Finite Difference Scheme with a Linearization Technique for Numerical Solution of (MRLW) Equation

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Abstract

In this paper, a numerical solution of the modified regularized long wave (MRLW) equation has been showed with help a linearization technique using Crank-Nicolson finite difference method . Eror norms norms L_2 and L_∞ have been calculated to show performance of present method. Calculated values are compared with study available in the literature.

Keywords: Crank-Nicolson finite difference technique, MRLW equation

Introduction

The modified regularized long wave (MRLW) equation plays an important role especially in physics and in physical phenomena such as nonlinear transverse waves in shallow water, phonon packets used in nonlinear crystals. Numerical solution of MRLW equation has been studied by many author. Benjamin et al. introduced (MRLW) equation using a mathematical theory of the equation. Bona and pryant have proposed the existence and uniqueness of the equation. Esen and Kutluay, Gardner and Gardner have examined with finite element method to solution of the MRLW equation. Gou and Cao have used pseudo-spectral method for he solution of the MRLW equation. Karakoç et al. obtained a numerical soltion of the modified regularized long wave (MRLW) equation using a numerical technique based on lumped Galerkin method using cubic B-spline finite elements. Karakoç and et al. applied a method based on collocation of quintic B- spline and Petrov Galerkin finite element method in which the element shape functions are cubic and weight fuctions are quadratic B-spline for numerical solutions of (MRLW) equation. MRLW equation have been applied two finite difference approximations for the space discretization and a multi-time step method for the time discretization by Keskin and Irk.

In this paper, we examined with help Crank-Nicolson finite difference method using a linearization technique numerical solution of the (MRLW) equation. We used mathlab program to obtain numerical results of the MRLW equation

2. Application of the Method

In this paper, we will consider the MRLW equation

$$U_t + U_x + 6U^2U_x - U_{xxt} = 0 \qquad (1)$$

with physical boundary conditions $U \to 0$ as $x \to \pm\infty$, where μ is a positive parameter and x is space step, t is time step.To apply numerical method the MRLW equation, we will take solution domain on interval $a \leq x \leq b$. The modified regularized long wave (MRLW) equation has boundary-initial conditions with following form

$$\mathrm{U(a,t) = 0, U(b,t) = 0} \qquad (2)$$
$$U(x,0) = \sqrt{c}\,\mathrm{sech}[p(x - x_0)] \qquad (3)$$

where $c = 1,\ p = \sqrt{\frac{c}{\mu(c+1)}}$ and $x_0 = 40.$ Exact solution of the MRLW equation

$$U(x,0) = \sqrt{c}\,\mathrm{sech}[p(x - (c+1)t - x_0)]$$

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The interval $[a, b]$ is divided into N equal subinterval such that $a < x_0 < x_1 < \ldots < x_N = b$ for $m = 0,1, \ldots, N$ at the nodal points x_m by selecting the space step size as

$$h = \frac{b-a}{N} = (x_{m+1} - x_m)$$

Using the forward difference approximation for U_t, U_{xxt},

$$U_t = \frac{U_m^{n+1} - U_m^n}{\Delta t}$$

and approximation central difference for U_x

$$U_x = \frac{{U_{m+1}}^n - {U_{m-1}}^n}{2h}$$

and the Crank-nicolsan difference approximation for $U^2 U_x$ in equation (1) lead to

$$\frac{U_m^{n+1} - U_m^n}{\Delta t} + \frac{1}{2}\left[\frac{U_{m+1}^{n+1} - U_{m-1}^{n+1}}{2h} + \frac{U_{m+1}^n - U_{m-1}^n}{2h}\right] + 6\left[\frac{(U^2U_x)^{n+1} + (U^2U_x)^n}{2}\right] - \frac{\mu}{k}\left[U_{xx}^{n+1} - U_{xx}^n\right] = 0$$

appling Rubin and Graves linearization technique to equation

$$(U^2U_x)^{n+1} = U^{n+1}U^nU_x^n + U^nU^{n+1}U_x^n + U^nU^nU_x^{n+1} - 2U^nU^nU_x^n$$

and we obtain

$$\left[-\frac{1}{4h} - 3\frac{(U_m^n)^2}{2h} - \frac{\mu}{kh^2}\right]U_{m-1}^{n+1} + \left[\frac{1}{k} + 3\frac{U_m^n}{2h}(U_{m+1}^n - U_{m-1}^n) + \frac{2\mu}{kh^2}\right]U_m^{n+1} + \left[\frac{1}{4h} + \frac{3(U_m^n)^2}{2h} - \frac{\mu}{kh^2}\right]U_{m+1}^{n+1}$$

$$= \frac{U_m^n}{k} + 3\frac{U(_m^n)^2}{2h}(U_{m+1}^n - U_{m-1}^n) - \frac{\mu}{kh^2}(U_{m-1}^n - 2U_m^n + U_{m+1}^n) - \frac{1}{4h}(U_{m+1}^n - U_{m-1}^n)$$

for $m = 1,2, \ldots, N$.

3. Numerical Examples and Results

The MEW equation (2) has three invariant conditions to be mass, momentum, and energy respectively [P.J.Olver]

$$I_1 = \int_a^b U dx \cong h\sum_{j=1}^{N}(U_j^n)$$

$$I_2 = \int_a^b U^2 + \mu(U_x)^2 dx \cong h\sum_{j=1}^{N}(U_j^n)^2 + \mu(U_x)_j^n)$$

$$I_3 = \int_a^b U^4 dx \cong h\sum_{j=1}^{N}(U_j^n)^4$$

to show the performence of the method, error norms L_2 and L_∞ are calculated

$$L_2 = \|U^{exact} - U_N\|_2 \cong \sqrt{h\sum_{j=0}^{N}\left|U_j^{exact} - (U_N)_j\right|^2}$$

and the error norm L_∞

$$L_\infty = \|U^{exact} - U_N\|_\infty \cong max\left|U^{exact} - (U_N)_j\right|$$

Table 1

tf=10, $0 \le x \le 100$

	finite diffrence		Keskin-Irk
h=Δt	L_2	L_∞	L_∞
05	0.3092	0.1598	0.1364
0.2	1.3131	0.6448	0.0029
0.1	0.0788	0.0407	$7.6x10^{-3}$
0.05	0.0195	0.0101	$1.9x10^{-3}$

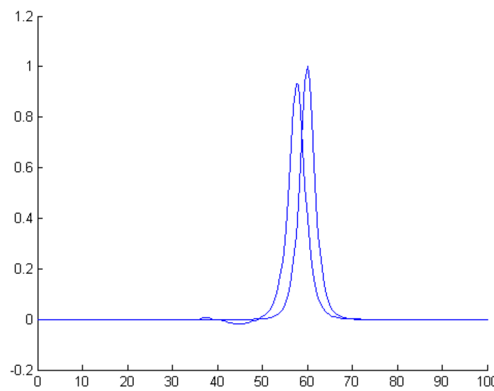

Figure 1 . for h=dt=0.1, h=dt=0.5 values.

Table 2

tf=10, $0 \le x \le 100$

h=0.2, Δt=0.02	L_2=0.0030	L_∞=0.0016

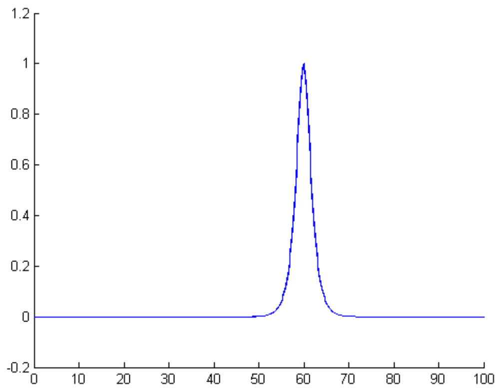

Figure 2. for h=0.2, dt=0.02 values.

4. Conclusion

In this paper, a numerical solution of the modified regularized long wave (MRLW) equation has been solved using Crank-Nicolson finite difference method with help a linearization techniques. In Table1 , when error norms norms L_2 and L_∞ calculated are compared with study available in the literatüre, we have find approximate values. But, as seen ın Table2 in decreasing time value , we have find more good results. We say that applied method is good of efficiency and performence.

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