tag{3.2}$$

where $u_0\left(x,t\right)$ and $u_1\left(x,t\right)$ are functions to be determined later. Setting Eq. (3.2) and its derivatives with the condition Eq. (2.3) into Eq. (3.1) and gathering all terms with the same power of $R\left(w\right),(i=0,1,...,4)$, we obtain the following system

$$R^4\left(w\right)\quad:\quad 6u_1w_x^3a_2^3-\alpha u_1^3w_xa_2=0, \tag{3.3}$$

$$\begin{aligned}R^3\left(w\right)\quad:\quad &-2\alpha u_0u_1^2w_xa_2+6\left(u_1\right)_xw_x^2a_2^2+12u_1w_x^3a_1a_2^2+6u_1w_xw_{xx}a_2^2\\ &-\alpha u_1^2\left(u_1\right)_x-\alpha u_1^3w_xa_1=0,\end{aligned} \tag{3.4}$$

$$\begin{aligned}R^2\left(w\right)\quad:\quad &9\left(u_1\right)_xw_x^2a_1a_2+8u_1w_x^3a_0a_2^2-\alpha u_0^2u_1w_xa_2+9u_1w_xw_{xx}a_1a_2\\ &-\alpha u_1^2\left(u_0\right)_x+u_1w_xa_2-2\alpha u_0u_1^2w_xa_1+u_1w_ta_2-\alpha u_1^3w_xa_0\\ &+7u_1w_x^3a_1^2a_2+3\left(u_1\right)_{xx}w_xa_2-2\alpha u_0u_1\left(u_1\right)_x\\ &+3\left(u_1\right)_xw_{xx}a_2+u_1w_{xxx}a_2=0,\end{aligned} \tag{3.5}$$

$$\begin{aligned}R^1\left(w\right)\quad:\quad &\left(u_1\right)_{xxx}+u_1w_ta_1+3\left(u_1\right)_xw_{xx}a_1+3u_1w_xw_{xx}a_1^2\\ &+\left(u_1\right)_t-\alpha u_0^2u_1w_xa_1+8u_1w_x^3a_1a_2a_0\\ &-2\alpha u_0u_1^2w_xa_0-2\alpha u_0u_1\left(u_0\right)_x+u_1w_x^3a_1^3+\left(u_1\right)_x\\ &+6\left(u_1\right)_xw_x^2a_2a_0+6u_1w_xw_{xx}a_2a_0+u_1w_{xxx}a_1\\ &+3\left(u_1\right)_{xx}w_xa_1+3\left(u_1\right)_xw_x^2a_1^2+u_1w_xa_1-\alpha u_0^2\left(u_1\right)_x=0,\end{aligned} \tag{3.6}$$

$$\begin{aligned}R^0\left(w\right)\quad:\quad &3\left(u_1\right)_xw_x^2a_1a_0+u_1w_{xxx}a_0+3u_1w_xw_{xx}a_1a_0\\ &+\left(u_0\right)_x+\left(u_0\right)_t+\left(u_0\right)_{xxx}+3\left(u_1\right)_{xx}w_xa_0+u_1w_ta_0\\ &+3\left(u_1\right)_xw_{xx}a_0-\alpha u_0^2\left(u_0\right)_x+u_1w_x^3a_1^2a_0\\ &+u_1w_xa_0+2u_1w_x^3a_2a_0^2-\alpha u_0^2u_1w_xa_0=0.\end{aligned} \tag{3.7}$$

From the Eq. (3.3), we get

$$u_1(x,t)=\sqrt{6}\sqrt{\frac{1}{\alpha}}a_2w_x. \tag{3.8}$$

If we substitute Eq. (3.8) in Eq. (3.4), we obtain

$$u_0(x,t)=\frac{\sqrt{6}\sqrt{\frac{1}{\alpha}}w_{xx}}{2w_x}+\sqrt{6}\sqrt{\frac{1}{\alpha}}a_1w_x-\frac{1}{2}w_x\alpha\sqrt{6}\sqrt{\left(\frac{1}{\alpha}\right)^3}a_1. \tag{3.9}$$

When we substitute Eq. (3.8) and Eq. (3.9) in Eq. (3.5), we get following partial differential equation

$$w_tw_x=-\frac{4a_2w_x^4a_0-w_x^4a_1^2-3w_{xx}^2+2w_xw_{xxx}+2w_x^2}{2}. \tag{3.10}$$

If we use Eq. (3.10) in Eq. (3.6) and Eq. (3.7), these Eqs. are equal to zero.
If w is a solution of Eq. (3.10), then

$$u=\frac{\sqrt{6}\sqrt{\frac{1}{\alpha}}w_{xx}}{2w_x}+\sqrt{6}\sqrt{\frac{1}{\alpha}}a_1w_x-\frac{1}{2}w_x\alpha\sqrt{6}\sqrt{\left(\frac{1}{\alpha}\right)^3}a_1+\sqrt{6}\sqrt{\frac{1}{\alpha}}a_2w_xR \tag{3.11}$$

is a solution of the DMBBM equation with $R\equiv R(w)$ being a solution of the Riccati equation (2.3).

We suppose that $w(x,t)$ be of the form

$$w(x,t)=a\cosh(kx+lt+\xi)+b\sinh(kx+lt+\xi)+r \tag{3.12}$$

where a,b,k,l and r are constants to be determined later and ξ is an arbitrary constant. Setting Eq. (3.12) into Eq. (3.10) , we obtain the following equations

$$-\frac{k\left(16a_2a_0a^3k^3b-4a_1^2ak^3b^3-4a_1^2a^3k^3b+16a_2a_0ak^3b^3\right)}{2}=0,$$

$$-\frac{k\left(-16a_2a_0a^3k^3b+4alb+4a_1^2a^3k^3b-2ak^3b+4akb\right)}{2}=0,$$

$$-\frac{k\left(-a_1^2a^4k^3+24a_2a_0a^2k^3b^2-a_1^2b^4k^3\right)}{2}$$

$$-\frac{k\left(-6a_1^2a^2k^3b^2+4a_2a_0b^4k^3+4a_2a_0a^4k^3\right)}{2}=0,$$

$$-\frac{k\left(2a_1^2a^4k^3+6a_1^2a^2k^3b^2-8a_2a_0a^4k^3+2a^2k+2b^2k\right)}{2}$$

$$-\frac{k\left(-b^2k^3-a^2k^3+2a^2l-24a_2a_0a^2k^3b^2+2b^2l\right)}{2}=0,$$

$$-\frac{k\left(-a_1^2a^4k^3-2a^2k-2a^2l-2a^2k^3+4a_2a_0a^4k^3+3b^2k^3\right)}{2}=0$$

Solving above system, we get the following two solutions.

State 1:

$$\begin{aligned}&a=b,\ a_0=\tfrac{a_1^2}{4a_2},\ a_1=a_1,\ a_2=a_2,\ b=b,\\&k=k,\ \xi=\xi,\ l=\tfrac{k\left(k^2-2\right)}{2},\ r=r.\end{aligned} \tag{3.13}$$

State 2:

$$\begin{aligned}&a=-b,\ a_0=\tfrac{a_1^2}{4a_2},\ a_1=a_1,\ a_2=a_2,\ b=b,\\&k=k,\ \xi=\xi,\ l=\tfrac{k\left(k^2-2\right)}{2},\ r=r.\end{aligned} \tag{3.14}$$

Combining Eq. (3.11), Eq. (3.12) with Eq. (3.13) and Eq. (3.14), two families of exact explicit solutions to the DMBBM equation are obtained

$$\begin{aligned} u(x,t) &= \frac{1}{2}\sqrt{6}\sqrt{\frac{1}{\alpha}}k\left(a_1 b\cosh(\beta)+1+a_1 b\sinh(\beta)\right) \\ &+\sqrt{6}\sqrt{\frac{1}{\alpha}}a_2\left(bk\sinh(\beta)+bk\cosh(\beta)\right) \\ &\times R\left(b\cosh(\beta)+b\sinh(\beta)+r\right) \end{aligned}$$

and

$$\begin{aligned} u(x,t) &= \frac{1}{2}\sqrt{6}\sqrt{\frac{1}{\alpha}}k\left(a_1 b\cosh(\beta)-1-a_1 b\sinh(\beta)\right) \\ &+\sqrt{6}\sqrt{\frac{1}{\alpha}}a_2\left(-bk\sinh(\beta)+bk\cosh(\beta)\right) \\ &\times R\left(-b\cosh(\beta)+b\sinh(\beta)+r\right). \end{aligned}$$

where $\beta = kx-kt+\dfrac{k^3 t}{2}+\xi$.

We suppose that $w(x,t)$ be of the form

$$w(x,t) = A\exp(k_1 x+l_1 t+\xi_1)+B\exp(k_2 x+l_2 t+\xi_2)+C \tag{3.15}$$

where A, B, C, k_i and l_i are constants to be determined later and ξ_i are an arbitrary constant. Setting Eq. (3.15) into Eq. (3.10) , we get the following system

$$\begin{aligned} \frac{a_1^2 B^4 k_2^4}{2} - 2a_2 a_0 B^4 k_2^4 &= 0, \\ -8a_2 a_0 A k_1 B^3 k_2^3 + 2a_1^2 A k_1 B^3 k_2^3 &= 0, \\ -12 a_2 a_0 A^2 k_1^2 B^2 k_2^2 + 3a_1^2 A^2 k_1^2 B^2 k_2^2 &= 0, \\ -B^2 k_2^2 + \tfrac{1}{2} B^2 k_2^4 - B^2 l_2 k_2 &= 0, \\ -8a_2 a_0 A^3 k_1^3 B k_2 + 2a_1^2 A^3 k_1^3 B k_2 &= 0, \\ -A l_1 B k_2 - A k_1^3 B k_2 - B l_2 A k_1 - B k_2^3 A k_1 + 3A k_1^2 B k_2^2 - 2A k_1 B k_2 &= 0, \\ \frac{a_1^2 A^4 k_1^4}{2} - 2a_2 a_0 A^4 k_1^4 &= 0, \\ \frac{A^2 k_1^4}{2} - A^2 k_1^2 - A^2 l_1 k_1 &= 0. \end{aligned}$$

Solving above system, one gets the following set of solution.

$$\begin{aligned} &A = A,\ B = B,\ C = C,\ a_0 = \tfrac{a_1^2}{4a_2},\ a_1 = a_1,\ a_2 = a_2,\ k_1 = k_2, \\ &k_2 = k_2,\ l_1 = -k_2 + \tfrac{k_2^3}{2},\ l_2 = -k_2 + \tfrac{k_2^3}{2},\ \xi_1 = \xi_1,\ \xi_2 = \xi_2 \end{aligned} \tag{3.16}$$

Combining Eq. (3.11), Eq. (3.15) with Eq. (3.16), exact explicit solution is obtained

$$\begin{aligned} u(x,t) &= \frac{k_2\sqrt{6}}{2}\sqrt{\frac{1}{\alpha}}\left(Aa_1\exp\left(\beta+\xi_1\right)+Ba_1\exp\left(\beta+\xi_2\right)+1\right) \\ &\quad +a_2\sqrt{6}\sqrt{\frac{1}{\alpha}}\left(Ak_2\exp\left(\beta+\xi_1\right)+Bk_2\exp\left(\beta+\xi_2\right)\right) \\ &\quad \times R\left(A\exp\left(\beta+\xi_1\right)+B\exp\left(\beta+\xi_2\right)+C\right) \end{aligned}$$

where $\beta = k_2x+\left(-k_2+\frac{k_2^3}{2}\right)t$.

3.2 Modified Korteweg-de Vries (mKdV)-Burgers equation

mKdV-Burgers equation is given by [15]

$$u_t+qu^2u_x+ru_{xx}-su_{xxx}=0 \tag{3.17}$$

where $q,\ r$ and s are arbitrary constants. According to the homogeneous balance method, we get the balancing number as $n=1$. From here, we infered that the exact solution of Eq. (3.17) is

$$u(x,t)=u_0\left(x,t\right)+u_1\left(x,t\right)R\left(w\left(x,t\right)\right) \tag{3.18}$$

where $u_0\left(x,t\right)$ and $u_1\left(x,t\right)$ are functions to be detected later. Setting Eq. (3.18) and its derivatives with the condition Eq. (2.3) into Eq. (3.17) and picking all terms with the same power of $R\left(w\right),\left(i=0,1,...,4\right)$, we have the following system

$$R^4\left(w\right)\ :\ qu_1^3w_xa_2-6su_1w_x^3a_2^3=0, \tag{3.19}$$

$$\begin{aligned} R^3\left(w\right)\ :\ & 2qu_0u_1^2w_xa_2-6s\left(u_1\right)_xw_x^2a_2^2+2ru_1w_x^2a_2^2-12su_1w_x^3a_1a_2^2 \\ & +qu_1^2\left(u_1\right)_x+qu_1^3w_xa_1-6su_1w_xw_{xx}a_2^2=0, \end{aligned} \tag{3.20}$$

$$\begin{aligned} R^2\left(w\right)\ :\ & qu_1^2\left(u_0\right)_x+2r\left(u_1\right)_xw_xa_2-su_1w_{xxx}a_2 \\ & +u_1w_ta_2-3s\left(u_1\right)_xw_{xx}a_2-8su_1w_x^3a_0a_2^2 \\ & +3ru_1w_x^2a_1a_2+2qu_0u_1^2w_xa_1-3s\left(u_1\right)_{xx}w_xa_2 \\ & +2qu_0u_1\left(u_1\right)_x-9s\left(u_1\right)_xw_x^2a_1a_2-9su_1w_xw_{xx}a_1a_2 \\ & +qu_1^3w_xa_0+qu_0^2u_1w_xa_2-7su_1w_x^3a_1^2a_2+ru_1w_{xx}a_2=0, \end{aligned} \tag{3.21}$$

$$\begin{aligned} R^1\left(w\right)\ :\ & qu_0^2\left(u_1\right)_x-6su_1w_xw_{xx}a_2a_0+2qu_0u_1^2w_xa_0+r\left(u_1\right)_{xx} \\ & +ru_1w_{xx}a_1-su_1w_{xxx}a_1+\left(u_1\right)_t-3s\left(u_1\right)_{xx}w_xa_1 \\ & -3s\left(u_1\right)_xw_x^2a_1^2-su_1w_x^3a_1^3+ru_1w_x^2a_1^2-3s\left(u_1\right)_xw_{xx}a_1 \\ & +qu_0^2u_1w_xa_1-s\left(u_1\right)_{xxx}+u_1w_ta_1-3su_1w_xw_{xx}a_1^2 \\ & -8su_1w_x^3a_1a_2a_0+2r\left(u_1\right)_xw_xa_1+2ru_1w_x^2a_2a_0 \\ & +2qu_0u_1\left(u_0\right)_x-6s\left(u_1\right)_xw_x^2a_2a_0=0, \end{aligned} \tag{3.22}$$

$$\begin{aligned} R^0\left(w\right)\ :\ & u_1w_ta_0-3s\left(u_1\right)_xw_x^2a_1a_0+2r\left(u_1\right)_xw_xa_0+ru_1w_x^2a_1a_0 \\ & +ru_1w_{xx}a_0-3s\left(u_1\right)_{xx}w_xa_0+qu_0^2u_1w_xa_0-su_1w_{xxx}a_0 \\ & +r\left(u_0\right)_{xx}-2su_1w_x^3a_2a_0^2-3s\left(u_1\right)_xw_{xx}a_0+qu_0^2\left(u_0\right)_x \\ & -3su_1w_xw_{xx}a_1a_0+\left(u_0\right)_t-s\left(u_0\right)_{xxx}-su_1w_x^3a_1^2a_0=0. \end{aligned} \tag{3.23}$$

From the Eq. (3.19), we get

$$u_1\left(x,t\right)=\frac{\sqrt{6}\sqrt{s}a_2w_x}{\sqrt{q}}. \tag{3.24}$$

If we substitute Eq. (3.24) in Eq. (3.20), we obtain

$$u_0\left(x,t\right)=\frac{\sqrt{6}\left(3sw_{xx}-rw_x+3sw_x^2a_1\right)}{6\sqrt{s}\sqrt{q}w_x}. \tag{3.25}$$

When we substitute Eq. (3.24) and Eq. (3.25) in Eq. (3.21), we get following partial differential equation

$$w_tw_x=-3sw_{xx}^2+2sa_2a_0w_x^4-\frac{sw_x^4a_1^2}{2}+sw_{xxx}w_x-\frac{w_x^2r^2}{6s}. \tag{3.26}$$

If we use Eq. (3.26) in Eq. (3.22), this Eq. is equal to zero. If we use Eq. (3.26) in Eq. (3.23), we obtain

$$\frac{r\sqrt{6s}\left(4w_{xx}a_2w_x^4a_0+3w_{xx}^3+w_{xxxx}w_x^2-4w_{xx}w_{xxx}w_x-w_x^4a_1^2w_{xx}\right)}{2\sqrt{q}w_x^3}=0. \tag{3.27}$$

If w is a solution of Eqs. (3.26) and (3.27), then

$$u=\frac{\sqrt{6}\left(3sw_{xx}-rw_x+3sw_x^2a_1\right)}{6\sqrt{s}\sqrt{q}w_x}+\frac{\sqrt{6}\sqrt{s}a_2w_x}{\sqrt{q}}R \tag{3.28}$$

is a solution of the Eq. (3.17) with $R\equiv R\left(w\right)$ being a solution of the Riccati equation (2.3).

We suppose that $w\left(x,t\right)$ be of the form

$$w\left(x,t\right)=a\cosh\left(kx+lt+\xi\right)+b\sinh\left(kx+lt+\xi\right)+r \tag{3.29}$$

where a,b,k,l and r are constants to be determined later and ξ is an arbitrary constant. Setting Eq. (3.29) into Eqs. (3.26) and

(3.27), we obtain the following equations

$$sa^2k^4-a^2kl-\frac{3sb^2k^4}{2}-\frac{r^2a^2k^2}{6s}=0,\ 2sa_2a_0a^4k^4-\frac{sa_1^2a^4k^4}{2}=0,$$

$$-3sa_1^2a^2k^4b^2+12sa_2a_0a^2k^4b^2=0,\ 8sa_2a_0a^3k^4b-2sa_1^2a^3k^4b=0,$$

$$-\frac{sa_1^2b^4k^4}{2}+2sa_2a_0b^4k^4=0,\ -sak^4b-2akbl-\frac{r^2ak^2b}{3s}=0,$$

$$\frac{2\sqrt{6}r\sqrt{s}bk^6a_2a^4a_0}{\sqrt{q}}-\frac{\sqrt{6}r\sqrt{s}a^4k^6ba_1^2}{2\sqrt{q}}=0,\ -\frac{r^2b^2k^2}{6s}-b^2kl-\frac{3sa^2k^4}{2}+sb^2k^4=0,$$

$$-\frac{\sqrt{6}r\sqrt{s}a^5k^6a_1^2}{2\sqrt{q}}-\frac{2\sqrt{6}r\sqrt{s}a^3k^6b^2a_1^2}{\sqrt{q}}+2a_0\left(\frac{4\sqrt{6}r\sqrt{s}a^3k^6b^2a_2}{\sqrt{q}}+\frac{\sqrt{6}r\sqrt{s}a^5k^6a_2}{\sqrt{q}}\right)=0,$$

$$-\frac{2\sqrt{6}r\sqrt{s}a^4k^6ba_1^2}{\sqrt{q}}-\frac{3\sqrt{6}r\sqrt{s}a^2k^6b^3a_1^2}{\sqrt{q}}+2a_0\left(\frac{6\sqrt{6}r\sqrt{s}a^2k^6b^3a_2}{\sqrt{q}}+\frac{4\sqrt{6}r\sqrt{s}a^4k^6a_2b}{\sqrt{q}}\right)=0,$$

$$-2sa_1^2ak^4b^3+8sa_2a_0ak^4b^3=0,\ -\frac{3\sqrt{6}r\sqrt{s}bk^6a^2}{2\sqrt{q}}+\frac{3\sqrt{6}r\sqrt{s}b^3k^6}{2\sqrt{q}}=0,$$

$$-\frac{3\sqrt{6}r\sqrt{s}a^3k^6b^2a_1^2}{\sqrt{q}}-\frac{2\sqrt{6}r\sqrt{s}ab^4k^6a_1^2}{\sqrt{q}}+2a_0\left(\frac{4\sqrt{6}r\sqrt{s}ak^6b^4a_2}{\sqrt{q}}+\frac{6\sqrt{6}r\sqrt{s}a^3k^6a_2b^2}{\sqrt{q}}\right)=0,$$

$$\frac{7\sqrt{6}r\sqrt{s}ak^6b^2}{2\sqrt{q}}+\frac{\sqrt{6}r\sqrt{s}a^3k^6}{2\sqrt{q}}-2ak\left(\frac{\sqrt{6}r\sqrt{s}k^5b^2}{\sqrt{q}}+\frac{\sqrt{6}r\sqrt{s}a^2k^5}{\sqrt{q}}\right)=0,$$

$$-\frac{2\sqrt{6}r\sqrt{s}a^2k^6b^3a_1^2}{\sqrt{q}}-\frac{\sqrt{6}r\sqrt{s}ab^5k^6a_1^2}{2\sqrt{q}}+2a_0\left(\frac{\sqrt{6}r\sqrt{s}b^5k^6a_2}{\sqrt{q}}+\frac{4\sqrt{6}r\sqrt{s}a^2k^6a_2b^3}{\sqrt{q}}\right)=0,$$

$$\frac{7\sqrt{6}r\sqrt{s}bk^6a^2}{2\sqrt{q}}+\frac{\sqrt{6}r\sqrt{s}b^3k^6}{2\sqrt{q}}-2bk\left(\frac{\sqrt{6}r\sqrt{s}k^5b^2}{\sqrt{q}}+\frac{\sqrt{6}r\sqrt{s}a^2k^5}{\sqrt{q}}\right)=0,$$

$$\frac{3\sqrt{6}r\sqrt{s}a^3k^6}{2\sqrt{q}}-\frac{3\sqrt{6}r\sqrt{s}ak^6b^2}{2\sqrt{q}}=0,\ \frac{2\sqrt{6}r\sqrt{s}ak^6b^4a_2a_0}{\sqrt{q}}-\frac{\sqrt{6}r\sqrt{s}ab^4k^6a_1^2}{2\sqrt{q}}=0,$$

Solving above system, we get the following two solutions.

State 1:

$$\begin{aligned}&a=b,\ a_0=\tfrac{a_1^2}{4a_2},\ a_1=a_1,\ a_2=a_2,\ b=b,\\&k=k,\ \xi=\xi,\ l=-\tfrac{k\left(3s^2k^2+r^2\right)}{6s},\ r=r.\end{aligned}\tag{3.30}$$

State 2:

$$\begin{aligned}&a=-b,\ a_0=\tfrac{a_1^2}{4a_2},\ a_1=a_1,\ a_2=a_2,\ b=b,\\&k=k,\ \xi=\xi,\ l=-\tfrac{k\left(3s^2k^2+r^2\right)}{6s},\ r=r.\end{aligned}\tag{3.31}$$

Combining Eq. (3.28), Eq. (3.29) with Eq. (3.30) and Eq. (3.31), two families of exact explicit solutions to the mKdV-Burgers equation are obtained

$$u(x,t) = \frac{\sqrt{6}b^3k^3(\cosh(\beta)-\sinh(\beta))^3(3sk-r+3sbka_1\cosh(\beta)-3sbka_1\sinh(\beta))}{6\sqrt{s}\sqrt{q}(-bk\sinh(\alpha)+bk\cosh(\alpha))^3}$$
$$+\frac{\sqrt{6s}}{\sqrt{q}}(-bk\sinh(\alpha)+bk\cosh(\alpha))a_2R(b\cosh(\alpha)-b\sinh(\alpha)+r)$$

and

$$u(x,t) = \frac{\sqrt{6}b^3k^3(\cosh(\beta)-\sinh(\beta))^3(-3sk-r+3sbka_1\cosh(\beta)+3sbka_1\sinh(\beta))}{6\sqrt{s}\sqrt{q}(bk\sinh(\alpha)+bk\cosh(\alpha))^3}$$
$$+\frac{\sqrt{6s}}{\sqrt{q}}(bk\sinh(\alpha)+bk\cosh(\alpha))a_2R(-b\cosh(\alpha)-b\sinh(\alpha)+r)$$

where $\alpha = -kx+\frac{k(3s^2k^2+r^2)t}{6s}-\xi,\ \beta = \frac{-6kxs+3k^3ts^2+ktr^2-6s\xi}{6s}$.

We suppose that $w(x,t)$ be of the form

$$w(x,t) = A+B\exp(k_1x+l_1t+\xi_1) \tag{3.32}$$

where A, B, k_1 and l_1 are constants to be determined later and ξ_1 are an arbitrary constant. Setting Eq. (3.32) into Eqs. (3.26) and (3.27), we get the following system

$$\frac{2\sqrt{6}r\sqrt{s}B^2k_1^3a_2a_0}{\sqrt{q}}-\frac{\sqrt{6}r\sqrt{s}B^2k_1^3a_1^2}{2\sqrt{q}} = 0,$$

$$2sB^3k_1^3a_2a_0-\frac{sB^3k_1^3a_1^2}{2} = 0,$$

$$-\frac{sBk_1^3}{2}-\frac{Bk_1r^2}{6s}-Bl_1 = 0,$$

Solving above system, one gets the following set of solution

$$\begin{array}{l} A = A,\ B = B,\ a_0 = \frac{a_1^2}{4a_2},\ a_1 = a_1,\ a_2 = a_2, \\ k_1 = k_1, l_1 = -\frac{k_1(3s^2k_1^2+r^2)}{6s},\ \xi_1 = \xi_1. \end{array} \tag{3.33}$$

Combining Eq. (3.28), Eq. (3.29) with Eq. (3.33), exact explicit solution is obtained

$$u(x,t) = \frac{\sqrt{6}\exp(\alpha)\left(3k_1s-r+3sBk_1a_1\exp\left(\frac{\alpha}{3}\right)\right)}{6\sqrt{sq}(\exp(\beta))^3}$$
$$+\frac{\sqrt{6}\sqrt{s}Bk_1a_2\exp(\beta)R(A+B\exp(\beta))}{\sqrt{q}}$$

where $\alpha = -\frac{-6k_1xs+3k_1^3ts^2+k_1tr^2-6\xi_1 s}{2s}, \beta = k_1x-\frac{k_1(3s^2k_1^2+r^2)t}{6s}+\xi_1$

4. Conclusions

In this paper, by introducing CRE method we apply to DMBBM and mKdV-Burgers equations. We had exact explicit solutions of given equations with the help of Riccati equation. The obtained exact solutions are consist of hyperbolic and exponential functions. We checked all solutions of given equations by the Maple.

It is also shown that the CRE method can be performed to other kinds of integrable systems and can be obtained other kind of solutions.

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