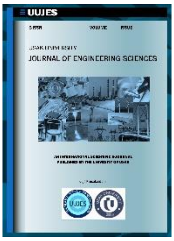

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Research article

NUMERICAL SOLUTION TECHNIQUES FOR CURRENT AND VOLTAGE VARIABLES IN ELECTRICAL (RLC) CIRCUITS

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Abstract

In this study, we present numerical solution approaches of second-order differential equations which is used as mathematical models of electrical circuits (RLC) consisting of a resistor, an inductor and a capacitor connected in series and parallel. The Differential Transformation Method (DTM) and Exponentially Fitted Collocation Approximation Method (EFCAM) were employed to obtain numerical solutions which are compared with the analytical solutions of the electrical circuits and are found to be accurate and compatible. Obtained voltage and current parameters are presented in tables and figures to show the efficiency of numerical techniques.

Keywords: *Electrical RLC circuits; Differential Transformation Method (DTM); Exponentially Fitted Collocation Approximation Method (EFCAM); voltages; current.*

1. Introduction

Differential equations are commonly used to describe physical phenomena in engineering sciences. Linear and non-linear differential equations have also common use to describe modelling of electrical circuits, heat transfer, mass load relationship and cracking of materials in solid dynamics. A lot of mathematical models in engineering and applied mathematics are expressed in terms of unknown quantities and their derivatives, particularly ODEs of different orders, which can be found in the mathematical modelling of real applications [1]. Electric circuits and electromagnetic theories are two major

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theories necessary to build all branches of electrical and electronics engineering. Many branches of electrical engineering and telecommunication sciences are based on electric circuit theories. Thus, circuits theories are important in specializing many branches of the physical sciences; because, circuits are appropriate for modelling energy flow in a circuit [2].

Ohm's law, Kirchhoff's Voltage and current laws are essential in analysis of linear electrical systems. These three laws are applied to resistive circuits where the only elements are voltage and/or current sources and resistors. In applying three laws, resistance of current through or voltage across resistor can be found if both are already known. Kirchhoff's Voltage Law (KVL) states that sum of all voltages in a closed loop has to be zero. In order to simplify the KVL equations, the polarities should satisfy the passive sign convention if possible [3]. In recent years, several researchers have worked on electrical (RLC) circuits problems. Atokolo [4] proposes iterative method to solve resistive electrical circuit problems. Axnuj [5] presents transient analysis of electrical circuits using Runge-Kutta method and its application. Reference [6] presents and studies on DC transients in R-L and R-C circuits. In this study, it is important to apply numerical solution techniques to compute current and voltage variables of the electric circuits as quick as possible. Thus, we employ a very easy, fast and accurate numerical techniques to obtain numerical solutions of both voltage and current variables in electrical circuits (RLC).

The outline of this paper is as follows: We present the resulting expressions of Kirchhoff's law dealing with the voltage and current in the circuit and Ohm's law relating voltage, current and resistance that eventually lead to Eq. (1). Then we present and employ two numerical techniques to obtain numerical solutions for both currents and voltages variables in electrical (RLC) circuit model. We conclude our study with three test problems to verify the efficiency and accuracy of proposed numerical techniques.

2. Electrical circuit models

In this study, we consider an electrical circuits consist of a resistor R, an inductor L, a capacitor C, a voltage source $v(t)$ and current $i(t)$ connected in series and parallel respectively.

Electrical models in Fig. 1 are expressed by the second order differential equations in following forms:

$$\begin{cases} L\dfrac{d^2i}{dt^2} + R\dfrac{di}{dt} + \dfrac{1}{C}i = \dfrac{dV(t)}{dt} \\ C\dfrac{d^2v}{dt^2} + \dfrac{1}{R}\dfrac{dv}{dt} + \dfrac{1}{L}v = \dfrac{dI(t)}{dt} \end{cases} \qquad t\epsilon[a,b] \tag{1}$$

Initial conditions are stated as follows:

$$\begin{cases} i(t_0) = \beta_1 \\ i^{/}(t_0) = \beta_2 \\ v(t_0) = \delta_1 \\ v^{/}(t_0) = \delta_2 \end{cases} \tag{2}$$

where $L, R, C, E, I\ and\ V$ are measured in Henrys, Ohms, Farad, Volts and Coulombs respectively. $\beta_1, \beta_2, \delta_1\ and\ \delta_2$ are arbitrary constants, t_0 is the time at initial condition, a and b are the domain of Eq. (1).

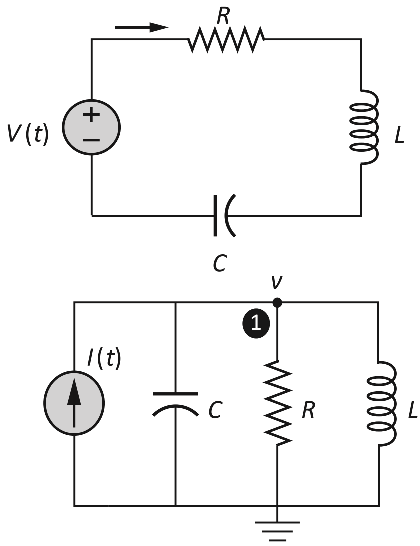

Fig. 1 Connected in serial and parallel electrical RLC circuit [3].

3. Description of techniques

3.1. Differential transformation method (DTM)

Zhou [7] was firstly proposed the concept of differential transformation method. It is an iterative procedure to obtain analytic Taylor series solutions of differential equations and/or partial differential equations. It is an effective technique for solving differential equations in various order. Chen and Liu [8] applied differential transform method to solve two-point boundary value problems. Ayaz [9] applied differential transform method for solving algebraic differential equations. Kangalgil and Ayaz [10] used DTM to obtain semi-analytical solutions of the KdV and mKdV equations. Ravi and Aruna [11] presented and employed two-dimensional differential transform method for solving linear and non-linear Schrödinger equations. Arikoglu and Ozkol [12] solved fractional differential equations by using differential transform method. The method was also used to solve Riccati differential equations [13]. The method is capable for reducing the computational work load while accuracy and convergence rate of the series solution are still provided.

Consider an arbitrary functions $i(t)$ and $v(t)$ which can be expanded in Taylor series around a point $t = 0$.

$$\begin{cases} i(t) = \displaystyle\sum_{k=0}^{\infty} \frac{t^k}{k!}\left[\frac{d^k i}{dt^k}\right]_{t=0} \\ v(t) = \displaystyle\sum_{k=0}^{\infty} \frac{t^k}{k!}\left[\frac{d^k v}{dt^k}\right]_{t=0} \end{cases} \tag{4}$$

The differential transforms of $i(t)$ and $v(t)$ are defined as follows:

$$\begin{cases} I(k) = \dfrac{1}{k!}\left[\dfrac{d^k i}{dt^k}\right]_{t=0} \\ V(k) = \dfrac{1}{k!}\left[\dfrac{d^k v}{dt^k}\right]_{t=0} \end{cases} \quad (5)$$

The inverse differential transforms are given as:

$$\begin{cases} i(t) = \sum_{k=0}^{\infty} t^k\, I(k) \\ v(t) = \sum_{k=0}^{\infty} t^k\, V(k) \end{cases} \quad (6)$$

The fundamental mathematical operations performed by differential transform method are tabulated in Table 1.

Table 1 One dimensional differential transformations.

Functional form	Transformed form
$i(t) = w(t) \pm z(t)$	$I(k) = W(k) \pm Z(k)$
$i(t) = \eta\, z(t)$	$I(k) = \eta\, Z(k)$, η is a constant
$i(t) = \frac{di(t)}{dt}$	$I(k) = (k+1)\, I(k+1)$
$i(t) = \frac{d^m i(t)}{dt^m}$	$I(k) = (k+1) \ldots\ldots (k+m) I(k+m)$
$i(t) = e^t$	$I(k) = \dfrac{1}{k!}$
$i(t) = i^m$	$I(k) = \delta(k\text{-}m) = \begin{cases} 1, & k = m \\ 0, & \text{otherwise} \end{cases}$
$i(t) = w(t) + v(t)$	$I(k) = \sum_{r=0}^{\infty} W(r)V(k-r)$

3.2. Exponentially fitted collocation approximation method (EFCAM)

The exponentially fitted collocation approximation method (EFCAM) was formulated by Falade [14] and it was applied to solve singular initial value problems and integro-differential equations. The whole idea of the method is to use power series as a basis function and its derivative substituted into a slightly perturbed equation in which perturbation term added to the right hand side of the equation. The addition of the perturbation term is to minimize the error of the problems in consideration.

3.2.1. Definition of Chebyshev polynomials

The Chebyshev polynomials of first kind can be defined by the recurrence relation given by

$$T_0(t) = 1, \;\; T_1(t) = 2t - 1$$

Thus, we have:

$$T_{N+1}(t) = 2(2t-1)T_N(t) - T_{N-1}(t) \qquad N \geq 1 \quad (7)$$

In order to employ exponentially collocation approximation technique for the numerical solution of Eq. (1), we consider power series of the form:

$$\begin{cases} i_N(t) = \sum_{k=0}^{N} p_k\, t^k \\ v_N(t) = \sum_{k=0}^{N} q_k\, t^k \end{cases} \tag{8}$$

and the exponentially fitted approximate solution of the form:

$$\begin{cases} i_N(t) \approx \sum_{k=0}^{N} p_k\, t^k + \tau_{2_{[I]}} e^t \\ v_N(t) \approx \sum_{k=0}^{N} q_k\, t^k + \tau_{2_{[V]}} e^t \end{cases} \tag{9}$$

where t represents the dependent variables in the problem, 2 represents highest derivative of the Eq. (1) and p_k , $i_N(t)$, q_k, $v_N(t)$ $(k \geq 0)$ are unknown constants to be determined. N is the length of computation and degree of Chebyshev polynomials in Table 2.

Table 2 The first fourteen Chebyshev polynomials.

$T_N(t)$	**Chebyshev polynomials**
$T_0(t)$	1
$T_1(t)$	$2t - 1$
$T_2(t)$	$8t^2 - 8t + 1$
$T_3(t)$	$32t^3 - 48t^2 + 18t - 1$
$T_4(t)$	$128t^4 - 258t^3 + 160t^2 - 32t + 1$
$T_5(t)$	$512t^5 - 1280t^4 + 1120t^3 - 400t^2 + 50t - 1$
$T_6(t)$	$2048t^6 - 6144t^5 + 6912t^4 - 3584t^3 + 640t^2 - 72t + 1$
$T_7(t)$	$8172t^7 - 28672t^6 + 39424t^5 - 26880t^4 + 9408t^3 - 1568t^2 + 98t - 1$
$T_8(t)$	$32768t^8 - 131072t^7 + 212992t^6 - 180224t^5 - 84480t^4 - 21504t^3 + 2688t^2$ $-128t + 1$
$T_9(t)$	$131072t^9 - 589824t^8 + 1105920t^7 - 1118208t^6 + 658944t^5 - 228096t^4 +$ $44352t^3 - 4320t^2 - 1$
$T_{10}(t)$	$52488t^{10} - 2621440t^9 + 5570560t^8 - 6553600t^7 + 4659200t^6 -$ $2050048t^5 + 549120t^4 - 84480t^3 + 6600t^2 - 200t + 1$
$T_{11}(t)$	$2097152t^{11} - 11534336t^{10} + 27394048t^9 - 36765696t^8 + 30638080t^7 -$ $16400384t^6 + 5637632t^5 - 1208064t^4 + 151008t^3 - 9680t^2 + 242t - 1$
$T_{12}(t)$	$8388608t^{12} - 50331648t^{11} + 13210576t^{10}$-$199229440t^9$+$190513152t^8 -$ $120324096t^7 + 50692096t^6 - 14057472t^5 + 2471040t^4 - 256256t^3 +$ $13728t^2 - 288t + 1$
$T_{13}(t)$	$33554432t_r{}^{13} - 218103808t^{12} + 5402263552t^{11} - 1049624576t^{10} +$ $1133117440t^9 - 825556992t^8 + 412778496t^7 - 1413213696t^6 +$ $2361471t^5 - 4759040t^4 + 416416t^3 - 18928t^2 + 338t - 1$
$T_{14}(t)$	$134217728t^{14} - 939524096t^{13} + 2936012800t^{12} - 5402263552t^{11} +$ $6499598336t^{10} - 5369233408t^9 + 3111714816t^8 - 1270087680t^7 +$ $+3611811184t^6 - 69701632t^5 \;\; 8712704t^4 - 652288t^3 + 25480t^2 - 372t + 1$

The first and second derivatives of Eq. (8) gives:

$$\begin{cases} i^{/}(t) = \sum_{k=0}^{N} kp_k\, t^{k-1} \\ v^{/}(t) = \sum_{k=0}^{N} kq_k\, t^{k-1} \end{cases} \tag{10}$$

$$\begin{cases} i^{//}(t) = \sum_{k=0}^{N} k(k-1)p_k\, t^{k-2} \\ v^{//}(t) = \sum_{k=0}^{N} k(k-1)q_k\, t^{k-2} \end{cases} \tag{11}$$

Substitution of Eqs. (8), (10) and (11) into Eq. (1) gives following equation.

$$\begin{cases} L\sum_{k=2}^{N} k(k-1)p_k t^{k-2} + R\sum_{k=1}^{N} kp_k t^{k-1} + \frac{1}{C}\sum_{k=0}^{N} p_k t^k = \frac{dV(t)}{dt} \\ C\sum_{k=2}^{N} k(k-1)q_k t^{k-2} + \frac{1}{R}\sum_{k=1}^{N} kq_k t^{k-1} + \frac{1}{L}\sum_{k=0}^{N} q_k\, t^k = \frac{dI(t)}{dt} \end{cases} \tag{12}$$

With the expansion of Eq. (12):

$$\begin{cases} L[2p_2 + 6tp_3 + 12t^2p_4 + \cdots + N(N-1)p_N t^{N-2}] + \\ R[p_1 + 2tp_2 + 3t^2p_3 + 4t^3p_4 + \cdots + Np_N t^{N-1}] + \\ \dfrac{1}{C}[p_0 + tp_1 + t^2p_2 + t^3p_3 + t^4p_4 \ldots + p_N t^N] = \dfrac{dV(t)}{dt} \end{cases}$$

$$\begin{cases} C[2q_2 + 6tq_3 + 12t^2q_4 + \cdots + N(N-1)q_N t^{N-2}] \\ \dfrac{1}{R}[q_1 + 2tq_2 + 3t^2q_3 + 4t^3q_4 + \cdots + Nq_N t^{N-1}] + \\ \dfrac{1}{L}[q_0 + tq_1 + t^2q_2 + t^3q_3 + t^4q_4 \ldots + q_N t^N] = \dfrac{dI(t)}{dt} \end{cases}$$

and collecting the terms, we obtain following expressions.

$$\begin{cases} \frac{1}{C}p_0 + [R+\frac{t}{C}]p_1 + \left[2L + 2tR + \frac{1}{C}t^2\right]p_2 + \left[6Lt + 3t^2R + \frac{1}{C}t^3\right]p_3 \\ \quad + \cdots + [L(N(N-1)t^{N-2} + R(N)t^{N-1} + \frac{1}{C}t^N]p_N = \frac{dV(t)}{dt} \end{cases} \tag{13}$$

$$\begin{cases} \frac{1}{L}q_0 + [\frac{1}{R}+\frac{t}{L}]q_1 + \left[2C + \frac{1}{R}2t + \frac{1}{L}t^2\right]q_2 + \left[6tC + \frac{1}{R}3t^2 + \frac{1}{L}t^3\right]q_3 \\ \quad + \cdots + [C(N(N-1)t^{N-2} + \frac{1}{R}(N)t^{N-1} + \frac{1}{L}t^N]q_N = \frac{dI(t)}{dt} \end{cases} \tag{14}$$

Slight perturbation and collocation of Eqs. (13) and (14) lead to:

$$\begin{cases} \frac{1}{C}p_0 + [R+\frac{t_r}{C}]p_1 + \left[2L + 2t_rR + \frac{1}{C}{t_r}^2\right]p_2 + \left[6Lt_r + 3{t_r}^2R + \frac{1}{C}{t_r}^3\right]p_3 \\ + \cdots + [L(N(N-1){t_r}^{N-2} + R(N){t_r}^{N-1} + \frac{1}{C}{t_r}^N]p_N - \tau_{1[I]}(t_r) - \tau_{2[I]}\mathrm{T}_{N-1}(t_r) = \frac{dV(t_r)}{dt} \end{cases} \tag{15}$$

$$\begin{cases} \frac{1}{L}q_0 + [\frac{1}{R}+\frac{t_r}{L}]q_1 + \left[2C + \frac{1}{R}2t_r + \frac{1}{L}{t_r}^2\right]q_2 + \left[6t_rC + \frac{1}{R}3{t_r}^2 + \frac{1}{L}{t_r}^3\right]q_3 \\ + \cdots + [C(N(N-1){t_r}^{N-2} + \frac{1}{R}(N){t_r}^{N-1} + \frac{1}{L}{t_r}^N]q_N - \tau_{1[V]}(t_r) - \tau_{2[V]}T_{N-1}(t_r) = \frac{dI(t_r)}{dt} \end{cases} \tag{16}$$

where $t_r = a + \frac{(b-a)r}{N+2}$; $r = 1,2,\ldots\ldots..N+1$. $\tau_{1[I]}, \tau_{1[V]}, \tau_{2[I]}$ and $\tau_{2[V]}$ are free tau parameters to be determined and $T_N(t), T_{N-1}(t)$ are the Chebyshev polynomials defined in Table 2. Eqs. (15) and (16) are called perturbed collocation current and perturbed collocation voltage equations respectively.

Applying initial conditions given in Eq.(2) on approximate solution given in Eq.(9) gives:

$$\begin{cases} i(t_0) = p_0 + \tau_{2[I]}\, e^{t_0} = \beta_1 \\ i'(t_0) = p_1 + \tau_{2[I]}\, e^{t_0} = \beta_2 \\ v(t_0) = q_0 + \tau_{2[V]}\, e^{t_0} = \delta_1 \\ v'(t_0) = q_1 + \tau_{2[V]}\, e^{t_0} = \delta_2 \end{cases} \tag{17}$$

Altogether, we obtain (N+3) linear algebraic equations in (N+3) unknown constants. MAPLE 18 software is used to obtain (N+3) unkown constants substituted into approximate solution (Eq.(9)).

3.3. Relative error

The relative error used in this study can be defined as:

$$E_t = \left|\frac{\text{Analytical solution} - \text{Numerical solution}}{\text{Analytical solution}}\right| \times 100 \tag{18}$$

4. Numerical applications

To illustrate the ability and efficiency of applied methods for the numerical solution of Eq. (1), three test problems are considered in which comparison was made between the analytical solutions and approximate solutions. The results revealed that the methods are effective and simple.

Problem 1: If Eq. (1) is considered with the following constant coefficients: Inductor (L) =2.0, Resistor (R) =4.0, Capacitor (C) =0.05 , $\frac{dV(t)}{dt} = \frac{dI(t)}{dt} = 10e^{-t}$ and N=14, Eq.(19) is obtained and initial conditions are taken as in Eq.(20).

$$\begin{cases} 2\dfrac{d^2 i}{dt^2} + 4\dfrac{di}{dt} + \dfrac{1}{0.05} i = 10e^{-t} \\ 0.05\dfrac{d^2 v}{dt^2} + \dfrac{1}{4}\dfrac{dv}{dt} + \dfrac{1}{2} v = 10e^{-t} \end{cases} \qquad t\epsilon[0,1] \tag{19}$$

$$\begin{cases} i(0) = -1 \\ i^{/}(0) = 0 \\ v(0) = -1 \\ v^{/}(0) = 0 \end{cases} \tag{20}$$

4.1. DTM technique

Taking the differential transform of Eq.(19) using Table1 yields following expressions:

$$2[(k+1)(k+2)I(k+2)] + 4[(k+1)I(k+1)] + \frac{1}{0.05}[I(k)] = -\frac{10}{k!}$$

$$0.05[(k+1)(k+2)V(k+2)] + \frac{1}{4}[(k+1)V(k+1)] + \frac{1}{2}[V(k)] = -\frac{10}{k!}$$

Above equations are arranged by taking $I(k+2)$ and $V(k+2)$ as subject of relations.

$$2I(k+2) = \frac{-4[(k+1)I(k+1)] - \frac{1}{0.05}[I(k)] - \frac{10}{k!}}{(k+1)(k+2)} \tag{21}$$

$$0.05V(k+2) = \frac{-\frac{1}{4}[(k+1)V(k+1)] - \frac{1}{2}[V(k)] - \frac{10}{k!}}{(k+1)(k+2)} \tag{22}$$

Transformed forms of the initial conditions given by Eq. (20) are obtained as follows:

$$\begin{cases} I(0) = -1 \\ I(1) = 0 \\ V(0) = -1 \\ V(1) = 0 \end{cases} \tag{23}$$

Substituting Eq. (23) into Eqs. (21) and (22) respectively by recursive approach for $k = 0,1,2,3,\dots,14$, gives the results that are listed in tabular form in Table 3.

Table 3 Transformed forms of current and voltage variables.

$\boldsymbol{I}(\mathbf{0})$	-1.0000000000	$\boldsymbol{I}(\mathbf{8})$	0.2808779762000
$\boldsymbol{V}(\mathbf{0})$	-1.0000000000	$\boldsymbol{V}(\mathbf{8})$	-10.21949405000
$\boldsymbol{I}(\mathbf{1})$	0.000000000000	$\boldsymbol{I}(\mathbf{9})$	0.0611634700200
$\boldsymbol{V}(\mathbf{1})$	0.000000000000	$\boldsymbol{V}(\mathbf{9})$	3.6225473990000
$\boldsymbol{I}(\mathbf{2})$	7.500000000000	$\boldsymbol{I}(\mathbf{10})$	-0.04343998016
$\boldsymbol{V}(\mathbf{2})$	105.0000000000	$\boldsymbol{V}(\mathbf{10})$	-0.675719246000
$\boldsymbol{I}(\mathbf{3})$	-6.00000000000	$\boldsymbol{I}(\mathbf{11})$	0.002337737494
$\boldsymbol{V}(\mathbf{3})$	-208.33333330	$\boldsymbol{V}(\mathbf{11})$	-0.02218238937
$\boldsymbol{I}(\mathbf{4})$	-3.12500000000	$\boldsymbol{I}(\mathbf{12})$	0.002901295110
$\boldsymbol{V}(\mathbf{4})$	181.250000000	$\boldsymbol{V}(\mathbf{12})$	0.060433931740
$\boldsymbol{I}(\mathbf{5})$	4.1250000000000	$\boldsymbol{I}(\mathbf{13})$	-0.000596208864
$\boldsymbol{V}(\mathbf{5})$	-78.7500000000	$\boldsymbol{V}(\mathbf{13})$	-0.02182190398
$\boldsymbol{I}(\mathbf{6})$	-0.32638888890	$\boldsymbol{I}(\mathbf{14})$	-0.0007424066730
$\boldsymbol{V}(\mathbf{6})$	16.45833333000	$\boldsymbol{V}(\mathbf{14})$	0.0044729937310
$\boldsymbol{I}(\mathbf{7})$	-0.88988095240		
$\boldsymbol{V}(\mathbf{7})$	14.79166667000		

Therefore, the closed form solution of current (I) and voltage (V) in the electrical circuits expressed by Eq. (19) can be written as:

$$I(t) \approx \begin{cases} -1.0000 + 7.50000t^2 - 6.0000t^3 - 3.125000t^4 + 4.125000\, t^5 - \\ 0.3263888889t^6 - 0.8898809524t^7 + 0.2808779762\, t^8 + \\ 0.06116347002t^9 - 0.04343998016t^{10} + 0.002337737494t^{11} + \\ 0.002901295110\, t^{12} - 0.0005962088644t^{13} - 0.0007424066730t^{14} \end{cases} \tag{24}$$

$$V(t) \approx \begin{cases} -1 + 105t^2 - 208.33333330\, t^3 + 181.250000000t^4 - \\ 78.7500000000t^5 + 16.45833333000t^6 + 14.79166667000t^7 - \\ 10.219494050t^8 + 3.62254739900t^9 - \ 0.6757192460t^{10} - \\ .02218238937t^{11} + \ 0.060433931740t^{12} - 0.02182190398t^{13} + \\ 0.004472993731t^{14} \end{cases} \tag{25}$$

4.2. EFCAM technique

Comparing Eq. (19) with Eqs. (15) and (16) respectively and taking computational length (Chebyshev polynomial) $N = 14$, yield followings.

$$\begin{cases} 20I(0)+(4+20t_r)I(1)+(4+8t_r+20t_r^2)I(2)+(12t_r+12t_r^2+20t_r^3)I(3)+ \\ (24t_r^2+16t_r^3+20t_r^4)I(4)(40t_r^3+20t_r^4+20t_r^5)I(5)+(60t_r^4+24t_r^5+20t_r^6)I(6)+ \\ (84t_r^5+28t_r^6+20t_r^7)I(7)+(112t_r^6+32t_r^7+20t_r^8)I(8)+(114t_r^7+36t_r^8+20t_r^9)I(9)+ \\ +(180t_r^8+40t_r^9+20t_r^{10})I(10)+(220t_r^9+44t_r^{10}+20t_r^{11})I(11)+ \\ (264t_r^{10}+48t_r^{11}+20t_r^{12})I(12)+(312t_r^{11}+52t_r^{12}+20t_r^{13})I(13)+ \\ (364t_r^{12}+56t_r^{13}+20t_r^{14})I(14)- \\ \begin{Bmatrix} 134217728t_r^{14}-939524096t_r^{13}+2936012800t_r^{12}-5402263552t_r^{11}+ \\ 6499598336t_r^{10}-5369233408t_r^9+3111714816t_r^8-1270087680t_r^7+ \\ 3611811184t_r^6-69701632t_r^5+8712704t_r^4-652288t_r^3+25480t_r^2- \\ 372t_r+1 \end{Bmatrix}\tau_{1[I]} \\ -\begin{Bmatrix} 33554432t_r^{13}-218103808t_r^{12}+5402263552t_r^{11}-1049624576t_r^{10}+ \\ 1133117440t_r^9-825556992t_r^8+412778496t_r^7-1413213696t_r^6+ \\ 32361471t_r^5-4759040t_r^4+416416t_r^3- \\ -18928t_r^2+338t-1 \end{Bmatrix}\tau_{2[I]}=10e^{-t_r} \end{cases} \quad (26)$$

$$\begin{cases} \frac{1}{2}V(0)+\left(\frac{1}{4}+\frac{1}{2}t_r\right)V(1)+\left(0.10+\frac{1}{2}t_r+\frac{1}{2}t_r^2\right)V(2)+\left(0.30t_r+\frac{3}{4}t_r^2+\frac{1}{2}t_r^3\right)V(3)+ \\ \left(0.60t_r^2+t_r^3+\frac{1}{2}t_r^4\right)V(4)+\left(1.0t_r^3+\frac{5}{4}t_r^4+\frac{1}{2}t_r^5\right)V(5)+\left(1.50t_r^4+\frac{3}{2}t_r^5+\frac{1}{2}t_r^6\right)V(6)+ \\ +\left(2.10t_r^5+\frac{7}{4}t_r^6+\frac{1}{2}t_r^7\right)V(7)+\left(2.80t_r^6+2t_r^7+\frac{1}{2}t_r^8\right)V(8) \\ +\left(3.60t_r^7+\frac{9}{4}t_r^8+\frac{1}{2}t_r^9\right)V(9)+\left(4.50t_r^8+\frac{5}{2}t_r^9+\frac{1}{2}t_r^{10}\right)V(10) \\ +\left(5.50t_r^9+\frac{11}{4}t_r^{10}+\frac{1}{2}t_r^{11}\right)V(11)+(6.60t_r^{10}+3t_r^{11}+20t_r^{12})V(12) \\ +\left(7.80t_r^{11}+\frac{13}{4}t_r^{12}+\frac{1}{2}t_r^{13}\right)V(13)+\left(9.10t_r^{12}+\frac{7}{2}t_r^{13}+\frac{1}{2}t_r^{14}\right)V(14) \\ -\begin{Bmatrix} 134217728t_r^{14}-939524096t_r^{13}+2936012800t_r^{12}-5402263552t_r^{11}+ \\ 6499598336t_r^{10}-5369233408t_r^9+3111714816t_r^8-1270087680t_r^7+ \\ 3611811184t_r^6-69701632t_r^5+8712704t_r^4-652288t_r^3+25480t_r^2- \\ 372t_r+1 \end{Bmatrix}\tau_{1[V]} \\ -\begin{Bmatrix} 33554432t_r^{13}-218103808t_r^{12}+5402263552t_r^{11}-1049624576t_r^{10}+ \\ 1133117440t_r^9-825556992t_r^8+412778496t_r^7-1413213696t_r^6+ \\ 32361471t_r^5-4759040t_r^4+416416t_r^3- \\ 18928t_r^2+338t-1 \end{Bmatrix}\tau_{2[V]}=10e^{-t_r} \end{cases} \quad (27)$$

Eqs. (26) and (27) are collocated as follows:

$$t_r = a + \frac{(b-a)r}{N+2};\ \ r = 1,2,3\ldots\ldots.N+1 \ \text{ where } \ a=0\ \ ,\ b=1\ ,\ N=14$$
$$t_1=\frac{1}{16}, t_2=\frac{2}{16}, t_3=\frac{3}{16}, t_4=\frac{4}{16}, t_5=\frac{5}{16}, t_6=\frac{6}{16},\ t_7=\frac{7}{16}, t_8=\frac{8}{16}, t_9=\frac{9}{16}$$
$$t_{10}=\frac{10}{16}, t_{11}=\frac{11}{16},\ t_{12}=\frac{12}{16}, t_{13}=\frac{13}{16},\ t_{14}=\frac{14}{16}, t_{15}=\frac{15}{16}$$

As a result of considering initial conditions (20) and using MAPLE 18 software to obtain seventeen unkown constants of Eqs. (26) and (27), eventually, we obtain the constants given in Table 4.

Substitution of the values in Table 4 into Eq.(9), the approximate solution of current $I(t)$ and voltage $V(t)$ in the electrical circuits given by Eq. (19) can be written as follow:

$$I(t) \approx \begin{cases} I(0)+I(1)t+I(2)t^2+I(3)t^3+I(4)t^4+I(5)t^5+I(6)t^6 \\ I(7)t^7+I(8)t^8+I(9)t^9+I(10)t^{10}+I(11)t^{11}+ \\ +I(12)t^{12}+I(13)t^{13}+I(14)t^{14}+\tau_{2[I]}e^t \end{cases}$$

$$V(t) \approx \begin{cases} V(0)+V(1)t+V(2)t^2+V(3)t^3+V(4)t^4+V(5)t^5+V(6)t^6+ \\ V(7)t^7+V(8)t^8+V(9)t^9+V(10)t^{10}+V(11)t^{11}+ \\ V(12)t^{12}+V(13)t^{13}+V(14)t^{14}+\tau_{2[V]}e^t \end{cases}$$

$$I(t) \approx \begin{cases} -1.0000001160000000 - 0.0000001158246719t + \\ 7.49996484900000000t^2 - 5.8327085980000000t^3 - \\ 3.1314962740000000t^4 + 4.16957039500000000t^5 - \\ 0.5362652712000000t^6 - 0.1946550513000000t^7 - \\ 1.36134887000000000\ t^8 + 2.83591997300000000t^9 - \\ 3.367929853000000t^{10} + 2.7580963510000000t^{11} - \\ 1.499035609000000t^{12} + 0.48332055340000003t^{13} \\ 0.069829910730000t^{14} + 0.000000115824671900e^t \end{cases} \quad (28)$$

$$V(t) \approx \begin{cases} -0.99999989740000 + 0.0000001026145435t \\ 104.99938400000000t^2 - 208.3221512000000t^3 + \\ 181.13421620000000t^4 - 77.96782916000000t^5 + \\ 37.2370587800000\ t^8 + 48.248794920000000t^9 - \\ 52.9747646100000t^{10} + 42.42102700000000t^{11} - \\ 22.6134190000000t^{12} + 7.150125049000000t^{13} - \\ 1.0139342670000t^{14} - 0.00000001026145435e^t \end{cases} \quad (29)$$

Table 4 Constants of Eqs. (26) and (27) for current and voltage variables.

$\boldsymbol{I(0)}$	−1.0000001160000000	$\boldsymbol{I(9)}$	2.8359199730000000
$\boldsymbol{V(0)}$	−0.999999897400000	$\boldsymbol{V(9)}$	48.248794920000000
$\boldsymbol{I(1)}$	−0.0000001158246719	$\boldsymbol{I(10)}$	−3.367929853000000
$\boldsymbol{V(1)}$	0.0000001026145430	$\boldsymbol{V(10)}$	−52.97476461000000
$\boldsymbol{I(2)}$	7.49996484900000000	$\boldsymbol{I(11)}$	2.7580963510000000
$\boldsymbol{V(2)}$	104.999384000000000	$\boldsymbol{V(11)}$	42.421027000000000
$\boldsymbol{I(3)}$	−5.8327085980000000	$\boldsymbol{I(12)}$	−1.499035609000000
$\boldsymbol{V(3)}$	−208.32215120000000	$\boldsymbol{V(12)}$	−22.61341900000000
$\boldsymbol{I(4)}$	−3.1314962740000000	$\boldsymbol{I(13)}$	0.4833205534000000
$\boldsymbol{V(4)}$	181.134216200000000	$\boldsymbol{V(13)}$	7.1501250490000000
$\boldsymbol{I(5)}$	4.16957039500000000	$\boldsymbol{I(14)}$	−0.069829910730000
$\boldsymbol{V(5)}$	−77.96782916000000	$\boldsymbol{V(14)}$	−1.01393426700000
$\boldsymbol{I(6)}$	−0.5362652712000000	$\boldsymbol{\tau_{1[I]}}$	0.0000005549290900
$\boldsymbol{V(6)}$	1.8754628620000000	$\boldsymbol{\tau_{1[V]}}$	0.0000002008025430
$\boldsymbol{I(7)}$	−0.1946550513000000	$\boldsymbol{\tau_{2[I]}}$	0.000000115824671900
$\boldsymbol{V(7)}$	26.492353170000000	$\boldsymbol{\tau_{2[V]}}$	−0.0000000102614544
$\boldsymbol{I(8)}$	−1.3613488700000000		
$\boldsymbol{V(8)}$	−37.2370587800000		

The above process was also repeated for other two cases stated below.

Problem 2 ($L = 4.0$, $R = 8.0$ and $C = 0.1$, $\frac{dV(t)}{dt} = 10e^{-t}$, $\frac{dI(t)}{dt} = 10e^{-t}$ and N=14)
Problem 3 ($L = 8.0$, $R = 16.0$ and $C = 0.2$, $\frac{dV(t)}{dt} = 10e^{-t}$, $\frac{dI(t)}{dt} = 10e^{-t}$ and N=14)

Moreover, we consider initial conditions given in Eq.(20). Eventually, for all the problems considered, we obtain numerical solutions as tabulated in Tables 5,6, and 7; and also as illustrated in Figs. 2 and 3.

Table 5 Numerical results of the Problem 1: (L = 2.0, R = 4.0, C = 0.05 , $\frac{dV(t)}{dt} = 10e^{-t}$).

t	Current I(t) and Voltage V(t)	Method			E_t (%)
		Analytical	DTM	EFCAM	
0	I(t)	-1.000000000	-1.000000000	-1.000000000	0.00000000
	V(t)	-1.000000000	-1.000000000	-1.000000000	0.000000000
0.1	I(t)	-0.931104996	-0.931104996	-0.931105081	0.000000003
	V(t)	-0.140988970	-0.140988967	-0.140990368	0.000002340
0.2	I(t)	-0.750378201	-0.750378200	-0.750378344	0.000000005
	V(t)	1.7986494000	1.7986494010	1.7986470740	0.000000055
0.3	I(t)	-0.498201923	-0.498201923	-0.4982020860	0.000000022
	V(t)	4.1083936200	4.1083936140	4.1083909020	0.000000150
0.4	I(t)	-0.213692506	-0.213692505	-0.213692651	0.000000005
	V(t)	6.3411537200	6.3411537250	6.3411509430	0.000000079
0.5	I(t)	0.068551053	0.068551052	0.068550956	0.000000018
	V(t)	8.243298230	8.243298251	8.243295634	0.000000255
0.6	I(t)	0.320706399	0.320706382	0.320706373	0.000000054
	V(t)	9.697230716	9.697230937	9.697228385	0.000002279
0.7	I(t)	0.522971495	0.522971321	0.522971552	0.000003315
	V(t)	10.67604850	10.67605073	10.67604647	0.000020888
0.8	I(t)	0.663864697	0.663863420	0.663864844	0.000001922
	V(t)	11.20905845	11.20907502	11.20905676	0.000147827
0.9	I(t)	0.739725052	0.739717653	0.739725293	0.000010001
	V(t)	11.35655982	11.35665715	11.35655839	0.000857038
1.0	I(t)	0.753602463	0.753566876	0.753602768	0.000047220
	V(t)	11.19220809	11.19268121	11.19220665	0.004227227

Table 6 Numerical results of the Problem 2: (L = 4.0, R = 8.0, C = 0.1 , $\frac{dV(t)}{dt} = 10e^{-t}$).

t	Current I(t) and Voltage V(t)	Method			E_t (%)
		Analytical	DTM	EFCAM	
0	I(t)	-1.00000000	-1.000000000	-1.000000000	0.0000000000
	V(t)	-1.00000000	-1.000000000	-1.000000000	0.0000000000
0.1	I(t)	-0.977020981	-0.977020981	-0.977020996	0.0000000030
	V(t)	-0.524958810	-0.524958806	-0.524958556	0.0000007239
0.2	I(t)	-0.915672725	-0.915672725	-0.915672756	0.0000000032
	V(t)	-0.755197280	-0.755197277	-0.755197778	0.0000004369
0.3	I(t)	-0.826242952	-0.826242953	-0.826243009	0.0000000012
	V(t)	2.635085800	2.635085792	2.6350865020	0.0000003036
0.4	I(t)	-0.717604372	-0.717604372	-0.717604456	0.0000000027
	V(t)	4.927077660	4.927077661	4.9270785360	0.0000000203
0.5	I(t)	-0.597288177	-0.597288176	-0.597288289	0.0000000010
	V(t)	7.462880000	7.462879995	7.4628809910	0.0000000669
0.6	I(t)	-0.471572866	-0.471572866	-0.471573009	0.0000000021
	V(t)	10.09454907	10.09454907	10.094550150	0.0000000000
0.7	I(t)	-0.345583772	-0.345583772	-0.345583949	0.0000000000
	V(t)	12.69496996	12.69496997	12.694971080	0.0000000079
0.8	I(t)	-0.223399261	-0.223399262	-0.223399476	0.0000000038
	V(t)	15.15784848	15.15784846	15.157849630	0.0000001319
0.9	I(t)	-0.108160106	-0.108160107	-0.108160356	0.0000000074
	V(t)	17.39726710	17.39726709	17.397268250	0.0000000574
1.0	I(t)	-0.002179105	-0.002179107	-0.002179398	0.0000000575
	V(t)	19.34686204	19.34686203	19.346863180	0.00000005160

Table 7 Numerical results of the Problem 3: (L = 8.0, R = 16.0, C = 0.2 , $\frac{dV(t)}{dt} = 10e^{-t}$).

t	Current I(t) and Voltage V(t)	Method			E_t (%)
		Analytical	DTM	EFCAM	
0	I(t)	-1.000000000	-1.000000000	-1.00000000	0.00000000
	V(t)	-1.000000000	-1.000000000	-1.00000000	0.00000000
0.1	I(t)	-0.991417820	-0.991417820	-0.99141782	0.00000000
	V(t)	-0.757684560	-0.757684559	-0.757684394	0.00000005
0.2	I(t)	-0.968540869	-0.968540869	-0.96854089	0.00000000
	V(t)	-0.072718520	-0.072718525	-0.072718176	0.000007398
0.3	I(t)	-0.935064079	-0.935064079	-0.935064110	0.000000000
	V(t)	0.994676750	0.994676743	0.9946772660	0.000000744
0.4	I(t)	-0.893979914	-0.893979915	-0.893979960	0.000000001
	V(t)	2.387814630	2.387814626	2.3878153190	0.000000168
0.5	I(t)	-0.847697582	-0.847697583	-0.847697650	0.000000001
	V(t)	4.053399240	4.053399249	4.0534000960	0.000000222
0.6	I(t)	-0.798143101	-0.798143101	-0.798143180	0.000000007
	V(t)	5.941401590	5.941401579	5.9414025780	0.000000185
0.7	I(t)	-0.746843207	-0.746843207	-0.746843320	0.000000000
	V(t)	8.004952340	8.004952330	8.0049534660	0.000000125
0.8	I(t)	-0.694995625	-0.694995625	-0.694995750	0.000000000
	V(t)	10.20024859	10.20024859	10.200249840	0.000000000
0.9	I(t)	-0.643527837	-0.643527837	-0.64352799	0.000000000
	V(t)	12.486471570	12.48647156	12.48647294	0.000000080
1.0	I(t)	-0.593146175	-0.593146174	-0.59314636	0.000000001
	V(t)	14.825712970	14.82571297	14.82571447	0.000000000

Numerical solutions enable researchers to obtain the effect of different variables or parameters on the function under study. Accordingly, this study shows that Differential Transformation Method (DTM) and Exponentially Fitted Collocation Approximate Method (EFCAM) are very efficient in solving numerical solutions of second-order differential equation of voltage and current variable in electrical RLC circuits. Figs. 2 and 3 show the plots of the current and voltage profiles which tend to zero as given time goes to ten seconds (10 sec.) except for voltage v(t) obtained in problem 3. All the solutions obtained by analytical method, differential transformation method and exponentially fitted collocation approximate method are in good agreement with relatively minor errors (Table 5, Table 6, Table 7).

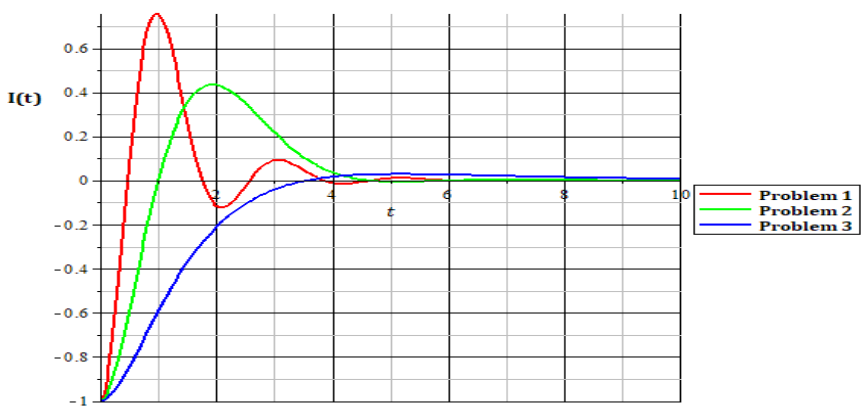

Fig. 2 Current, I(t), flow profile (10 sec).

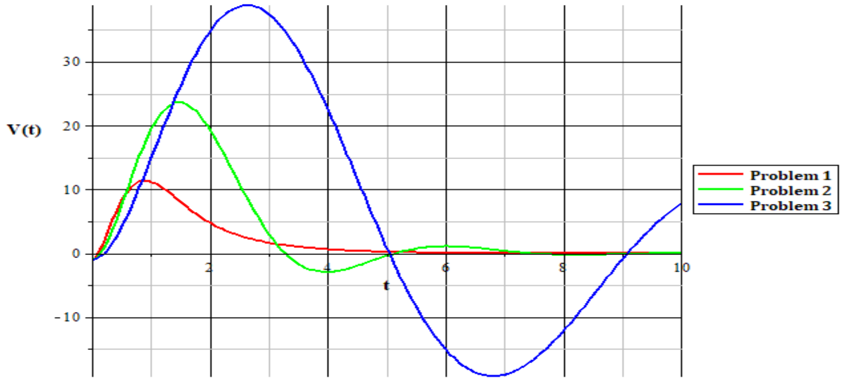

Fig. 3 Voltage, V(t), flow profile (10 sec).

5. Conclusion

The general conclusion is that this study will serve as a good alternative to experimental laboratory measurements and make an improvement in computational engineering and technology. Thus, we hereby suggest for further study when the functions $\frac{dI(t)}{dt}$ and $\frac{\mathrm{dV(t)}}{\mathrm{dt}}$ are trigonometric and logarithmic functions. Moreover, the proposed method (EFCAM) is equally recommended in finding numerical solution of several differential equations in other engineering and applied sciences.

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