Mass Pattern of the SM Fermions: Flavor Democracy Revisited

Umit Kaya\textsuperscript{1,2}, Saleh Sultansoy\textsuperscript{2,3*}

\textsuperscript{1}Ankara University, Science Faculty, Department of Physics, Ankara, Turkey
\textsuperscript{2}TOBB University of Economics and Technology, Ankara, Turkey
\textsuperscript{3}ANAS Institute of Physics, Baku, Azerbaijan

Geliş: 18 Mayıs 2019            Kabul: 31 Mayıs 2019 / Received: 18 May 2019           Accepted: 31 May 2019

Abstract

Mass pattern of the SM fermions is one of the most important mysteries in particle physics. Flavor Democracy could shed light on this mystery. Addition of isosinglet quark and isosinglet lepton give opportunity to obtain masses of charged leptons and quarks of the 2nd and 3rd family due to small deviations from full Flavor Democracy.

Keywords: Flavor democracy, isosinglet quark, isosinglet leptons, standard model.

Özet

SM fermiyonlarının kütle ve karışımları parçacık fiziğinin en önemli gizemlerinden biridir. Çeşni Demokrasisi bu gizemi aynıldabilir. Izosinglet kuarkı ve izosinglet leptonun eklenmesi 2. e 3. aile kuarklarının ve yüklü leptonların kütlelerinin Çeşni Demokrasisinden küçük sapmalar sayesinde elde edilmesini sağlar.

Anahtar Kelimeler: Çeşni demokrasisi, izosinglet kuark, izosinglet lepton, standart model.

1. Introduction

Mass and mixing patterns of the SM fermions are among the most important issues, which should be clarified in particle physics. In recent interview published in CERN Courier [1] Steven Weinberg emphasized this point: "Asked what single mystery, if he could choose, he would like to see solved in his lifetime, Weinberg doesn't have to think for long: he wants to be able to explain the observed pattern of quark and lepton masses". In our opinion, Flavor Democracy (see reviews [2-5] and references therein) could provide an important key to solve this mystery.

In this paper, we deal with mass pattern of the SM fermions (mixing pattern will be considered separately). In Section 2, mass pattern of the SM fermions is summarized.
Section 3 is devoted to the current status of the Chiral Fourth Family. Possible solution of the mass pattern mystery due to adding new isosinglet down quark and charged lepton has been considered in Section 4. Finally, we summarized our results in Section 5.

2. Mass Pattern of the SM Fermions

Masses of known charged leptons and quarks are given in Table 1 [6]. We do not consider neutrino masses since their values are not assigned experimentally and probably have specific mechanism, namely, see-saw. It should be noted that right-handed components of neutrinos are counterparts of the right-handed components of up quarks, therefore their inclusion does not mean BSM (Beyond the Standard Model) physics.

It is seen from Table 1 that masses of the first family fermions are much less than masses of second family ones and the latters are much lighter than the masses of the third family fermions (fermion mass hierarchy). The second important point is that mass of t quark is much greater than masses of tau lepton and b quark. This point excludes Flavor Democracy for 3 SM family case.

Table 1 Mass pattern of charged leptons and quarks

<table>
<thead>
<tr>
<th></th>
<th>charged leptons</th>
<th>Up type quarks</th>
<th>Down type quarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Family</td>
<td>$0.510998928 \pm 1.1 \times 10^{-8}$ MeV</td>
<td>$2.3^{+0.5}_{-0.5}$ MeV</td>
<td>$4.8^{+0.5}_{-0.3}$ MeV</td>
</tr>
<tr>
<td>2nd Family</td>
<td>$105.6583715 \pm 3.5 \times 10^{-6}$ MeV</td>
<td>$1.275 \pm 0.025$ GeV</td>
<td>$95 \pm 5$ MeV</td>
</tr>
<tr>
<td>3rd Family</td>
<td>$1776.82 \pm 0.16$ MeV</td>
<td>$173.21 \pm 0.51 \pm 0.71$ GeV</td>
<td>$4.18 \pm 0.03$ GeV</td>
</tr>
</tbody>
</table>

3. Status of the Chiral Fourth Family (C4F)

It is known that the Standard Model does not fix the number of fermion families. This number should be less than 9 in order to preserve asymptotic freedom and more than 2 in order to provide CP violation. According to the LEP data on Z decays, number of chiral families with light neutrinos ($\mathcal{m}_\nu << \mathcal{m}_Z$) is equal to 3, whereas extra families with heavy neutrinos are not forbidden. The fourth chiral family was widely discussed thirty years ago (see, for example [7, 8]). However, the topic was pushed off the agenda due to the misinterpretation of the LEP data.

Twenty years later 3 workshops on the fourth SM family [9–11] were held (for summary of the first and third workshops see [12] and [13], respectively). Main motivation was Flavor Democracy [14–16], which naturally provides heavy fourth family fermions including neutrino (consequences of Flavor Democracy Hypothesis for different models, including MSSM and E6, have been considered in [17, 18]). In addition, fourth family gives opportunity to explain baryon asymmetry of universe; it can accommodate emerging possible hints of new physics in rare decays of heavy mesons etc. (see [12] and references therein). Phenomenological papers on direct production (including anomalous resonant production) of the SM4 fermions at different colliders are reviewed in [19] (see tables VI and VII in [19]).

This activity has almost ended due to misinterpretation of the LHC data on the Higgs decays. It should be emphasized that these data exclude the minimal SM4 with one Higgs
doublet, whereas non-minimal SM4 with extended Higgs sector is still allowed [20, 21]. On the other hand, partial wave unitarity puts an upper limit around 700 GeV on the masses of fourth SM family quarks [22], which is almost excluded by the ATLAS and CMS data on search for pair production. For example, ATLAS $\sqrt{s} = 8$ TeV data with 20.3 fb$^{-1}$ integrated luminosity excludes new chiral quarks with mass below 690 GeV at 95% confidence level assuming $\text{BR}(Q \rightarrow W q)=1$ [23].

Even if SM4 may be excluded by the LHC soon, this is not the case for the general chiral fourth family (C4F). Therefore, ATLAS and CMS should continue a search for C4F up to kinematical limits. Concerning pair production, rescaling of the ATLAS lower bound using collider reach framework [24] shows that LHC will give opportunity to cover Mu4 up to 1.50 and 2.13 TeV with integrated luminosities 300 and 3000 fb$^{-1}$, respectively.

4. New Weak Isosinglet Fermions

As mentioned in [2, 3, 5], large difference between $m_b$ and $m_t$ can be explained by the addition of isosinglet quarks. Here we consider an addition of one isosinglet quark, so the quark sector is determined as:

\begin{equation}
\begin{pmatrix}
  u_L \\
  d_L \\
  c_L \\
  t_L \\
  s_L \\
  b_L
\end{pmatrix}
\begin{pmatrix}
  u_L \\
  d_L \\
  c_L \\
  t_L \\
  s_L \\
  b_L
\end{pmatrix}
\begin{pmatrix}
  u_R \\
  d_R \\
  c_R \\
  t_R \\
  s_R \\
  b_R \\
  D_L \\
  D_R
\end{pmatrix}
\end{equation}

where D denotes new isosinglet quark.

In the case of full Flavor Democracy, the mass matrix of the up type quarks can be written as

\begin{equation}
\begin{pmatrix}
  u_R & c_L & t_L \\
  u_L & \alpha & \alpha & \alpha \\
  c_L & \alpha & \alpha & \alpha \\
  t_L & \alpha & \alpha & \alpha
\end{pmatrix}
\end{equation}

and mass matrix of down type quarks is

\begin{equation}
\begin{pmatrix}
  d_R & s_R & b_R & D_R \\
  d_L & \alpha & \alpha & \alpha & \alpha \\
  s_L & \alpha & \alpha & \alpha & \alpha \\
  b_L & \alpha & \alpha & \alpha & \alpha \\
  D_L & M & M & M & M
\end{pmatrix}
\end{equation}

where $M (M >> \eta)$ is the new physics scale that determines the mass of isosinglet quark. In this case $m_u = m_c = 0$ and $m_t = 3\alpha\eta$ for up type quarks, $m_u = m_c = m_t = 0$ and $m_d = 3\alpha\eta + M = m_t + M$ for down type quarks.
In order to obtain mass of b quark, small deviation from matrix (3) is involved, namely

\[
\begin{pmatrix}
    d_r & s_r & b_r & D_r \\
    d_L & a\eta & a\eta & (1-\alpha_s)a\eta \\
    s_L & a\eta & a\eta & (1-\alpha_s)a\eta \\
    b_L & a\eta & a\eta & (1-\alpha_s)a\eta \\
    D_L & (1-\beta_b)M & (1-\beta_b)M & M
\end{pmatrix}
\]

(4)

At this stage, for numerical calculations we assume \(\alpha_s = \beta_b\). The masses of d and s quarks remain as \(m_d = m_s = 0\). On the other hand, masses of b and D quarks are as follows

\[
m_b = \frac{1}{2}(3a\eta + M - \sqrt{\ldots}) ; \quad m_D = \frac{1}{2}(3a\eta + M + \sqrt{\ldots})
\]

\[
\sqrt{\ldots} = \sqrt{(a\eta)^2 + 12a\eta M\alpha_s\beta_b - 12a\eta M\alpha_s - 12a\eta M\beta_b + 6a\eta M + M^2}
\]

(5)

For \(\alpha_s = \beta_b << 1\)

\[
\sqrt{\ldots} \approx (3a\eta + M) - \frac{12a\eta M\alpha_b}{(3a\eta + M)}
\]

(6)

Therefore \(m_s, m_b\) and \(\alpha_s\) are given by

\[
m_b \approx \frac{6a\eta M\alpha_b}{(3a\eta + M)}
\]

(7)

\[
m_D \approx (3a\eta + M) - \frac{6a\eta M\alpha_b}{(3a\eta + M)}
\]

(8)

\[
\alpha_b \approx \frac{(m_t + M)m_b}{2m_t M}
\]

(9)

Taking \(m_b = 4.18\) GeV and \(m_t = 173\) GeV we obtain \(\alpha_s\) and \(m_D\) corresponding to the different values of \(M\) which are given in Table 2.

Table 2 \(\alpha_s\) and \(m_D\) corresponding to different values of \(M\)

<table>
<thead>
<tr>
<th>M(GeV)</th>
<th>1000</th>
<th>2000</th>
<th>5000</th>
<th>10000</th>
<th>20000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_s)(10^{-3})</td>
<td>1.42</td>
<td>1.31</td>
<td>1.25</td>
<td>1.23</td>
<td>1.22</td>
</tr>
<tr>
<td>(m_D)</td>
<td>1169</td>
<td>2169</td>
<td>5169</td>
<td>10169</td>
<td>20169</td>
</tr>
<tr>
<td>(\alpha_s)(10^{-3})</td>
<td>6.02</td>
<td>5.58</td>
<td>5.31</td>
<td>5.22</td>
<td>5.18</td>
</tr>
<tr>
<td>(m_t)</td>
<td>1171</td>
<td>2171</td>
<td>5171</td>
<td>10171</td>
<td>20171</td>
</tr>
</tbody>
</table>
Similarly, tau lepton mass can be determined by adding an isosinglet lepton. In this case lepton sector is

\[
\begin{pmatrix}
e_L \\
v_e_L \\
\mu_L \\
v_\mu_L \\
t_L \\
v_t_L \\
u_e R \\
u_\mu R \\
u_\tau R \\
u_e R \\
u_\mu R \\
u_\tau R \\
t_L L \\
\end{pmatrix}
\]

\[L_L, L_R \]  \tag{10}

Then, the mass matrix becomes

\[
\begin{pmatrix}
e_e & \mu_e & \tau_e & L_e \\
e_e & a_\eta & a_\eta & a_\eta & (1-a_e)a_\eta \\
\mu_e & a_\eta & a_\eta & a_\eta & (1-a_e)a_\eta \\
\tau_e & a_\eta & a_\eta & a_\eta & (1-a_e)a_\eta \\
L_e & (1-\beta_e)M & (1-\beta_e)M & (1-\beta_e)M & M \\
\end{pmatrix}
\]  \tag{11}

For \( a_e = \beta_e << 1 \), \( m_e, m_\mu \) and \( a_\tau \) are given by

\[
m_\tau = \frac{6a_\eta M a_\tau}{(3a_\eta + M)} \tag{12}
\]

\[
m_L = (3a_\eta + M) - \frac{6a_\eta M a_\tau}{(3a_\eta + M)} \tag{13}
\]

\[
\alpha_\tau \approx \frac{(m_\tau + M)m_\tau}{2m_\tau M} \tag{14}
\]

With \( m_\tau = 1.777 \text{ GeV} \) we obtain \( \alpha_\tau \) and \( m_\tau \) corresponding to the different values of \( M \) which are given in the last two rows of Table 2.

Because the masses of the e, u and d quarks are very small, we do not comment on them at this stage. Masses of s quark, muon and c quark can also be obtained due to small deviations from full democracy. Concerning c quark let us consider following modification of the mass matrix of up quarks

\[
\begin{pmatrix}
u_R & c_L & t_L \\
u_L & a_\eta & a_\eta & a_\eta \\
c_L & a_\eta & a_\eta & a_\eta \\
t_L & a_\eta & a_\eta & (1+a_\tau)a_\eta \\
\end{pmatrix}
\]  \tag{15}

For \( a_\tau << 1 \)

\[
m_u = 0; m_c = \frac{1}{2} a_\eta [a_\tau + 3 - \sqrt{...}] ; m_t = \frac{1}{2} a_\eta [a_\tau + 3 - \sqrt{...}]
\]

\[
\sqrt{...} = \sqrt{(3+a_\tau)^2 - 8a_\tau a_c} \tag{16}
\]
For $\alpha_c \ll 1$

$$\alpha_c \approx \frac{9m_e}{2m_c} = 3.3 \times 10^{-2}$$

(17)

In order to obtain muon mass, we consider following modification of the Eq. 4

$$\begin{align*}
\begin{bmatrix}
\epsilon^L & \mu^L & \tau^L & L^L \\
L^L & (1+\beta_\tau)M & ((1+\beta_\tau)M & (1-\beta_\tau)M & M
\end{bmatrix}
\end{align*}$$

(18)

For $M = 2000$ GeV, $\alpha_\tau = \beta_\tau = 5.58 \times 10^{-3}$ and $\alpha_\mu = 2.73 \times 10^{-4}$ this mass matrix lead to

$$m_\mu = 2171 \text{ GeV}, \ m_\tau = 1.777 \text{ GeV}, \ m_\mu = 104.7 \text{ MeV}$$

(19)

Similarly, for down type quarks

$$\begin{align*}
\begin{bmatrix}
d^R & s^R & b^R & D^R \\
d^L & a^\eta & a^\eta & (1-\alpha_\nu)a^\eta \\
s^L & a^\eta & a^\eta & (1-\alpha_\nu)a^\eta \\
b^L & a^\eta & a^\eta & (1+\alpha_\nu)a^\eta & (1-\alpha_\nu)a^\eta \\
D^L & (1-\beta_\nu)M & (1-\beta_\nu)M & (1-\beta_\nu)M & M
\end{bmatrix}
\end{align*}$$

(20)

For $M = 2000$ GeV, $\alpha_\nu = \beta_\nu = 1.32 \times 10^{-2}$ and $\alpha_\mu = 2.48 \times 10^{-4}$ we obtain

$$m_d = 2168 \text{ GeV}, \ m_s = 4.18 \text{ GeV}, \ m_s = 95.2 \text{ MeV}$$

(21)

5. Conclusion

It is shown that masses of 2nd and 3rd SM family fermions can be obtained due to small deviations of Flavor Democracy, if new heavy isosinglet quark and isosinglet lepton exist in nature. These new quark and lepton have approximately same masses. Main decay channels of isosinglet quarks are $D \rightarrow Wq \ (q = u,c,t)$ with BR~0.5, $D \rightarrow Zq \ (q = d,s,b)$ with BR~0.25 and $D \rightarrow Hq \ (q = d,s,b)$ with BR~0.25. Isosinglet lepton will decay into $L \rightarrow Wl$ with BR~0.5, $L \rightarrow Zl \ (l = e,\mu,\tau)$ with BR~0.25 and $L \rightarrow Hl \ (l = e,\mu,\tau)$ with BR~0.25. The ATLAS experiment excludes the mass smaller than 1.21 TeV from the decay channel $D \rightarrow Hb$ [25].
References


23. Aad G. et al. Search for pair production of a new heavy quark that decays into a W boson and a light quark in pp collisions at \( \sqrt{s} = 8 \text{ TeV} \) with the ATLAS detector. Physical Review D, 2015; 92(11):112007.
