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SCREEN SLANT LIGHTLIKE SUBMANIFOLDS

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Abstract. Recently, the present author [10] studied slant lightlike submanifolds and concluded that, contrary to the Riemannian slant submanifolds [5], slant lightlike submanifolds do not include invariant and real subcases. To fill this missing gap, we introduce a new class of lightlike submanifolds, called screen slant lightlike submanifolds, of an indefinite Hermitian manifold, which contain screen real and invariant subcases. We give characterization theorems with examples and study minimal screen slant lightlike submanifolds supported by an example.

1. Introduction

In 1990, Chen [4, 5] defined a slant submanifold (M, g) of an almost Hermitian manifold $(\bar{M}, \bar{g}, \bar{J})$ as a real submanifold verifying that the Wirtinger angle, i.e., the angle between $\bar{J}X$ and T_xM is constant for every vector $X \in T_xM$ and $x \in M$. In 1996, Duggal-Bejancu presented the theory of lightlike submanifolds in [6] , but they did not discuss the slant lightlike case.

Motivated by Chen's work (followed by many others) on the geometry of slant submanifolds, recently, the present author introduced a new class, called slant lightlike submanifolds of Hermitian manifolds [10] and concluded that these submanifolds do not include invariant and real sub cases.

Since there is a natural existence of invariant (complex) and real submanifolds (see Ogiue [9], Yano-Kon [11], Bejancu [3]) in the complex differential geometry, the objective of this paper is to introduce another new class of lightlike submanifolds, called screen slant lightlike submanifolds, of an indefinite Hermitian manifolds which include invariant and screen real lightlike submanifolds. Roughly speaking, this new lightlike submanifold is a lightlike version of Chen's slant submanifolds.

Note that one can not define slant submanifolds in the lightlike geometry as usual, because the angle between a vector field and the tangent space of a lightlike submanifold can not be defined for a proper degenerate case. A lightlike submanifold of an indefinite Kaehler manifold has two distributions, namely, radical (totally lightlike) and non-degenerate screen distributions. Thus, one way to define a slant submanifold is to find Riemannian screen distribution. Indeed, we show that the

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screen distribution is Riemannian under a condition on nullity degree of radical distribution (Lemma 3.1). This enables us to define slant notion on the Riemannian screen distribution of a lightlike submanifold (Definition 3.1).

The remaining of the paper is arranged as follows. In section 2, we brief basic formulas and definitions for an indefinite Kaehler manifold and its lightlike submanifolds which we shall use later. In section 3, we introduce screen slant lightlike submanifold and give a characterization theorem. We show that screen slant lightlike submanifolds include invariant and screen real lightlike submanifolds. Then we give three examples of proper screen slant submanifolds. After we find integrability conditions for both screen and radical distributions, we obtain a necessary condition for the induced connection ∇ to be a metric connection. We note that the induced connection ∇ is not a metric connection on a lightlike submanifold, in general. In section 4, we obtain characterizations for minimal screen slant lightlike submanifolds and give an example.

2. Preliminaries

Let $(\bar{M}, \bar{g})$ be a $2k$-dimensional semi-Riemannian manifold with the semi-Riemannian metric $\bar{g}$ of constant index q, $0 < q < 2k$. An almost complex structure on $\bar{M}$ is a tensor field $\bar{J}$ of type (1,1) on $\bar{M}$ such that at every $p \in \bar{M}$ we have $\bar{J}^2 = -I$ where I denotes the identity transformation of $T_p\bar{M}$. A manifold $\bar{M}$ endowed with an almost complex structure is called an almost complex manifold. A Hermitian metric on $\bar{M}$ is a semi-Riemannian metric $\bar{g}$ satisfying

$$\bar{g}(X, Y) = \bar{g}(\bar{J}X, \bar{J}Y), \forall X, Y \in \Gamma(T\bar{M}). \tag{2.1}$$

An almost complex manifold endowed with a Hermitian metric is called an indefinite almost Hermitian manifold, denoted by $(\bar{M}, \bar{J}, \bar{g})$. Denote the Levi-Civita connection on $\bar{M}$ with respect to $\bar{g}$ by $\bar{\nabla}$. Then $\bar{M}$ is called an indefinite Kaehler manifold if $\bar{J}$ is parallel with respect to $\bar{\nabla}$,i.e.,

$$(\bar{\nabla}_X \bar{J})Y = 0, \forall X, Y \in \Gamma(T\bar{M}). \tag{2.2}$$

We now recall basic materials for the geometry of lightlike submanifolds from [6]. A submanifold M^m immersed in a semi-Riemannian manifold $(\bar{M}^{m+n}, \bar{g})$ is called a lightlike submanifold if it is a lightlike manifold w.r.t. the metric g induced from $\bar{g}$ and the radical distribution $Rad(TM)$ is of rank r, where $1 \leq r \leq m$. Let $S(TM)$ be a screen distribution which is a semi-Riemannian complementary distribution of $Rad(TM)$ in TM, i.e., $TM = Rad\,(TM) \perp S(TM)$.

Consider a screen transversal vector bundle $S(TM^{\perp})$, which is a semi-Riemannian complementary vector bundle of $Rad(TM)$ in $TM^{\perp}$. Since, for any local basis $\{\xi_i\}$ of $Rad(TM)$, there exists a local null frame $\{N_i\}$ of sections with values in the orthogonal complement of $S(TM^{\perp})$ in $[S(TM)]^{\perp}$ such that $\bar{g}(\xi_i, N_j) = \delta_{ij}$, it follows that there exists a lightlike transversal vector bundle $ltr(TM)$ locally spanned by $\{N_i\}$ [6, page 144]. Let $tr(TM)$ be complementary (but not orthogonal) vector bundle to TM in $T\bar{M}|_M$. Then, we have the following decompositions

$$\begin{aligned} tr(TM) &= ltr(TM) \perp S(TM^{\perp}), \\ T\bar{M}|_M &= S(TM) \perp [Rad(TM) \oplus ltr(TM)] \perp S(TM^{\perp}). \end{aligned}$$

Following are four subcases of a lightlike submanifold $(M, g, S(TM), S(TM^{\perp})$.

Case 1: r - lightlike if $r < min\{m, n\}$;

Case 2: Co - isotropic if $r = n < m$; $S(TM^{\perp}) = \{0\}$;

Case 3: Isotropic if $r = m < n$; $S(TM) = \{0\}$;
Case 4: Totally lightlike if $r = m = n$; $S(TM) = \{0\} = S(TM^{\perp})$.
The Gauss and Weingarten equations are:

$$\bar{\nabla}_X Y = \nabla_X Y + h(X,Y), \forall X, Y \in \Gamma(TM), \tag{2.3}$$
$$\bar{\nabla}_X V = -A_V X + \nabla^t_X V, \forall X \in \Gamma(TM), V \in \Gamma(tr(TM)), \tag{2.4}$$

where $\{\nabla_X Y, A_V X\}$ and $\{h(X,Y), \nabla^t_V V\}$ belong to $\Gamma(TM)$ and $\Gamma(ltr(TM))$, respectively. ∇ and ∇^t are linear connections on M and on the vector bundle $ltr(TM)$, respectively. The second fundamental form h is a symmetric $\mathcal{F}(M)$-bilinear form on $\Gamma(TM)$ with values in $\Gamma(tr(TM))$ and the shape operator A_V is a linear endomorphism of $\Gamma(TM)$. Then we have

$$\bar{\nabla}_X Y = \nabla_X Y + h^l(X,Y) + h^s(X,Y), \tag{2.5}$$
$$\bar{\nabla}_X N = -A_N X + \nabla^l_X(N) + D^s(X,N), \tag{2.6}$$
$$\bar{\nabla}_X W = -A_W X + \nabla^s_X(W) + D^l(X,W), \quad \forall X, Y \in \Gamma(TM), \tag{2.7}$$

$N \in \Gamma(ltr(TM))$ and $W \in \Gamma(S(TM^{\perp}))$. Denote the projection of TM on $S(TM)$ by P. Then, by using (2.3), (2.5)-(2.7) and taking account that $\bar{\nabla}$ is a metric connection we obtain

$$\bar{g}(h^s(X,Y),W) + \bar{g}(Y, D^l(X,W)) = g(A_W X, Y), \tag{2.8}$$
$$\bar{g}(D^s(X,N),W) = \bar{g}(N, A_W X). \tag{2.9}$$

We set

$$\nabla_X PY = \nabla^*_X PY + h^*(X,PY), \tag{2.10}$$
$$\nabla_X \xi = -A^*_\xi X + {\nabla^*}^t_X \xi, \tag{2.11}$$

for $X, Y \in \Gamma(TM)$ and $\xi \in \Gamma(RadTM)$. By using above equations we obtain

$$\bar{g}(h^l(X,PY),\xi) = g(A^*_\xi X, PY), \tag{2.12}$$
$$\bar{g}(h^*(X,PY),N) = g(A_N X, PY), \tag{2.13}$$
$$\bar{g}(h^l(X,\xi),\xi) = 0 \quad , \quad A^*_\xi \xi = 0. \tag{2.14}$$

In general, the induced connection ∇ on M is not metric connection. Since $\bar{\nabla}$ is a metric connection, by using (2.5) we get

$$(\nabla_X g)(Y,Z) = \bar{g}(h^l(X,Y),Z) + \bar{g}(h^l(X,Z),Y). \tag{2.15}$$

However, it is important to note that $\nabla^\star$ is a metric connection on $S(TM)$. Finally, we recall the following result which will be used later.

Theorem 2.1. *([6], P:161) Let M be an $r-$ lightlike submanifold with $r < min\{m,n\}$ or a coisotropic submanifold $\bar{M}$. Then the induced connection ∇ on M is a metric connection if and only if one of the following conditions is fulfilled:*

(i) *A^*_ξ vanish on $\Gamma(TM)$ for any $\xi \in \Gamma(RadTM)$.*
(ii) *$RadTM$ is a Killing distribution.*
(iii) *$RadTM$ is a parallel distribution with respect to ∇.*

3. Screen slant lightlike submanifolds

In this section, we introduce screen slant lightlike submanifolds, give examples and obtain characterizations. But we first recall the definitions of invariant lightlike submanifolds and screen real submanifolds from [7]. Let $\bar{M}$ be an indefinite Kaehler manifold and M a real lightlike submanifold of $\bar{M}$, then M is called invariant lightlike submanifold of $\bar{M}$ if

$$\bar{J}(Rad\,TM) = Rad\,TM \quad \text{and} \quad \bar{J}(S(TM)) = S(TM).$$

A lightlike submanifold M is called screen real submanifold if

$$\bar{J}(Rad\,TM) = Rad\,TM \quad \text{and} \quad \bar{J}(S(TM)) \subseteq S(TM^{\perp}).$$

We now give the following lemma which will be useful to define slant notion on the screen distribution.

Lemma 3.1. *Let M be a $2q-$ lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$ with constant index $2q$ such that $2q < dim(M)$. Then the screen distribution $S(TM)$ of lightlike submanifold M is Riemannian.*

Proof. Let $\bar{M}$ be a real $2k = m + n-$ dimensional indefinite Kaehler manifold and $\bar{g}$ be a semi-Riemannian metric on $\bar{M}$ of index $2q$. Let us assume that M be an $m-$ dimensional and $2q(< m)-$ lightlike submanifold of $\bar{M}$. Then we have a local quasi orthonormal field of frames on $\bar{M}$ along M

$$\{\xi_i,\, N_i,\, X_\alpha,\, W_a\}, i \in \{1, ..., 2q\},\ \alpha \in \{2q+1, ..., m\},\ a \in \{2q+1, ..., n\},$$

where $\{\xi_i\}$ and $\{N_i\}$ are lightlike basis of $RadTM$ and $ltr(TM)$, respectively and $\{X_\alpha\}$ and $\{W_a\}$ are orthonormal basis of $S(TM)$ and $S(TM^{\perp})$, respectively. From the null basis $\{\xi_1, ..., \xi_{2q}, N_1, ..., N_{2q}\}$ of $ltr(TM) \oplus RadTM$, we can construct an orthonormal basis $\{U_1, ..., U_{4q}\}$ as follows

$$\begin{array}{cc} U_1 = \frac{1}{\sqrt{2}}(\xi_1 + N_1) & U_2 = \frac{1}{\sqrt{2}}(\xi_1 - N_1) \\ U_3 = \frac{1}{\sqrt{2}}(\xi_2 + N_2) & U_4 = \frac{1}{\sqrt{2}}(\xi_2 - N_2) \\ ... & ... \\ ... & ... \\ U_{4q-1} = \frac{1}{\sqrt{2}}(\xi_{2q} + N_{2q}) & U_{4q} = \frac{1}{\sqrt{2}}(\xi_{2q} - N_{2q}). \end{array}$$

Hence, $Span\{\xi_i,\, N_i\}$ is a non-degenerate space of constant index $2q$. Thus we conclude that $RadTM \oplus ltr(TM)$ is non-degenerate and it has constant index $2q$ on $\bar{M}$. Since

$$index(T\bar{M}) = index(RadTM \oplus ltr(TM)) + index(S(TM^{\perp}) \perp S(TM)),$$

we obtain that $S(TM) \perp S(TM^{\perp})$ is constant index zero, that is, $S(TM)$ and $S(TM^{\perp})$ are Riemannian vector bundles. Thus proof is complete. □

Thus Lemma 3.1 enables us to give the following definition.

Definition 3.1. Let $(M, g, S(TM))$ be a $2q-$ lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$ with constant index $2q < dim(M)$. Then we say that M is a screen slant lightlike submanifold of $\bar{M}$ if the following conditions are satisfied:

(i) $RadTM$ is invariant with respect to $\bar{J}$, i.e. $\bar{J}(RadTM) = RadTM$.

(ii) For each non-zero vector field X tangent to $S(TM)$ at $x \in U \subset M$, the angle $\theta(X)$ between $\bar{J}X$ and $S(TM)$ is constant, i.e., it is independent of the choice of x and $X \in \Gamma(S(TM))$.

We note that $\theta(X)$ is called the slant angle. We point out the following features:

(a) $RadTM$ is even dimensional.
(b) Screen slant lightlike submanifolds do not include real hypersurface.

From now on, we suppose that $(M, g, S(TM))$ is a $2q(< dim(M))-$ lightlike submanifold of an indefinite Kaehler manifold with constant index $2q$ and denote it by M.

Proposition 3.1. *Let M be a screen slant lightlike submanifold of $\bar{M}$. Then M is invariant (resp.screen real) if and only if $\theta = 0$ (resp. $\theta = \frac{\pi}{2}$).*

Proof. If M is invariant, then $\bar{J}(RadTM) = RadTM$ and $\bar{J}(S(TM)) = S(TM)$, thus $\theta = 0$. Conversely, if M is screen slant lightlike with $\theta = 0$, then it is clear that $\bar{J}(S(TM)) = S(TM)$. Since $RadTM$ is invariant with respect to $\bar{J}$, proof is complete. The other assertion can be proved in a similar way. □

Thus it follows that a screen slant lightlike submanifold is a natural generalization of invariant and screen real lightlike submanifolds. A screen slant lightlike submanifold is said to be proper if it is neither invariant nor screen real lightlike submanifold.

For any vector field $X \in \Gamma(S(TM))$, we write

$$\bar{J}X = TX + \omega X, \tag{3.1}$$

where $TX \in \Gamma(TM)$ and $\omega X \in \Gamma(tr(TM))$.

Corollary 3.1. *Let M be a screen slant lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. Then, for any $X \in \Gamma(TM)$, we have*

(i) *If $X \in \Gamma(S(TM))$ then $\omega X \in \Gamma(S(TM^{\perp})$.*
(ii) *If $X \in \Gamma(RadTM)$, then $\omega X = 0$.*

Proof. It is easy to see that $ltr(TM)$ is invariant with respect to $\bar{J}$ due to $\bar{J}(RadTM) = RadTM$. (ii) is clear. □

Proposition 3.1 implies that invariant and screen real lightlike submanifolds are examples of screen slant lightlike submanifolds. Now, we want to present some examples of proper screen slant lightlike submanifolds. Let $\{\frac{\partial}{\partial x_1}, \frac{\partial}{\partial y_1}, ..., \frac{\partial}{\partial x_n}, \frac{\partial}{\partial y_n}\}$ be a canonical basis for $\mathbf{R}^{2n}_q$. Then we may define $\bar{J}$ such that

$$\bar{J}(\frac{\partial}{\partial x_i}) = \frac{\partial}{\partial y_i},\ \bar{J}(\frac{\partial}{\partial y_i}) = -\frac{\partial}{\partial x_i}.$$

Example 3.1. For any $\alpha > 0$, we consider the following immersion in $\mathbf{R^8_2}$:

$$x(u, v, t, s) = (t,\ s,\ u\cos\alpha,\ -v\cos\alpha,\ u\sin\alpha,\ v\sin\alpha,\ t,\ s,).$$

Then $RadTM$ is spanned by

$$\xi_1 = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_7},\ \xi_2 = \frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_8}$$

and $S(TM)$ is spanned by

$$X_1 = cos\,\alpha\,\frac{\partial}{\partial\,x_3} + sin\,\alpha\,\frac{\partial}{\partial\,x_5},\; X_2 = -cos\,\alpha\,\frac{\partial}{\partial\,x_4} + sin\,\alpha\,\frac{\partial}{\partial\,x_6}.$$

Then we can see that $RadTM$ is invariant with respect to $\bar{J}$ and $S(TM)$ is a slant distribution with slant angle 2α. Thus M is a screen slant hyperplane in $\mathbf{R}^{\mathbf{8}}_{\mathbf{2}}$. Moreover, we obtain screen transversal vector bundle $S(TM^{\perp})$

$$S(TM^{\perp}) = Span\{W_1 = sin\,\alpha\,\frac{\partial}{\partial\,x_4} + cos\,\alpha\,\frac{\partial}{\partial\,x_6},\; W_2 = -sin\,\alpha\,\frac{\partial}{\partial\,x_3} + cos\,\alpha\,\frac{\partial}{\partial\,x_5}\}$$

and lightlike transversal bundle $ltr(TM)$

$$ltr(TM) = Span\{N_1 = \frac{1}{2}\{-\frac{\partial}{\partial\,x_1} + \frac{\partial}{\partial\,x_7}\},\; N_2 = \frac{1}{2}\{-\frac{\partial}{\partial\,x_2} + \frac{\partial}{\partial\,x_8}\}\}.$$

Example 3.2. Consider in $\mathbf{R}^8_2$ the submanifold M given by

$$x(u,v,t,s) = (u, v, s\,sint,\ s\,cost,\ sin\,s,\ cos\,s,\ u\,cos\,\alpha - v\,sin\,\alpha,\ u\,sin\,\alpha + v\,cos\,\alpha)$$

for $\alpha,\ t, s \in (0, \frac{\pi}{2})$. Then TM is spanned by derive

$$\begin{aligned}
\xi_1 &= \frac{\partial}{\partial\,x_1} + cos\,\alpha\,\frac{\partial}{\partial\,x_7} + sin\,\alpha\frac{\partial}{\partial\,x_8}\\
\xi_2 &= \frac{\partial}{\partial\,x_2} - sin\,\alpha\,\frac{\partial}{\partial\,x_7} + cos\,\alpha\frac{\partial}{\partial\,x_8}\\
X_1 &= s\,cos\,t\,\frac{\partial}{\partial\,x_3} - s\,sin\,t\,\frac{\partial}{\partial\,x_4},\\
X_2 &= sin\,t\,\frac{\partial}{\partial\,x_3} + cos\,t\,\frac{\partial}{\partial\,x_4} + cos\,s\,\frac{\partial}{\partial\,x_5} - sin\,s\,\frac{\partial}{\partial\,x_6}.
\end{aligned}$$

It follows that $RadTM = Span\{\xi_1, \xi_2\}$, hence M is a $2-$lightlike submanifold. Since $\bar{J}RadTM = RadTM$, $RadTM$ is invariant. Moreover, we can choose $S(TM) = Span\{X_1, X_2\}$ which is Riemannian vector subbundle and it can be easily proved that $S(TM)$ is a slant distribution with slant angle $\theta = \frac{\pi}{4}$. Finally, the screen transversal vector bundle $S(TM^{\perp})$ is spanned by

$$\begin{aligned}
W_1 &= sin\,s\frac{\partial}{\partial\,x_5} + cos\,s\,\frac{\partial}{\partial\,x_6}\\
W_2 &= sin\,t\,\frac{\partial}{\partial\,x_3} + cos\,t\,\frac{\partial}{\partial\,x_4} - cos\,s\,\frac{\partial}{\partial\,x_5} + sin\,s\frac{\partial}{\partial\,x_6}
\end{aligned}$$

and the lightlike transversal bundle $ltr(TM)$ is spanned by

$$N_1 = \frac{1}{2}\{-\frac{\partial}{\partial\,x_1} + cos\,\alpha\,\frac{\partial}{\partial\,x_7} + sin\,\alpha\frac{\partial}{\partial\,x_8}\},\ N_2 = \frac{1}{2}\{-\frac{\partial}{\partial\,x_2} - sin\,\alpha\,\frac{\partial}{\partial\,x_7} + cos\,\alpha\frac{\partial}{\partial\,x_8}\}.$$

Example 3.3. For any $k, \alpha > 0$, consider in $\mathbf{R}^{12}_2$ the submanifold M given by

$$x(u,v,t,s) = (u\,ch\,\alpha,\ v\,ch\,\alpha, u, v,\ t,\ s,\ k\,cos\,t,\ k\,sin\,t,\ k\,cos\,s,\ k\,sin\,s,\ u\,sh\,\alpha,\ v\,sh\,\alpha)$$

Then TM is spanned by

$$\begin{aligned}
\xi_1 &= ch\,\alpha\,\frac{\partial}{\partial\,x_1} + \frac{\partial}{\partial\,x_3} + sh\,\alpha\,\frac{\partial}{\partial\,x_{11}},\ \xi_2 = ch\,\alpha\,\frac{\partial}{\partial\,x_2} + \frac{\partial}{\partial\,x_4} + sh\,\alpha\,\frac{\partial}{\partial\,x_{12}}\\
X_1 &= \frac{\partial}{\partial\,x_5} - k\,sin\,t\frac{\partial}{\partial\,x_7} + k\,cos\,t\,\frac{\partial}{\partial\,x_8},\ X_2 = \frac{\partial}{\partial\,x_6} - k\,sin\,s\frac{\partial}{\partial\,x_9} + k\,cos\,s\,\frac{\partial}{\partial\,x_{10}}.
\end{aligned}$$

Then we can see that $RadTM = Span\{\xi_1, \xi_2\}$. Thus M is a $2-$ lightlike submanifold. It is easy to see $\bar{J}RadTM = RadTM$, that is, $RadTM$ is invariant. Choose $S(TM) = Span\{X_1, X_2\}$ and we obtain that $S(TM)$ is slant distribution with slant angle $\theta = cos^{-1}(\frac{1}{1+k^2})$. Thus M is a screen slant lightlike submanifold of $\mathbf{R}^{12}_2$.

Chen ([5]) studied the following important problem in complex geometry: *Given a surface M of a Kaehler manifold $\bar{M}$, when is M slant in $\bar{M}$?*

Based on the above problem, there are several interesting results on the geometry of a slant surface of a Euclidean space R^4, see: ([5]). For the screen slant lightlike case, we have the following result.

Proposition 3.2. *There exist no screen slant lightlike surface of an indefinite Hermitian (or Kaehler) manifold with index* 2.

Proof. Let M be a screen slant lightlike surface of an indefinite Hermitian manifold $\bar{M}$ with index 2. Then M is $2-$ lightlike or $1-$ lightlike. If M is $2-$ lightlike then $S(TM) = 0$ and M is totally lightlike submanifold. Hence M is invariant. Now, suppose that M is $1-$ lightlike, then $RadTM = span\{\xi\}$. This is not possible, because $RadTM$ is invariant with respect to $\bar{J}$. □

Let M be a screen slant lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. We denote the projection morphisms on the distributions $RadTM$ and $S(TM)$ by Q and P, respectively. Then we have

$$X = QX + PX \tag{3.2}$$

for any $X \in \Gamma(TM)$, where QX denotes the component of X in $RadTM$ and PX denotes the component of X in $S(TM)$. Applying $\bar{J}$ on (3.2) we obtain

$$\bar{J}X = \bar{J}QX + \bar{J}PX = TQX + TPX + \omega PX. \tag{3.3}$$

Thus we derive

$$\bar{J}QX = TQX,\ \omega QX = 0 \tag{3.4}$$

and

$$TPX \in \Gamma(S(TM)). \tag{3.5}$$

On the other hand, the screen transversal bundle $S(TM^{\perp})$ has the following decomposition

$$S(TM^{\perp}) = \omega P(S(TM)) \perp \upsilon. \tag{3.6}$$

Then, for any $W \in \Gamma(S(TM^{\perp}))$ we write

$$\bar{J}W = BW + CW \tag{3.7}$$

where $BV \in \Gamma(S(TM)$ and $CV \in \Gamma(\upsilon)$.

Next, we give an useful characterization of screen slant lightlike submanifolds:

Theorem 3.1. *Let M be a $2q-$ lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$ with constant index $2q < dim(M)$. Then M is a screen slant lightlike submanifold if and only if*

(i) *the lightlike transversal bundle $ltr(TM)$ is invariant with respect to $\bar{J}$.*

(ii) *There exists a constant* $\lambda \in [-1,0]$ *such that*

$$(P \circ T)^2 X = \lambda X \tag{3.8}$$

for any $X \in \Gamma(S(TM))$. *Moreover, in this case* $\lambda = -cos\theta \mid_{S(TM)}$.

Proof. Let M be a $2q-$ lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$ with constant index $2q$. Then Lemma 3.1 guarantees that $S(TM)$ is a Riemannian vector bundle. Let M be a screen slant lightlike submanifold of $\bar{M}$. Then its radical distribution is invariant with respect to $\bar{J}$ and from Corollary 3.1, we have $\omega PX \in \Gamma(S(TM))$. Thus, using (2.1) and (3.3) we have

$$\bar{g}(\bar{J}N, X) = -\bar{g}(N, \bar{J}X) = -\bar{g}(N, TPX) - \bar{g}(N, \omega PX) = 0$$

for $X \in \Gamma(S(TM))$. Hence we conclude that $\bar{J}N$ does not belong to $S(TM)$. On the other hand, from (2.1) and (3.7) we obtain

$$\bar{g}(\bar{J}N, W) = -\bar{g}(N, \bar{J}W) = -\bar{g}(N, BW) - \bar{g}(N, CW) = 0$$

for $W \in \Gamma(S(TM^{\perp}))$. Thus $\bar{J}N$ does not belong to $S(TM^{\perp})$. Now suppose that $\bar{J}N \in \Gamma(RadTM)$. Then we obtain $\bar{J}\bar{J}N = -N \in \Gamma(ltrTM)$, since $RadTM$ is invariant with respect to $\bar{J}$ we get a contradiction which proves (i). With regards to statement (ii), since M is a screen slant lightlike submanifold, there is a constant angle θ which is independent $X \in \Gamma(STM)$ and $x \in \mathbf{U} \subset M$. Thus we derive

$$cos\theta(X) = \frac{\bar{g}(\bar{J}X, TPX)}{\mid \bar{J}X \mid\mid TPX \mid} = -\frac{\bar{g}(X, \bar{J}TPX)}{\mid \bar{J}X \mid\mid TPX \mid} = -\frac{\bar{g}(X, (P \circ T)^2 X)}{\mid X \mid\mid TPX \mid}. \tag{3.9}$$

On the other hand,we have

$$cos\theta(X) = \frac{\mid TPX \mid}{\mid \bar{J}X \mid}. \tag{3.10}$$

Thus, from (3.9) and (3.10) we obtain

$$cos^2\theta(X) = -\frac{g(X, (P \circ T)^2 X)}{\mid X \mid^2}.$$

Since $\theta(X)$ is a constant, we conclude that $(PoT)^2 X = \lambda X, \lambda \in [-1,0]$, which proves (ii).
$(\Leftarrow)$ The converse can be obtained in a similar way. □

From Theorem 3.1 we obtain the following corollary:

Corollary 3.2. *Let* M *be a screen slant lightlike submanifold of* $\bar{M}$. *Then*

$$g(TPX, TPY) = cos^2\theta \mid_{S(TM)} g(X, Y) \tag{3.11}$$

and

$$\bar{g}(\omega PX, \omega PY) = sin^2\theta \mid_{S(TM)} g(X, Y) \tag{3.12}$$

for any $X, Y \in \Gamma(TM)$.

Proof. From (2.1) and (3.3) we obtain

$$g(TPX, TPY) = -g(X, (PoT)^2 Y)$$

for any $X, Y \in \Gamma(S(TM))$. Then from Theorem 3.1 we derive

$$g(TPX, TPY) = cos^2\theta g(X, Y),$$

which proves (3.11). In a similar way we obtain (3.12). □

Differentiating (3.3) and comparing the tangent and transversal parts we have

$$(\nabla_X T)Y = A_{\omega PY}X + Bh^s(X,Y) \tag{3.13}$$

$$\bar{J}h^l(X,Y) = h^l(X,\bar{J}QY) + h^l(X,TPY) + D^l(X,\omega PY) \tag{3.14}$$

$$(\nabla_X \omega)Y = -h^s(X,\bar{J}QY) - h^s(X,TPY) + Ch^s(X,Y) \tag{3.15}$$

for $X, Y \in \Gamma(TM)$, where $(\nabla_X T)Y = \nabla_X \bar{J}QY + \nabla_X TPY - \bar{J}Q\nabla_X Y - TP\nabla_X Y$ and $(\nabla_X \omega)Y = \nabla^s_X \omega PY - \omega P\nabla_X Y$.

Theorem 3.2. *Let M be a screen slant lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. Then*

(i) *the radical distribution $RadTM$ is integrable if and only if the screen transversal second fundamental form of M satisfies*

$$h^s(X,\bar{J}Y) = h^s(\bar{J}X,Y), \forall X,Y \in \Gamma(RadTM)$$

(ii) *The screen distribution $S(TM)$ is integrable if and only if*

$$Q(\nabla_X TPY - \nabla_Y TPX) = Q(A_{\omega PY}X - A_{\omega PX}Y), \forall X,Y \in \Gamma(S(TM))$$

Proof. From (3.15) we obtain

$$h^s(X,\bar{J}Y) - Ch^s(X,Y) = \omega\nabla_X Y$$

for any $X, Y \in \Gamma(RadTM)$. Thus we obtain $h^s(X,\bar{J}Y) - h^s(\bar{J}X,Y) = \omega P[X,Y]$ which proves assertion (i). On the other hand, from (3.13) we derive

$$\nabla_X TPY - A_{\omega PY}X = \bar{J}Q\nabla_X Y + TP\nabla_X Y + Bh^s(X,Y)$$

for any $X, Y \in \Gamma(STM))$. Hence we get

$$\nabla_X TPY - \nabla_Y TPX + A_{\omega PX}Y - A_{\omega PY}X = \bar{J}Q[X,Y] + TP[X,Y].$$

Thus we obtain

$$Q(\nabla_X TPY - \nabla_Y TPX) + Q(A_{\omega PX}Y - A_{\omega PY}X) = \bar{J}Q[X,Y],$$

which proves (ii) □

Theorem 3.3. *Let M be a screen slant lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. Then the screen distribution defines a totally geodesic foliation if and only if $\bar{J}A_{\omega PY}X - A_{\omega PTPY}X$ has no components in $RadTM$ for $X, Y \in \Gamma(S(TM))$.*

Proof. Using (2.2) and (2.5) we have $\bar{g}(\nabla_X Y, N) = \bar{g}(\bar{\nabla}_X \bar{J}Y, \bar{J}N)$, for $X, Y \in \Gamma(S(TM))$ and $N \in \Gamma(ltr(TM))$. Thus from (3.3) and (2.7) we obtain $\bar{g}(\nabla_X Y, N) = \bar{g}(\bar{\nabla}_X TPY, \bar{J}N) - \bar{g}(A_{\omega PY}X, \bar{J}N)$. Using again (2.2), (2.5), (3.3) and (2.7) in the first expression in the above equation we derive

$$\bar{g}(\nabla_X Y, N) = g(\nabla_X (P \circ T)^2 Y, N) - \bar{g}(A_{\omega PTPY}X, N) - \bar{g}(A_{\omega PY}X, \bar{J}N).$$

Thus from Theorem 3.1 we get

$$\bar{g}(\nabla_X Y, N) = -cos^2\theta \bar{g}(\nabla_X Y, N) - \bar{g}(A_{\omega PTPY}X, N) - \bar{g}(A_{\omega PY}X, \bar{J}N).$$

Hence we obtain

$$(1 + cos^2\theta)\bar{g}(\nabla_X Y, N) = -\bar{g}(A_{\omega PTPY}X, N) - \bar{g}(A_{\omega PY}X, \bar{J}N).$$

This completes the proof. □

Next we investigate $\nabla_X T = 0$ on a screen slant lightlike submanifold. In the non-degenerate complex geometry, if a slant submanifold satisfies the above property, it is called a Kaehlerian slant submanifold [5].

Theorem 3.4. *Let M be a screen slant lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. Then T is parallel if and only if $D^s(X, N) \in \Gamma(v)$ and*

$$\bar{g}(h^s(X, Y), \omega PZ) = g(h^s(X, Z), \omega PY)$$

for $X \in \Gamma(TM)$ and $Y, Z \in \Gamma(S(TM))$.

Proof. From (3.13) we obtain $\bar{g}((\nabla_X T)Y, N) = 0$, for $Y \in \Gamma(RadTM)$ and $X \in \Gamma(TM)$. For $Y \in \Gamma(S(TM))$, we get $\bar{g}((\nabla_X T)Y, N) = \bar{g}(A_{\omega PY} X, N)$. Using (2.9) we derive

$$\bar{g}((\nabla_X T)Y, N) = \bar{g}(D^s(X, N), \omega PY) \tag{3.16}$$

for any $X \in \Gamma(S(TM))$. On the other hand, from (2.12), (3.1) and (2.1) we obtain

$$g((\nabla_X T)Y, Z) = g(A_{\omega PY} X, Z) - g(h^s(X, Y), \omega PZ)$$

for any $X, Y \in \Gamma(TM)$ and $Z \in \Gamma(S(TM))$. By using (2.8) we get

$$g((\nabla_X T)Y, Z) = g(h^s(X, Z), \omega PY) - g(h^s(X, Y), \omega PZ) \tag{3.17}$$

for any $X, Y \in \Gamma(TM)$ and $Z \in \Gamma(S(TM))$. Thus from (3.16) and (3.17) we obtain our assertion. □

It is known that the induced connection ∇ of a lightlike submanifold is not a metric connection, in general. Next, we give a necessary condition for the induced connection ∇ to be a metric connection.

Theorem 3.5. *Let M be a screen slant lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. If $(\nabla_X T)Y = 0$ for $X \in \Gamma(TM)$ and $Y \in \Gamma(RadTM)$, then the induced connection ∇ is a metric connection.*

Proof. If $(\nabla_X T)Y = 0$ for $X \in \Gamma(TM)$ and $Y \in \Gamma(RadTM)$, then from (3.13) we have $Bh^s(X, Y) = 0$, hence $g(Bh^s(X, Y), Z) = 0$ for $X \in \Gamma(TM)$, $Y \in \Gamma(RadTM)$ and $Z \in \Gamma(TM)$. Thus we obtain

$$\bar{g}(\bar{J}h^s(X, Y), Z) = 0 \tag{3.18}$$

$$\bar{g}(h^s(X, Y), \omega PZ) = 0. \tag{3.19}$$

Now, by using (2.5) we get

$$\bar{g}(\omega P\nabla_X Y, \bar{J}h^s(X, Y)) = \bar{g}(\omega P\nabla_X Y, \bar{J}\bar{\nabla}_X Y - \bar{J}\nabla_X Y - \bar{J}h^l(X, Y))$$

for $X \in \Gamma(TM)$, $Y \in \Gamma(RadTM)$, since $ltr(TM)$ is invariant, from (2.1) and (3.3) we get

$$\bar{g}(\omega P\nabla_X Y, \bar{J}h^s(X, Y)) = \bar{g}(\omega P\nabla_X Y, \bar{\nabla}_X \bar{J}Y) - \bar{g}(\omega P\nabla_X Y, \omega P\nabla_X Y).$$

Then using (2.5) we derive

$$\bar{g}(\omega P\nabla_X Y, \bar{J}h^s(X, Y)) = \bar{g}(\omega P\nabla_X Y, h^s(X, \bar{J}Y) - \bar{g}(\omega P\nabla_X Y, \omega P\nabla_X Y).$$

Thus from (3.19) we have

$$\bar{g}(\omega P\nabla_X Y, \bar{J}h^s(X, Y)) = -\bar{g}(\omega P\nabla_X Y, \omega P\nabla_X Y).$$

Then using (3.12) we obtain

$$\bar{g}(\omega P\nabla_X Y, \bar{J}h^s(X, Y)) = -sin^2\theta g(P\nabla_X Y, P\nabla_X Y) \tag{3.20}$$

for any $X \in \Gamma(TM)$ and $Y \in \Gamma(RadTM)$. On the other hand, from (2.1) and (3.3) we have

$$\bar{g}(\omega P\nabla_X Y, \bar{J}h^s(X,Y)) = -\bar{g}(TP\nabla_X Y, \bar{J}h^s(X,Y))$$

for any $X \in \Gamma(TM)$ and $Y \in \Gamma(RadTM)$. Then, from (3.18) we derive

$$\bar{g}(\omega P\nabla_X Y, \bar{J}h^s(X,Y)) = 0. \tag{3.21}$$

Then (3.20) and (3.21) imply

$$sin^2\,\theta g(P\nabla_X Y, P\nabla_X Y) = 0.$$

Since M is a proper screen slant lightlike submanifold and $S(TM)$ is Riemannian we obtain $P\nabla_X Y = 0$, hence $\nabla_X Y \in \Gamma(RadTM)$,i.e. the radical distribution $RadTM$ is parallel. Thus the assertion of theorem follows from Theorem 2.1. □

Remark 3.1. It is clear that radical distribution and screen distribution are orthogonal. However, we note that if $RadTM$ is parallel , then it doesn't imply that the screen distribution $S(TM)$ is parallel contrary to the non-degenerate case.

4. **Minimal Screen Slant Lightlike Submanifolds**

A general notion of minimal lightlike submanifold M of a semi-Riemannian manifold $\bar{M}$ has been introduced by Bejan-Duggal in [2] as follows:

Definition 4.1. We say that a lightlike submanifold $(M, g, S(TM))$ isometrically immersed in a semi-Riemannian manifold $(\bar{M}, \bar{g})$ is minimal if:

(i) $h^s = 0$ *on* $Rad(TM)$ *and*
(ii) $trace\,h = 0$, *where trace is written w.r.t.* g *restricted to* $S(TM)$.

In the case 2, the condition (i) is trivial. Moreover, it has been shown in [2] that above definition is independent of $S(TM)$ and $S(TM^{\perp})$, but it depends on the choice of the transversal bundle $tr(TM)$. As in the semi-Riemannian case, any lightlike totally geodesic M is minimal.

Example 4.1. Let $\bar{M} = \mathbf{R}^8_2$ be a semi-Euclidean space of signature $(-,-,+,+,+,+,+,+)$ with respect to the canonical basis $\{\partial\, x_1,\, \partial\, x_2,\, \partial\, x_3,\, \partial\, x_4,\, \partial\, x_5,\, \partial\, x_6,\, \partial\, x_7,\, \partial\, x_8\}$. Consider a complex structure J_1 defined by

$$\begin{aligned} J_1(x_1,\, x_2,\, x_3,\, x_4,\, x_5,\, x_6,\, x_7,\, x_8) \;\; = \;\; & (-x_2,\, x_1,\, -x_4,\, x_3,\, -x_7\, cos\,\alpha - x_6\, sin\,\alpha, \\ & -x_8 cos\,\alpha + x_5\, sin\,\alpha,\, x_5\, cos\,\alpha + x_8\, sin\,\alpha, \\ & x_6\, cos\,\alpha - x_7\, sin\,\alpha). \end{aligned}$$

for $\alpha \in (0, \frac{\pi}{2})$. Let M be a submanifold of $(\mathbf{R}^8_2, J_1)$ given by

$$\begin{aligned} x_1 = u_1, x_2 = u_2,\, x_3 = u_1\, cos\,\theta \;\; & - \;\; u_2\, sin\,\theta,\, x_4 = u_1\, sin\,\theta + u_2\, cos\,\theta \\ x_5 = u_3,\, x_6 = sin\,u_3\, sinhu_4 \;\; & , \;\; x_7 = u_4,\, x_8 = cos\,u_3\, cosh\,u_4. \end{aligned}$$

Then TM is spanned by

$$\begin{aligned} Z_1 &= \partial\, x_1 + cos\,\theta\, \partial\, x_3 + sin\,\theta\, \partial\, x_4 \\ Z_2 &= \partial\, x_2 - sin\,\theta\, \partial\, x_3 + cos\,\theta\, \partial\, x_4 \\ Z_3 &= \partial\, x_5 + cos\,u_3\, sinh\,u_4\, \partial\, x_6 + sinu_3\, cosh\,u_4\, \partial\, x_8 \\ Z_3 &= sin\,u_3\, cosh\,u_4\, \partial\, x_6 \partial\, x_6 + \partial\, x_7 + cos\,u_3\, sinh\,u_4\, \partial\, x_8. \end{aligned}$$

Then M is a $2-$ lightlike submanifold and $RadTM = \{Z_1, Z_2\}$. It follows that $RadTM$ is J_1- invariant. It is easy to see that $S(TM) = span\{Z_3, Z_4\}$ is a slant distribution with respect to J_1 with slant angle α. The screen transversal bundle is spanned by

$$\begin{aligned} W_1 &= -cosh\, u_4\, sinh\, u_4\, \partial\, x_5 + cos\, u_3\, cosh\, u_4\, \partial\, x_6 - sin\, u_3\, cos\, u_3\, \partial\, x_7 \\ &\quad -sin\, u_3\, sinh\, u_4\, \partial\, x_8 \\ W_2 &= sin\, u_3\, cos\, u_3\, \partial\, x_5 + sin\, u_3\, sinh\, u_4\, \partial\, x_6 - cosh\, u_4\, sinh\, u_4\, \partial\, x_7 \\ &\quad +cos\, u_3\, cosh\, u_4\, \partial\, x_8 \end{aligned}$$

and the lightlike transversal bundle is spanned by

$$\begin{aligned} N_1 &= \frac{1}{2}\{-\partial\, x_1 + cos\,\theta\, \partial\, x_3 + sin\,\theta\, \partial\, x_4\} \\ N_2 &= \frac{1}{2}\{-\partial\, x_2 - sin\,\theta\, \partial\, x_3 + cos\,\theta\, \partial\, x_4\}. \end{aligned}$$

Now, by direct calculations, using Gauss formulas we get

$$h^l = 0,\ h^s(X, Z_1) = 0,\ h^s(X, Z_2) = 0, \forall X \in \Gamma(TM)$$

and

$$\begin{aligned} h^s(Z_3, Z_3) &= \frac{-(sinh^2\, u_4 + cos^2\, u_3)}{cosh^4\, u_4 - sin^4\, u_3} W_2,\ h^s(Z_4, Z_4) = \frac{sinh^2\, u_4 + cos^2\, u_3}{cosh^4\, u_4 - sin^4\, u_3} W_2 \\ h^s(Z_3, Z_4) &= \frac{sinh^2\, u_4 + cos^2\, u_3}{cosh^4\, u_4 - sin^4\, u_3} W_1. \end{aligned}$$

Hence the induced connection is a metric connection, M is not totally geodesic and it is not also totally umbilical, but it is a minimal proper screen slant lightlike submanifold of $\mathbf{R}^8_2$.

Next, we prove two characterization results for minimal slant lightlike submanifolds. First we give the following lemma which will be useful later.

Lemma 4.1. *Let M be a proper screen slant lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$ such that $F(S(TM)) = S(TM^\perp)$. If $\{e_1, ..., e_m\}$ is a local orthonormal basis of $S(TM)$, then $\{\csc\,\theta F e_1,\ ...,\ \csc\,\theta\, F e_m\}$ is a orthonormal basis of $S(TM^\perp)$.*

Proof. Since $e_1, ..., e_m$ is a local orthonormal basis of $S(TM)$ and $S(TM)$ is Riemannian , from Corollary 3.1, we obtain

$$\bar{g}(\csc\,\theta F e_i, \csc\,\theta F e_j) = \csc^2\,\theta\, sin^2\,\theta\, g(e_i, e_j) = \delta_{ij},$$

which proves the assertion. □

Theorem 4.1. *Let M be a proper screen slant lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. Then M is minimal if and only if*

$$traceA^*_{\xi_j}\mid_{S(TM)} = 0,\ traceA_{W_\alpha}\mid_{S(TM)} = 0$$

and

$$\bar{g}(D^l(X, W), Y) = 0, \forall X, Y \in \Gamma(RaDTM),$$

where $\{\xi_j\}_{j=1}^r$ is a basis of $RadTM$ and $\{W_\alpha\}_{\alpha=1}^m$ is a basis of $S(TM^\perp)$.

Proof. From Proposition 3.1 in [2], we have $h^l = 0$ on $RadTM$. Thus M is minimal if and only if

$$\sum_{k=1}^{m} h(e_k, e_k) = 0$$

and $h^s = 0$ on $RadTM$ Using (2.8) and (2.12) we obtain

$$\sum_{k=1}^{m} h(e_k, e_k) = \sum_{k=1}^{m} \frac{1}{r} \sum_{j=1}^{r} g(A^*_{\xi_j} e_k, e_k) N_j + \frac{1}{m} \sum_{\alpha=1}^{m} g(A_{W_\alpha} e_k, e_k) W_\alpha. \tag{4.1}$$

On the other hand, from (2.8) we obtain $\bar{g}(h^s(X, Y), W) = \bar{g}(D^l(X, W), Y)$ for $X, Y \in \Gamma(RadTM)$. Thus our assertion follows from (4.1). □

Theorem 4.2. *Let M be a proper slant lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$ such that $F(S(TM)) = S(TM^\perp)$. Then M is minimal if and only if*

$$traceA^*_{\xi_j} \mid_{S(TM)} = 0, \; traceA_{Fe_i} \mid_{S(TM)} = 0$$

and

$$\bar{g}(D^l(X, Fe_i), Y) = 0, \forall X, Y \in \Gamma(RadTM)$$

where $\{e_1, ..., e_m\}$ is a basis of $S(TM)$.

Proof. From Lemma 4.1, $\{\csc\,\theta Fe_1, ..., \csc\,\theta\, Fe_m\}$ is a orthonormal basis of $S(TM^\perp)$. Thus we can write

$$h^s(X, X) = \sum_{i=1}^{m} A_i \csc\,\theta Fe_i, \forall X \in \Gamma(TM).$$

for some functions A_i, $i \in \{1, ..., m\}$. Hence we obtain

$$h^s(X, X) = \sum_{i=1}^{m} \csc\,\theta g(A_{Fe_i} X, X)$$

for $X \in \Gamma(S(TM))$. Then the assertion of theorem comes from Theorem 4.1. □

Remark 4.1. (a) Observe that between slant lightlike and screen slant lightlike submanifolds there exists no inclusion relation because a lightlike real hypersurface is a slant lightlike submanifold and it is not a screen slant lightlike submanifold. Moreover, invariant and screen real lightlike submanifolds are screen slant lightlike submanifolds, but they are not slant lightlike submanifolds.

(b) Notice that it follows from the Proposition 3.2 that the screen slant lightlike geometry is different than its counter part of Chen's Riemannian case. For example, there does not exist any screen slant lightlike surface of a semi-Euclidean space R^4_2.

(c) Finally, it is important to mention that, as per ([6], Page 157), the second fundamental forms of a lightlike submanifold M do not depend on the vector bundles $S(TM^\perp)$ and $ltr(TM)$. Thus, our results of this paper are stable with respect to any change in above vector bundles.

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