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ANNIHILATORS OF TOP LOCAL COHOMOLOGY MODULES DEFINED BY A PAIR OF IDEALS

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ABSTRACT. Let R be a commutative Noetherian ring, I, J two proper ideals of R and let M be a non-zero finitely generated R-module with $c = \mathrm{cd}(I, J, M)$. In this paper, we first introduce $T_R(I, J, M)$ as the largest submodule of M with the property that $\mathrm{cd}(I, J, T_R(I, J, M)) < c$ and we describe it in terms of the reduced primary decomposition of zero submodule of M. It is shown that $\mathrm{Ann}_R(H^d_{I,J}(M)) = \mathrm{Ann}_R(M/T_R(I, J, M))$ and $\mathrm{Ann}_R(H^d_I(M)) = \mathrm{Ann}_R(H^d_{I,J}(M))$, whenever R is a local ring, M has dimension d with $H^d_{I,J}(M) \neq 0$ and $J^tM \subseteq T_R(I, M)$ for some positive integer t.

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1. Introduction

Local cohomology theory has been a significant tool in commutative algebra and algebraic geometry. As a generalization of the usual local cohomology modules, Takahashi, Yoshino and Yoshizawa [12] introduced the local cohomology modules with respect to a pair of ideals. To be more precise, suppose that R is a commutative Noetherian ring and I, J are two ideals of R. Let $W(I, J) = \{\mathfrak{p} \in \mathrm{Spec}(R) : I^n \subseteq \mathfrak{p} + J$ for some positive integer $n\}$. For an R-module M, the (I, J)-torsion submodule $\Gamma_{I,J}(M)$ of M, which consists of all elements x of M with $\mathrm{Supp}(Rx) \subseteq W(I, J)$, is considered. Let i be an integer, the local cohomology functor $H^i_{I,J}$ with respect to (I, J) is defined to be the i-th right derived functor of $\Gamma_{I,J}$. The i-th local cohomology module of M with respect to (I, J) is denoted by $H^i_{I,J}(M)$. When $J = 0$, then $H^i_{I,J}$ coincides with the usual local cohomology functor H^i_I with the support in the closed subset $V(I)$.

One of the basic problems concerning local cohomology modules is to determine the annihilators of them. This problem for ordinary local cohomology modules has been studied by several authors, see [6,7,8,9,11], and has led to some interesting results. In particular, Bahmanpour et al. in [2] proved an interesting result about

the annihilator of $H^d_{\mathfrak{m}}(M)$ the d-th local cohomology module of M, when $(R, \mathfrak{m})$ is a complete local ring and M is a non-zero finitely generated R-module with $d = \dim M$. Then Atazadeh et. al. [1] generalized this fact to the local cohomology modules with respect to an arbitrary ideal I.
They first defined $T_R(I, M)$ as the largest submodule of M such that $\mathrm{cd}(I, T_R(I, M)) < c$, in which $c = \mathrm{cd}(I, M)$, see [1, Definition 2.2], and then they proved the following fact.

Theorem 1.1. [1, Theorem 2.3] *Let R be a Noetherian ring and I be an ideal of R. Let M be a non-zero finitely generated R-module of dimension d such that $H^d_I(M) \neq 0$. Then* $\mathrm{Ann}_R(H^d_I(M)) = \mathrm{Ann}_R(M/T_R(I, M))$.

The purpose of the present paper is to introduce $T_R(I, J, M)$ as the largest submodule of M with the property that $\mathrm{cd}(I, J, T_R(I, J, M)) < c$, in which $c = \mathrm{cd}(I, J, M)$. Next in Corollary 2.3 we relate this submodule of M with another special submodules $T_R(I, M)$ and $T_R(\mathfrak{m}, M)$ of M. Then we describe in more detail the structure of $T_R(I, J, M)$ in terms of the reduced primary decomposition of the zero submodule of M in Theorem 2.4. Namely, if $0 = \cap_{i=1}^n N_i$ denotes a reduced primary decomposition of the zero submodule in M such that N_i is a $\mathfrak{p}_i$-primary submodule of M, for all $i = 1, \cdots, n$, then

$$T_R(I, J, M) = \cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)=c} N_i.$$

Pursuing this point of view further we establish some results about the annihilator of top local cohomology modules with respect to a pair of ideals I, J. More precisely, as a main result of this paper, we derive the following consequence.

Theorem 1.2. *Let R be a Noetherian local ring and I, J be two ideals of R. Let M be a non-zero finitely generated R-module with dimension d such that $H^d_{I,J}(M) \neq 0$. Then*

(i) $\mathrm{Ann}_R(H^d_{I,J}(M)) = \mathrm{Ann}_R(M/T_R(I, J, M))$.
(ii) *If $J^t M \subseteq T_R(I, M)$ for some positive integer t, then* $\mathrm{Ann}_R(H^d_I(M)) = \mathrm{Ann}_R(H^d_{I,J}(M))$.

For notations and terminologies not given in this paper, the reader is referred to [3] if necessary.

2. Annihilators of local cohomology modules

Throughout this section R is a commutative Noetherian ring, I, J are two proper ideals of R and M is a non-zero finitely generated R-module. Let $\mathrm{cd}(I, J, M)$ be the supremum of all integers r for which $H^r_{I,J}(M) \neq 0$. We call this integer the cohomological dimension of R-module M with respect to I, J. When $J = 0$, we have that $\mathrm{cd}(I, 0, M) = \mathrm{cd}(I, M)$, which is just the supremum of all integers r for which $H^r_I(M) \neq 0$. In [5, Corollary 3.3] a characterization for $\mathrm{cd}(I, J, M)$ is provided

$$\mathrm{cd}(I, J, M) = \inf\{\ i |\ H^i_{I,J}(R/\mathfrak{p}) = 0 \text{ for all } \mathfrak{p} \in \mathrm{Supp}_R(M)\} - 1.$$

Lemma 2.1. [5, Proposition 3.2] *Let M and N be two finitely generated R-modules such that* $\mathrm{Supp}_R N \subseteq \mathrm{Supp}_R M$. *Then* $\mathrm{cd}(I, J, N) \leq \mathrm{cd}(I, J, M)$.

Definition 2.2. Let R be a Noetherian ring and I, J be two ideals of R. Let M be a non-zero finitely generated R-module. We denote by $T_R(I, J, M)$ the largest submodule of M such that $\mathrm{cd}(I, J, T_R(I, J, M)) < \mathrm{cd}(I, J, M)$. It is easy to see that $T_R(I, J, M) = \cup\{N : N \leq M \text{ and } \mathrm{cd}(I, J, N) < \mathrm{cd}(I, J, M)\}$. When $J = 0$ this definition coincides with that of [1, Definition 2.2].

Corollary 2.3. *Let $(R, \mathfrak{m})$ be a Noetherian local ring and M be a non-zero finitely generated R-module of dimension d such that $H^d_{I,J}(M) \neq 0$. Then $T_R(\mathfrak{m}, M) \subseteq T_R(I, M) \subseteq T_R(I, J, M)$.*

Proof. For the first inclusion let $x \notin T_R(I, M)$. Then $\mathrm{cd}(I, Rx) = d$ and so $H^d_I(Rx) \neq 0$. Thus $\dim Rx = d$ and by [3, Theorem 6.1.4], $H^d_{\mathfrak{m}}(Rx) \neq 0$. Hence, $x \notin T_R(\mathfrak{m}, M)$ and therefore $T_R(\mathfrak{m}, M) \subseteq T_R(I, M)$. Now, let $x \notin T_R(I, J, M)$. Then $\mathrm{cd}(I, J, Rx) = d$ so that $H^d_{I,J}(Rx) \neq 0$. Now, by [4, Theorem 2.1] we have $\emptyset \neq \mathrm{Att}_R(H^d_{I,J}(Rx)) \subseteq \mathrm{Att}_R(H^d_I(Rx))$. Hence, $H^d_I(Rx) \neq 0$ and $\mathrm{cd}(I, Rx) = d$. Therefore, $x \notin T_R(I, M)$ we have the desired result. □

Theorem 2.4. *Let M be a non-zero finitely generated R-module with cohomological dimension $c = \mathrm{cd}(I, J, M)$. Then*

$$T_R(I, J, M) = \cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)=c} N_i.$$

Here, $0 = \cap_{i=1}^n N_i$ denotes a reduced primary decomposition of the zero submodule of M and N_i is a $\mathfrak{p}_i$-primary submodule of M.

Proof. We first show that $T_R(I, J, M) \subseteq \cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)=c} N_i$ and then we have the desired result whenever $\mathrm{cd}(I, J, \cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)=c} N_i) < c$. Let $x \in T_R(I, J, M)$. Then $\mathrm{cd}(I, J, Rx) < c$ and so $H^c_{I,J}(Rx) = 0$. Thus for each $\mathfrak{p} \in \mathrm{Ass}_R(Rx)$ we have $H^c_{I,J}(R/\mathfrak{p}) = 0$. Hence, $\mathrm{Ass}_R(Rx) \subseteq \{\mathfrak{p}_i : \mathfrak{p}_i \in \mathrm{Ass}_R M, \mathrm{cd}(I, J, R/\mathfrak{p}_i) < c\}$ and therefore,

$$\bigcap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)<c} \mathfrak{p}_i \subseteq \bigcap_{\mathfrak{p}\in \mathrm{Ass}_R(Rx)} \mathfrak{p} = \sqrt{\mathrm{Ann}_R(Rx)}.$$

So that there exists a positive integer m such that $(\cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)<c}\mathfrak{p}_i)^m x = 0$. We claim that $x \in \cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)=c} N_i$. Assume contrary that there is an integer t such that $\mathrm{cd}(I,J,R/\mathfrak{p}_t) = c$ and $x \notin N_t$. Then by $(\cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)<c}\mathfrak{p}_i)^m x = 0 \in N_t$ and $x \notin N_t$ it follows that $(\cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)<c}\mathfrak{p}_i)^m \subseteq \mathfrak{p}_t$ since N_t is $\mathfrak{p}_t$-primary. Thus there exists some $\mathfrak{p}_j$ such that $\mathrm{cd}(I,J,R/\mathfrak{p}_j) < c$ and $\mathfrak{p}_j \subseteq \mathfrak{p}_t$. Hence, in view of Lemma 2.1, $\mathrm{cd}(I,J,R/\mathfrak{p}_t) \leq \mathrm{cd}(I,J,R/\mathfrak{p}_j) < c$, which is a contradiction. So we have the desired result.

For the second part assume contrarily $H^c_{I,J}(N) \neq 0$, where $N = \cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)=c} N_i$. Then $0 :_R N = \cap_{i=1}^n (N_i :_R N)$ and so $\sqrt{0 :_R N} = \cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)<c}\sqrt{N_i :_R M} = \cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)<c}\mathfrak{p}_i$. Assume that $\mathfrak{p} \in \mathrm{Supp}(N)$, thus there is some $\mathfrak{p}_j$ with $\mathrm{cd}(I,J,R/\mathfrak{p}_j) < c$ such that $\mathfrak{p}_j \subseteq \mathfrak{p}$. Hence, $\mathrm{cd}(I,J,R/\mathfrak{p}) \leq \mathrm{cd}(I,J,R/\mathfrak{p}_j) < c$. Then by the first paragraph of this section $\mathrm{cd}(I,J,N) < c$, which is a contradiction. Therefore, $H^c_{I,J}(N = \cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)=c} N_i) = 0$. □

Lemma 2.5. *Let R be a Noetherian local ring and M be a non-zero finitely generated R-module of dimension d such that $H^d_{I,J}(M) \neq 0$. Then there exists a positive integer t such that $J^t M \subseteq T_R(I,J,M)$.*

Proof. Let $0 = \cap_{i=1}^n N_i$ denote a reduced primary decomposition of the zero submodule of M where N_i's are $\mathfrak{p}_i$-primary submodules of M. By Theorem 2.4 we know that

$$T_R(I,J,M) = \bigcap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)=d} N_i.$$

If $\mathrm{cd}(I,J,R/\mathfrak{p}_i) = d$, then $H^d_{I,J}(R/\mathfrak{p}_i) \neq 0$ and so by [4, Theorem 2.1], it follows that $J \subseteq \mathfrak{p}_i = \sqrt{\mathrm{Ann}_R(M/N_i)}$. Hence, there exists a positive integer t_i such that $J^{t_i} M \subseteq N_i$. Set $t := \max\{t_i : \mathrm{cd}(I,J,R/\mathfrak{p}_i) = d\}$. Then $J^t M \subseteq \cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)=d} N_i$ and so we have the desired result by Theorem 2.4. □

Corollary 2.6. *Let R be a Noetherian local ring, I_1, I_2, J_1, J_2 be ideals of R and M be a non-zero finitely generated R-module of dimension d. If $\mathrm{Att}_R H^d_{I_1,J_1}(M) = \mathrm{Att}_R H^d_{I_2,J_2}(M)$, then the following statements are true:*

(i) $T_R(I_1,J_1,M) = T_R(I_2,J_2,M)$.

(ii) *There exist positive integers t, s such that $H^d_{I_1,J_1}(M) \cong H^d_{I_1,J_1}(M/J_2^t M)$ and $H^n_{I_2,J_2}(M) \cong H^n_{I_2,J_2}(M/J_1^s M)$.*

Proof. (i) By assumption and [10, Theorem 3.1] for each $\mathfrak{p} \in \mathrm{Supp}_R(M)$ we have $\mathrm{cd}(I_1, J_1, R/\mathfrak{p}) = d$ if and only if $\mathrm{cd}(I_2, J_2, R/\mathfrak{p}) = d$. Now, in view of Theorem 2.4,

$$T_R(I_1, J_1, M) = \cap_{\mathrm{cd}(I_1,J_1,R/\mathfrak{p}_i)=d} N_i = \cap_{\mathrm{cd}(I_2,J_2,R/\mathfrak{p}_i)=c} N_i = T_R(I_2, J_2, M),$$

where $0 = \cap_{i=1}^{n} N_i$ denotes a reduced primary decomposition of the zero submodule of M and N_i is a $\mathfrak{p}_i$-primary submodule of M.

(ii) In view of Lemma 2.5 and (i) we have $J_2^t M \subseteq T_R(I_1, J_1, M)$, for some positive integer t. Now by applying the functor $\Gamma_{I_1,J_1}(-)$ on the exact sequence

$$0 \to J_2^t M \to M \to M/J_2^t M \to 0$$

the desired result follows that is $H^d_{I_1,J_1}(M) \cong H^d_{I_1,J_1}(M/J_2^t M)$. □

Theorem 2.7. *Let R be a Noetherian local ring and M be a non-zero finitely generated R-module of dimension d such that $H^d_{I,J}(M) \neq 0$. Then $T_R(I, M/J^t M) = T_R(I, J, M)/J^t M$, for some positive integer t.*

Proof. By Lemma 2.5 there exists an integer $t \geq 1$ such that $J^t M \subseteq T_R(I, J, M)$. We show that $T_R(I, M/J^t M) = T_R(I, J, M)/J^t M$. Let $x + J^t M \in T_R(I, M/J^t M)$. Then $0 = H^d_I(R(x + J^t M)) \cong H^d_{I,J^t}(Rx/Rx \cap J^t M)$ and so in view of [12, Proposition 1.4(8)], $H^d_{I,J}(Rx/Rx \cap J^t M) = 0$. Also, as $Rx \cap J^t M \subseteq T_R(I, J, M)$ it follows that $H^d_{I,J}(Rx \cap J^t M) = 0$. The exact sequence

$$0 \to Rx \cap J^t M \to Rx \to \frac{Rx}{Rx \cap J^t M} \to 0$$

induces an exact sequence

$$\cdots \to H^d_{I,J}(Rx \cap J^t M) \to H^d_{I,J}(Rx) \to H^d_{I,J}(\frac{Rx}{Rx \cap J^t M}) \to 0. \qquad (*)$$

Hence, it follows that $H^d_{I,J}(Rx) = 0$ and therefore, $x \in T_R(I, J, M)$. If $x \in T_R(I, J, M)$, then $H^d_{I,J}(Rx) = 0$. Thus

$$\begin{aligned} H^d_{I,J}(Rx/Rx \cap J^t M) &\cong H^d_{I,J}(R(x + J^t M)) \\ &\cong H^d_{I,J^t}(R(x + J^t M) \cong H^d_I(R(x + J^t M) = 0 \end{aligned}$$

by $(*)$. Therefore, $x + J^t M \in T_R(I, M/J^t M)$. □

Corollary 2.8. *Let R be a Noetherian local ring and M be a non-zero finitely generated R-module of dimension d such that $H^d_{I,J}(M) \neq 0$ and let $J^t M \subseteq T_R(I, M)$ for some positive integer t. Then $T_R(I, J, M) = T_R(I, M)$.*

Proof. By a similar argument to Theorem 2.7 one can show that $T_R(I, M/J^t M) = T_R(I, M)/J^t M$ so that the result follows by Theorem 2.7. □

Theorem 2.9. *Let R be a Noetherian local ring and M be a non-zero finitely generated R-module of dimension d such that $H^d_{I,J}(M) \neq 0$. Then the following statements are true:*

(i) $\mathrm{Ann}_R(H^d_{I,J}(M)) = \mathrm{Ann}_R(M/T_R(I,J,M))$.

(ii) *If $J^tM \subseteq T_R(I,M)$ for some positive integer t, then* $\mathrm{Ann}_R(H^d_I(M)) = \mathrm{Ann}_R(H^d_{I,J}(M))$.

Proof. (i) By Lemma 2.5 there exists a positive integer t such that $J^tM \subseteq T_R(I,J,M)$. Now, from the exact sequence

$$0 \to J^tM \to M \to \frac{M}{J^tM} \to 0$$

we have the exact sequence

$$\cdots \to H^d_{I,J}(J^tM) \to H^d_{I,J}(M) \to H^d_{I,J}(\frac{M}{J^tM}) \to 0.$$

Since $J^tM \subseteq T_R(I,J,M)$, it follows that $H^d_{I,J}(J^tM) = 0$ and so $H^d_{I,J}(M) \cong H^d_{I,J}(M/J^tM)$. Thus $H^d_{I,J}(M) \cong H^d_{I,J^t}(M/J^tM) \cong H^d_I(M/J^tM)$. Hence, by [1, Theorem 2.3],

$$\begin{aligned}\mathrm{Ann}_R(H^d_{I,J}(M)) &= \mathrm{Ann}_R(H^d_I(M/J^tM)) = \mathrm{Ann}_R(\frac{M/J^tM}{T_R(I,M/J^tM)})\\ &= \mathrm{Ann}_R(\frac{M/J^tM}{T_R(I,J,M)/J^tM}) = \mathrm{Ann}_R(M/T_R(I,J,M)).\end{aligned}$$

(ii) The result follows by (i), Corollary 2.8 and [1, Theorem 2.3]. □

Corollary 2.10. *Let R be a Noetherian local ring with dimension d such that $H^d_{I,J}(R) \neq 0$. Then $\mathrm{Ann}_R(H^d_{I,J}(R)) = T_R(I,J,R)$. So that $\mathrm{Ann}_R(H^d_{I,J}(R))$ is the largest submodule of R such that $\mathrm{cd}(I,J,\mathrm{Ann}_R(H^d_{I,J}(R))) < d$.*

Proof. It follows easily from Theorem 2.9(i) and Definition 2.2. □

Corollary 2.11. *Let R be a Noetherian local ring with dimension d such that $H^d_{I,J}(R) \neq 0$. Then $\mathrm{Ann}_R(H^d_{I,J}(R)) = \cap_{\mathrm{cd}(I,J,R/\mathfrak{p}_i)=d}\mathfrak{q}_i$, where $0 = \bigcap_{i=1}^n \mathfrak{q}_i$ is a reduced primary decomposition of the zero ideal of R and $\mathfrak{q}_i$ is a $\mathfrak{p}_i$-primary ideal of R for all i with $1 \leq i \leq n$.*

Corollary 2.12. *Let R be a Noetherian local ring of dimension $d \geq 1$ such that $H^d_{I,J}(R) \neq 0$. Then*

$$\dim R = \dim R/\mathrm{Ann}_R(H^d_{I,J}(R)) = \dim R/\Gamma_{I,J}(R).$$

Proof. The first equality follows by Corollary 2.10 and the second equality follows by $H^d_{I,J}(R) \cong H^d_{I,J}(R/\Gamma_{I,J}(R))$, see [12, Corollary 1.13(4)]. So $\Gamma_{I,J}(R) \subseteq \mathrm{Ann}_R(H^d_{I,J}(R))$. □

Corollary 2.13. *Let $(R, \mathfrak{m})$ be a Noetherian local ring and M be a non-zero finitely generated R-module of dimension d such that $H^d_{I,J}(M) \neq 0$ and $T_R(I, J, M) = 0$. Then the following statements are true:*

(i) $H^d_{I,J}(M) \cong H^d_I(M)$.

(ii) $H^d_{I,J}(R/\mathfrak{p}) \cong H^d_I(R/\mathfrak{p})$ *for each* $\mathfrak{p} \in \mathrm{Ass}_R(M)$ *also* $H^d_{I,J}(R/\mathfrak{p}) \cong H^d_{\mathfrak{m}}(R/\mathfrak{p})$ *whenever R is a complete ring.*

(iii) $\mathrm{Att}_R(H^d_{I,J}(M)) = \mathrm{Att}_R(H^d_{\mathfrak{m}}(M)) = \mathrm{Assh}(M)$ *and* $\mathrm{Ann}_R(H^d_{I,J}(M)) = \mathrm{Ann}_R(H^d_{\mathfrak{m}}(M))$ *whenever R is a complete ring.*

(iv) $\mathrm{Supp}_R M = V(\mathrm{Ann}_R H^d_{I,J,}(M))$.

Proof. (i) By assumption and Lemma 2.5 it follows that $J^t M = 0$ for some positive integer t. Now the assertion follows by [12, Proposition 1.4(8)].

(ii) By assumption $\mathrm{cd}(I, J, R/\mathfrak{p}) = d$ for all $\mathfrak{p} \in \mathrm{Ass}_R(M)$ thus $H^d_{I,J}(R/\mathfrak{p}) \neq 0$. On the other hand, if $\mathfrak{p} \in \mathrm{Ass}_R(M) \setminus V(J)$, then $\dim R/(\mathfrak{p} + J) < d$ and so $H^d_{I,J}(R/\mathfrak{p}) = 0$, by [12, Theorem 4.3]. Hence, for each $\mathfrak{p} \in \mathrm{Ass}_R(M)$ we have $J \subseteq \mathfrak{p}$ and therefore $R/\mathfrak{p}$ is a J-torsion R-module and so the first desired result. The second part follows by Lichtenbaum–Hartshorne Vanishing Theorem, see [3, Theorem 8.2.1].

(iii) The first part follows from (ii), [4, Theorem 2.1], [1, Corollary 3.4] and [3, Theorem 7.3.2]. By Corollary 2.3 we have $T_R(\mathfrak{m}, M) = T_R(I, M) = 0$. Now the result follows from Theorem 2.9(i), [1, Corollary 2.7] and [2, Theorem 2.6].

(iv) It follows from Theorem 2.9(i). □

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