AN EVALUATION OF RESOURCE UTILISATION IN PALM OIL INDUSTRY USING THE MODIFIED COBB-DOUGLAS DECISION MODEL

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Abstract - Aggregate production functions, particularly the Cobb-Douglas model have been widely used in modeling input and output relationships in various organisations and at national economic levels. Most commonly used is the two-factor model where all inputs are aggregated as capital and labour factors of production. In this paper, a six-factor Cobb-Douglas model has been fitted to a ten-year production data obtained from a palm-oil producer. By logarithmic transformation, the normal equations obtained from the model were solved by the Least Squares method to obtain the output elasticities. The bootstrapping technique was used to establish their validity. The input components were aggregated and used in the traditional aggregate Cobb-Douglas model to obtain comparative results. For the disaggregated model, the output elasticities for the six input components were found to be 0.1653, 0.0457, 0.0864,-0.3136, 0.0403 and 0.3845 respectively, resulting in a decreasing return to scale of 0.4086. In the case of aggregate model, we have output elasticities of 0.4578 and 0.2730 for aggregate capital and labour respectively, also indicating a decreasing return to scale of 0.7308. However it was found that while the aggregate model gave a generalized result, the disaggregated approach pointed to specific aspects of the inputs that were adversely affecting the productivity of the organization and thus requiring stringent management control. Thus the study showed that the disaggregated Cobb-Douglas production function is superior to the traditional two-factor model. The model developed which was statistically tested, is novel and provides robust decision support for budding and seasoned firms.

Keywords: Cobb-Douglas Production function, Disaggregated, Return to scale, Output Elasticity, Urn, Bootstrapping.

1. Introduction

Management is about setting goals and establishing measures to be taken for their attainment, in the overall interest of the organisation. There is no gainsaying the fact that any organisation that desires to remain in business must have a means of evaluating how well it is achieving set goals. In other words, it must find out if returns are sufficiently high to reward the risks taken and, therefore, continue in business.

In essence, organisational performance must be continuously evaluated and necessary feedback sent to appropriate sections in order to remedy any perceived adverse occurrence [1].

Bagus et. at. [2] assesses and analyses the resilience of the metal sector in Indonesia using the Cobb-Douglas Production Function. The analysis shows how labour expenses, capital investment and total factor productivity contribute to sector growth. The study and its findings contribute to studies resilience measurement in both the metal sector and other industries.

Zaijian [3] did a study to analyse the temporal and spatial variation of the agricultural input-output and the relationship between agricultural output and input factors in Hebei Province using cobb-douglas production function in which cultivated area, effective irrigation area, chemical fertilizer usage, agricultural machinery power and rural electricity consumption had an upward trend from 1999 to 2008. With the help of this analysis he was able to advise Hebei Province to pay attention to effective use of water resources and accelerate investment in technology and mechanisation to promote agricultural sustainable development.

Every production organisation, be it small, medium or large scale, is characterised by the use of a group of inputs (resources) which are transformed by one form of activity or the other, into desired outputs. These resources which are largely materials, manpower, plant and machinery, money and time, must be well proportioned in the transformation process in order to obtain desired products (goods or services). In this paper, a new approach has been introduced in the production transformation process in the effective and efficient utilisation of input resources for production of output that satisfies customers' demands, while positioning the organisation on a profitable platform.

2. Material and Methods

This study is a new approach to empirical estimation of the Cobb-Douglas production function. Various existing models developed for the estimation of production functions for various purposes and applications were examined. These models include the Transcendental Logarithmic (Translog) production function [4]. Furthermore, Constant Elasticity of Substitution (CES) was developed [5] and this was further elaborated [6]. A nested CES function was proposed in a bid to introduce a four-input production function [7]. However without arbitrarily normalising the coefficients, not all the coefficients of the entire nested CES function can be readily identified. The authors further noted that arbitrarily normalising the coefficients gives rise to infinite number of non-normalised coefficients in the same output quantity. These limitations impacts negatively on the use of the nested CES function, in practical terms [6].

Relatedly Leontief introduced a constant return to scale function which was employed [8], in its generalized form in modeling the producer behaviour for United States of America's manufacturing sector for the 1947-1971 period. It is pertinent to observe that the Cobb-Douglas model is empirically more flexible than the others. Furthermore, it works in practice and provides practical ways of representing the relationship between the availability of inputs and the capacity to produce output. The Cobb-Douglas model also has the merit of allowing input factors to change in magnitude in response to factor price changes instead of fixing ratios of the inputs [9]. It was shown that the Cobb-Douglas has universal application when they employed the principle to develop a house-hold production model as a utility function to relate effective leisure of husband and wife to family output in the form [10]:

$$U = U(C, M_L L_L, M_W L_W) \tag{1}$$

where C = goods obtained in the market or produced at home MLLL and MWLW are effective leisure of husband and wife respectively

$$C = X_m + Z \tag{2}$$

where: Xm represents goods purchased in the market, and Z represents goods produced at home (and measured in the same units as market – purchased goods).

Furthermore, the flexibility of the Cobb-Douglas production function makes it amenable for use in combination with other production functions. To this end, the Cobb-Douglas, Translog Stochastic production function was employed to model the Brazilian main grain production [11]. Given the stated merits, the Cobb-Douglas Production function was selected for use in this paper. However, a major departure from the traditional two-factor is the decomposition of input resources into cost elements to enable their utilization as estimates in the production function with more dependable outcome.

2.1. Strategy for Data Collection and Input Data Structure

In this paper, the primary method of data collection with qualitative attributes was employed using a well-articulated data collection instrument. The input resources were broken down into various cost elements of fixed capital, working capital, direct labour, indirect labour, machinery and energy costs to enable the utilization of these resources to be effectively represented in estimating the production function. Using these elemental costs basis, historical production and input data were obtained from the organization under investigation. The restructured data are shown in table 1.

Table 1: Total Output and Input (Ten – year period)

	Q	K_1	K_2	L_1	L_2	M	E
	$x10^6$	$x10^6$	$x10^6$	$x10^6$	$x10^6$	$x10^6$	$x10^6$
1	71.5	29	20.100	2.2	5	7.5	10.6
2	65	47.4	28.100	2.7	5.2	11.6	10.6
3	76.1	26.9	27.800	2.7	6.5	14	10.8
4	69.1	28.9	53.100	2.7	8	12.9	13
5	71.1	53.8	47.100	3	10.4	19.1	18.5
6	95.03	59.2	63.300	3.3	10.9	76.7	25
7	103.2	81.6	108.900	4	11.1	121.1	35
8	152.7	134.8	107.800	4.2	12.3	175.8	41.9
9	176	139.4	106.200	4.7	13.8	96.1	38.6
10	112.4	134.8	78.300	5.5	15.8	89	39.3
Total	992.13	735.8	640.700	35	99	623.8	243.3

2.2. Equation Formulation and Computation

The six-factor model developed is of the form:

$$Q = f\left(K_1, K_2, L_1, L_2, M, E\right) = AK_1^{\alpha}.K_2^{\beta}L_1^{\gamma}L_2^{\mu}M^{\varphi}E^{\omega} \quad (3)$$

Q = Total output in monetary value of goods produced per annum

A = Total Factor Productivity, K1= Fixed capital, K2 = Working Capital,

L1 = Direct labour cost, L2 = Indirect labour cost, M = Machinery cost, E = Energy cost

α, β, γ, μ, φ and ω are output elasticities of the various inputs.

Taking logarithms of equation (3) we have:

$$\log Q = \log A + \alpha \log K_1 + \beta \log K_2 + \gamma \log L_1 + \mu \log L_2 + \varphi \log M + \omega \log E \quad (4)$$

This is of the form

$$y = \beta_o + \alpha x_1 + \beta x_2 + \gamma x_3 + \mu x_4 + \varphi x_5 + \omega x_6 + \varepsilon \quad (5)$$

Where

$$y = \log Q, \beta_0 = \log A, x_1 = \log K_1, x_2 = \log K_2, x_3 = \log L_1, x_4 = \log L_2, x_5 = \log M, x_6 = \log E$$

and ε is the random error which measures the discrepancy in the estimation of y as an approximate function of x [12]. From multivariate linear regression, we have the following seven normal equations.

$$\Sigma y = \Sigma\beta_0 + \alpha\Sigma x_1 + \beta\Sigma x_2 + \gamma\Sigma x_3 + \mu\Sigma x_4 + \varphi\Sigma x_5 + \omega\Sigma x_6 \quad (6)$$

$$\Sigma x_1 y = \beta_0\Sigma x_1 + \alpha\Sigma x_1^2 + \beta\Sigma x_1 x_2 + \gamma\Sigma x_1 x_3 + \mu\Sigma x_1 x_4 + \varphi\Sigma x_1 x_5 + \omega\Sigma x_1 x_6 \quad (7)$$

$$\Sigma x_2 y = \beta_0\Sigma x_2 + \alpha\Sigma x_1 x_2 + \beta\Sigma x_2^2 + \gamma\Sigma x_2 x_3 + \mu\Sigma x_2 x_4 + \varphi\Sigma x_2 x_5 + \omega\Sigma x_2 x_6 \quad (8)$$

$$\Sigma x_3 y = \beta_0\Sigma x_3 + \alpha\Sigma x_1 x_3 + \beta\Sigma x_2 x_3 + \gamma\Sigma x_3^2 + \mu\Sigma x_3 x_4 + \varphi\Sigma x_3 x_5 + \omega\Sigma x_3 x_6 \quad (9)$$

$$\Sigma x_4 y = \beta_0\Sigma x_4 + \alpha\Sigma x_1 x_4 + \beta\Sigma x_2 x_4 + \gamma\Sigma x_3 x_4 + \mu\Sigma x_4^2 + \varphi\Sigma x_4 x_5 + \omega\Sigma x_4 x_6 \quad (10)$$

$$\Sigma x_5 y = \beta_0\Sigma x_5 + \alpha\Sigma x_1 x_5 + \beta_0\Sigma x_2 x_5 + \gamma\Sigma x_3 x_5 + \mu\Sigma x_4 x_5 + \varphi\Sigma x_5^2 + \omega\Sigma x_5 x_6 \quad (11)$$

$$\Sigma x_6 y = \beta_0\Sigma x_6 + \alpha\Sigma x_1 x_6 + \beta\Sigma x_2 x_6 + \gamma\Sigma x_3 x_6 + \mu\Sigma x_4 x_6 + \varphi\Sigma x_5 x_6 + \omega\Sigma x_6^2 \quad (12)$$

The ten-year data obtained from the organization were transformed and substituted into equations 6-12. From this and using standard coefficient matrix and adjoint canonical form, we obtained the matrix

$$M_0 = \begin{pmatrix} 10 & 17.8473 & 17.385 & 5.2699 & 9.6592 & 15.6223 & 13.2104 & : & 19.7116 \\ 17.8473 & 32.5856 & 31.6073 & 9.7115 & 17.6190 & 29.0489 & 24.1975 & : & 35.5205 \\ 17.385 & 31.6073 & 30.8749 & 9.4276 & 17.1745 & 28.3075 & 23.5504 & : & 34.5683 \\ 5.2699 & 9.7115 & 9.4276 & 2.9227 & 5.2738 & 8.7500 & 7.2365 & : & 10.5321 \\ 9.6592 & 17.6190 & 17.1745 & 5.2738 & 9.6049 & 15.7878 & 13.1376 & : & 19.223 \\ 15.6223 & 29.0489 & 28.3078 & 8.7500 & 15.7878 & 26.7502 & 21.7806 & : & 31.3993 \\ 13.2104 & 24.1975 & 23.5504 & 7.2365 & 13.1376 & 21.7806 & 18.0464 & : & 26.3493 \end{pmatrix}$$

Using Matlab to solve the matrix, we have the solution

$$\beta_0 = 1.2831$$
$$= \log A$$
$$\therefore \quad A = 19.19111 \times 10^6$$
$$\alpha = 0.1653$$
$$\beta = 0.0457$$
$$\gamma = 0.0864$$
$$\mu = -0.3136$$
$$\varphi = 0.0403$$
$$\omega = 0.3843$$

Substituting these values in equation (3), we have the production function:

$$Q = 19.19111 K_1^{.1653} . K_2^{.0457} . L_1^{.0864} . L_2^{-.3136} . M^{.0403} . E^{.3842} \quad (13)$$

Various values of K1, K2, L1, L2, M and E can be inputted to obtain corresponding values of Q. Therefore the function, in addition to being a monitoring device can be used as a forward-planning tool to project into the future, particularly for the allocation of resources.

2.3. Comparative Computation

In the traditional two-factor Cobb-Douglas model, all inputs are aggregated as capital and labour inputs and presented as a production function of the form;

$$Q = f(K, L) = AK^{\alpha} L^{\beta} \quad (14)$$

The Cobb-Douglas model was fitted into the production data in the traditional two-factor for comparative analysis to obtain the following results:

β_0 = .6053 = log A; $\therefore A = 4.029953$
α = 0.4578; β = 0.2730

2.4. 2.4. Computer Programme

A flexible computer programme code was developed to customize the Matlab package, as a means of further enhancing the benefit of the disaggregated model, in the computation of the parameters of the proposed production function. A portion of the computer code is as follows:

```
% Script to compute the Total Factor Productivity of an industrial process
% Using a six-factor Cobb-Douglas Production function
load total_output; %load 11-by-7 data matrix
% Q = total output per annum
% A = total factor productivity
% k1 = fixed capital
% k2 = working capital
% L1 = direct labour
% L2 = indirect labour
% m = machinery cost
% e = energy cost
%reassign working data from input matrix
Q = total_output(1:10,1);
K1 = total_output(1:10,2);
K2 = total_output(1:10,3);
L1 = total_output(1:10,4);
L2 = total_output(1:10,5);
m = total_output(1:10,6);
e = total_output(1:10,7);
%taking logarithm of the working data
y = log10(Q);
beta0 = ones(10,1);
```

3. Statistical Validation

With the aid of the bootstrap resampling technique and using the ten-year data of inputs and outputs obtained from the organisation, thirty new sets of input and output data were generated with the use of a suitably selected urn.

This is essentially a manual process of resampling with replacement. Thereafter and in a fashion similar to the earlier stated computation procedure, the output elasticities were computed. In order to further establish the reliability of the results obtained, the output elasticities were subjected to more detailed statistical analysis. To this end, the results obtained from the previous manual bootstrapping analysis were used as base data to generate 10,000 new sets of data using the bootstrapping technique. The required computations were achieved with the aid of a computer program developed to adapt Matlab Computer package to suit the desired purpose. The results obtained from this analysis are presented in table 2.

Table 2: Results of Bootstrap Analysis

	A	B	γ	M	φ	Ω
Mean	0.5146	0.3317	0.0698	0.3104	0.2718	-0.9105
UCL	0.9651	0.6205	0.4963	0.88126	0.5510	0.0090
LCL	0.0858	0.0450	-0.4024	-0.1490	0.0608	-2.0676
Std. Error	0.2262	0.1468	0.2288	0.2463	0.1208	0.5295

In order to validate the parameters, they were statistically tested to establish the confidence limit for 95% confidence interval Using the relationship:

$$UCL = \bar{x} + 1.96\frac{\sigma}{\sqrt{n}} \quad (15)$$

And

$$LCL = \bar{x} - 1.96\frac{\sigma}{\sqrt{n}} \quad (16)$$

where UCL is the Upper 95% Confidence Limit and LCL is the Lower 95% Confidence Limit,

$\bar{x}$ = Sample mean of each output elasticity of the Cobb-Douglas model using the thirty sets of data.

σ = population standard deviation which was estimated from the sample standard deviation; valid for n ☐30. [13]

The population standard deviation σ, was computed from the relationship

$$\sigma = \sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n}} \quad (17)$$

where x = parameter value,

$\bar{x}$ = sample mean

n = sample size

Using Microsoft Excel package, we obtained means and standard deviations for each of the six output elasticities of the production function. Using these values in equations(15) and (16), UCL and LCL values were computed. The mean values of the six output elasticities were Using Microsoft Excel package, we obtained means and standard deviations for each of the six output elasticities of the production function. Using these values in equations(15) and (16), UCL and LCL values were computed. The mean values of the six output elasticities were found to be within the 95% confidence limits.

4. Result and Discussion

A new approach has been introduced in the empirical estimation of the Cobb-Douglas production function and found to be superior to the traditional aggregate model. It was specifically found that:

i. The output elasticities obtained from the analysis were 0.1653, 0.0457, 0.0864, -0.3136, 0.043 and 0.3845 for the input factors of fixed capital, working capital, direct labour, indirect labour, machinery cost and energy cost respectively. A combination of these values amount to decreasing

ii. returns to scale of 0.4086. These output elasticities represent the amounts by which the organisation's output will increase for every unit increase in investment for the input factor concerned.

iii. From a cursory look at the results obtained from the computations of UCL and LCL (as shown in table 2), it can be seen that the mean values of the ouput elasticities are dependable, since they were found to be within the 95% confidence limit.

iv. These values represent very marginal gains for the organization. For example, if for every unit increase in fixed capital investment in capital projects, we expect only 0.1653, or for machinery we expect only 0.0403, the situation calls for serious concern and urgent management attention. The case of indirect labour (non-production staff) is even worse as increase in investment amounts to undesirable drain on investment profile of the organization.

v. The scenario paints a grave picture in that while the classical accounting periodic reports may be showing some profits, the situation in the real sense is that the fortunes of the organization are going down a steep hill. This is similar to an observation made in the study of a Nigerian-based production company which eventually folded up [14].

vi. The new disaggregated approach is managerially superior to the traditional Cobb-Douglas model in terms of specificity.

5. Conclusion

The Cobb-Douglas Production Function has been disaggregated up to the extent of six input variables. This is a novel development because this new approach unlike the traditional Cobb-Douglas function, uncovers certain hidden details in the production transformation process as it affects specific input components. The disaggregated model is suitable for monitoring trend/patterns in the production process and is therefore a veritable management guide to action.

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