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# THE M-POLYNOMIAL OF LINE GRAPH OF SUBDIVISION GRAPHS

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ABSTRACT. Three composite graphs Ladder graph ($L_n$), Tadpole graph ($T_{n,k}$) and Wheel graph ($W_n$) are graceful graphs, which have different applications in electrical, electronics, wireless communication etc. In this report, we first determine M-polynomial of the Line graph of those three graphs using subdivision idea and then compute some degree based indices of the same.

## 1. INTRODUCTION

Throughout this article we use molecular graph, a connected graph having no loops and no parallel edges where vertices and edges are correspond to atoms and chemical bonds of the compound. Consider a molecular graph $G$ having $V(G)$ and $E(G)$ as vertex set and edge set respectively. The degree of a vertex $v \in V(G)$ of a graph $G$, denoted by $d_v$, is the total number of edges incident with $v$.

Mathematical modelling play significant role to analyze important concepts in chemistry. Mathematical chemistry has excellent tools such as polynomials, functions which can predict properties of chemical compounds successfully. Topological indices are functions $T : \sum \rightarrow R^+$ with the property $T(G) = T(H)$ for every graph $G$ isomorphic to $H$, where $\sum$ is the class of all graphs. These numerical quantity corresponding to a molecular graph are effective in correlating the structure with different physicochemical properties, chemical reactivity, and biological activities. These indices are evaluated by formal definitions. Instead of calculating different topological indices of a specific category [1, 2], we can use a compact general method related to polynomial for calculating the same. For instance, Wiener polynomial is a general polynomial in the field of distance-based topological indices whose derivatives at 1 yield Weiner and Hyper Weiner indices [2]. There are many

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such polynomials such as Pi polynomial [3], Theta polynomial [4] etc.
In the area of degree-based topological indices, M-polynomial [1] perform similar role to compute closed expressions of many degree based topological indices [1, 5]. Thus computation of degree based topological indices reduce to evaluation of a single polynomial. Moreover, detailed analysis of this polynomial can yield new insights in the knowledge of degree based topological indices.

The subdivision graph [6, 7, 8] $S(G)$ is obtained by inserting an additional vertex to each edge of $G$ [9]. The Line graph [10] $L(G)$ is the simple graph whose vertices are the edges of $G$ and $e_1e_2 \in E(L(G))$ if $e_1$, $e_2 \in E(G)$ share a common end point in $G$. The Ladder graph $L_n$ is obtained by the Cartesian product of the Path graph $P_n$ and the complete graph $K_2$. The Tadpole graph $T_{n,k}$[11] is obtained by joining a cycle $C_n$ to a path $P_k$. Wheel graph is obtained by joining $K_1$ to $C_n$.

Ranjini et al.[12] investigate the Zagreb indices of the Line graphs of those three graphs using the subdivision concepts. G Su et al.[13] derived topological indices of the same graphs. Here we calculate the degree based topological indices of $L(S(L_n))$, $L(S(T_{n,k}))$, $L(S(W_n))$ using $M$-polynomial.

## 2. Basic definitions and literature review

**Lemma 1.** *(Handshaking lemma) For a graph $G$, we have*

$$\sum_{v \in V(G)} d_v = 2|E(G)|$$

**Definition 1.** *The M-polynomial of a graph $G$ is defined as,*

$$M(G; x, y) = \sum_{\leqslant i \leqslant j} m_{ij}(G) x^i y^j.$$

*where $m_{ij}(G)$ is the number of edges $uv \in E(G)$ such that $\{d_u, d_v\} = \{i, j\}$.*

For a graph $G$, a degree-based topological index is a graph invariant of the form

$$I(G) = \sum_{uv \in E(G)} f(d_u, d_v),$$

where $f = f(x, y)$ is a function appropriately selected for possible chemical applications [14]. Collecting edges with the same set of end-degrees we can rewrite the above result as

$$I(G) = \sum_{i \leqslant j \leqslant} m_{ij}(G) f(i, j).$$

Gutman and Trinajstić introduced Zagreb indices [15].

The first Zagreb index is defined as,

$$M_1(G) = \sum_{v \in V(G)} (d_v)^2$$

The second Zagreb index is defined as,

$$M_2(G) = \sum_{uv\in E(G)} d_u d_v.$$

For more details about this indices see.
The second modified Zagreb index is defined as,

$$^{m}M_2(G) = \sum_{uv\in E(G)} \frac{1}{d_u d_v}$$

Bollobas and Erdos [16] and Amic et al. [17] presented the idea of the generalized Randić index and discussed widely in both chemistry and mathematics [18]. For more discussion, readers are referred [20, 19].

The general Randić index is defined as,

$$R_\alpha(G) = \sum_{uv\in E(G)} (d_u d_v)^\alpha$$

The inverse Randić index is defined as,

$$RR_\alpha(G) = \sum_{uv\in E(G)} \frac{1}{(d_u d_v)^\alpha}$$

symmetric division index of a connected graph $G$, is defined as:

$$SDD(G) = \sum_{uv\in E(G)} \{\frac{min(d_u, d_v)}{max(d_u, d_v)} + \frac{max(d_u, d_v)}{min(d_u, d_v)}\}.$$

The Harmonic index [21] is defined as,

$$H(G) = \sum_{uv\in E(G)} \frac{2}{d_u + d_v}.$$

The inverse sum index [22] is given by:

$$I(G) = \sum_{uv\in E(G)} \frac{d_u d_v}{d_u + d_v}.$$

The augmented Zagreb index of $G$ proposed by Furtula et al. [23] is defined as,

$$A(G) = \sum_{uv\in E(G)} \{\frac{d_u d_v}{d_u + d_v - 2}\}^3.$$

The relations of some degree-based topological indices with the M-polynomial are shown in the table below.

TABLE 1. Derivation of some degree based topological indices

| Topological Index | f(x,y) | Derivation from $M(G;x,y)$ |
|---|---|---|
| First Zagreb Index | $x+y$ | $(D_x+D_y)(M(G;x,y))\|_{x=y=1}$ |
| Second Zagreb Index | $xy$ | $(D_xD_y)(M(G;x,y))\|_{x=y=1}$ |
| Modified Second Zagreb Index | $\frac{1}{xy}$ | $(S_xS_y)(M(G;x,y))\|_{x=y=1}$ |
| Randić Index($\alpha \in N$) | $(xy)^\alpha$ | $(D_x^\alpha D_y^\alpha)(M(G;x,y))\|_{x=y=1}$ |
| Inverse Randić Index($\alpha \in N$) | $\frac{1}{(xy)^\alpha}$ | $(S_x^\alpha S_y^\alpha)(M(G;x,y))\|_{x=y=1}$ |
| Symmetric Division Index | $\frac{x^2+y^2}{xy}$ | $(D_xS_y+S_xD_y)(M(G;x,y))\|_{x=y=1}$ |
| Harmonic Index | $\frac{2}{x+y}$ | $2S_xJ(M(G;x,y))\|_{x=1}$ |
| Inverse sum Index | $\frac{xy}{x+y}$ | $2S_xJD_xD_y(M(G;x,y))\|_{x=1}$ |
| Augumented Zagreb Index | $\frac{xy}{(x+y-2)^3}$ | $2S_x^3Q_{-2}JD_x^3D_y^3(M(G;x,y))\|_{x=1}$ |

where,

$$D_x(f(x,y)) = x\frac{\partial(f(x,y))}{\partial x}, D_y(f(x,y)) = y\frac{\partial(f(x,y))}{\partial y},$$
$$S_x(f(x,y)) = \int_0^x \frac{\partial(f(t,y))}{t}dt, S_y(f(x,y)) = \int_0^y \frac{\partial(f(x,t))}{t}dt$$
$$J(f(x,y)) = f(x,x), Q_\alpha(f(x,y)) = x^\alpha f(x,y).$$

## 3. Main Results

In this part, we give our main computational results and divide the section in three subsections.

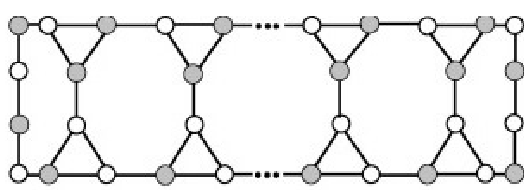

FIGURE 1. The line graph of subdivision graph of the Ladder graph.

### 3.1. **Computational aspects of the Line graph of subdivision of Ladder graph.**

We compute the $M$-polynomial of the line graph of subdivision graph of the Ladder graph in the following theorem.

**Theorem 1.** *Let $L(S(L_n))$ be the line graph of subdivision graph of the Ladder graph. Then we have,*

$$M(L(S(L_n))) = 6x^2y^2 + 4x^2y^3 + (9n-20)x^3y^3.$$

*Proof.* From the figure of $L(S(L_n))$, we can say that its vertices have two partitions,

$$|V_{\{2\}}| = |\{d_u \in V(L(S(L_n))) : d_u = 2\}| = 8.$$

$$|V_{\{3\}}| = |\{u \in V(L(S(L_n))) : d_u = 3\}| = 6n - 12.$$

By handshaking lemma, we have $|E(L(S(L_n)))| = 9n-10$. The edge set of $L(S(L_n))$ can be partitioned as,

$$|E_{\{2,2\}}| = |\{uv \in E(L(S(L_n))) : d_u = 2, d_v = 2\}| = 6.$$

$$|E_{\{2,3\}}| = |\{uv \in E(L(S(L_n))) : d_u = 2, d_v = 3\}| = 4.$$

$$|E_{\{3,3\}}| = |\{uv \in E(L(S(L_n))) : d_u = 3, d_v = 3\}| = 9n - 20.$$

From the definition, the $M$-polynomial of $L(S(L_n))$ is obtained bellow,

$$\begin{aligned} M(L(S(L_n))) &= \sum_{i \leqslant j} m_{ij}(L(S(L_n)))x^i y^j \\ &= \sum_{2 \leqslant 2} m_{ij}(L(S(L_n)))x^2 y^2 + \sum_{2 \leqslant 3} m_{ij}(L(S(L_n)))x^2 y^3 + \\ &\quad + \sum_{3 \leqslant 3} m_{ij}(L(S(L_n)))x^3 y^3 \\ &= |E_{\{2,2\}}|x^2 y^2 + |E_{\{2,3\}}|x^2 y^3 + |E_{\{3,3\}}|x^3 y^3 \\ &= 6x^2 y^2 + 4x^2 y^3 + (9n - 20)x^3 y^3. \end{aligned}$$

This completes the proof. □

Now using this $M$-polynomial, we calculate some degree based topological index of the line graph of subdivision of the Ladder graph in the following theorem.

**Theorem 2.** *Let $L(S(L_n))$ be the line graph of subdivision of the Ladder graph. Then we have,*

1. $M_1(L(S(L_n))) = 54n - 76$.
2. $M_2(L(S(L_n))) = 81n - 132$.
3. $M_2^m(L(S(L_n))) = \frac{18n-1}{18}$.
4. $R_\alpha(L(S(L_n))) = 6(4)^\alpha + 4(6)^\alpha + (9n - 20)(9)^\alpha$.
5. $RR_\alpha(L(S(L_n))) = \frac{6}{4^\alpha} + \frac{4}{6^\alpha} + \frac{9n-20}{9^\alpha}$.
6. $SDD(L(S(L_n))) = 18n - \frac{58}{3}$.
7. $H(L(S(L_n))) = 3n - \frac{31}{15}$.
8. $I(L(S(L_n))) = \frac{27n+48}{10}$.
9. $A(L(S(L_n))) = \frac{6561n-9460}{64}$.

*Proof.* Let $M(L(S(L_n)); x, y) = f(x, y) = 6x^2y^2 + 4x^2y^3 + (9n - 20)x^3y^3$. Then we have,

$$\begin{aligned} D_x(f(x,y)) &= 12x^2y^2 + 8x^2y^3 + 3(9n - 20)x^3y^3 \\ D_y(f(x,y)) &= 12x^2y^2 + 12x^2y^3 + 3(9n - 20)x^3y^3. \end{aligned}$$

$$\begin{aligned}
D_xD_y(f(x,y)) &= 24x^2y^2+24x^2y^3+9(9n-20)x^3y^3.\\
S_x(f(x,y)) &= 3x^2y^2+2x^2y^3+\frac{9n-20}{3}x^3y^3.\\
S_y(f(x,y)) &= 3x^2y^2+\frac{4}{3}x^2y^3+\frac{9n-20}{3}x^3y^3.\\
S_xS_y(f(x,y)) &= \frac{3}{2}x^2y^2+\frac{2}{3}x^2y^3+\frac{9n-20}{9}x^3y^3.\\
D_x^\alpha D_y^\alpha(f(x,y)) &= 6(4^\alpha)x^2y^2+4(6^\alpha)x^2y^3+(9n-20)(9^\alpha)x^3y^3.\\
S_x^\alpha S_y^\alpha(f(x,y)) &= \frac{6}{4^\alpha}x^2y^2+\frac{4}{6^\alpha}x^2y^3+\frac{9n-20}{9^\alpha}x^3y^3.\\
D_xS_y(f(x,y)) &= 6x^2y^2+\frac{8}{3}x^2y^3+(9n-20)x^3y^3.\\
S_xD_y(f(x,y)) &= 6x^2y^2+6x^2y^3+(9n-20)x^3y^3.\\
S_xJ(f(x,y)) &= 24x^4y^2+20x^5+6(9n-20)x^6.\\
S_xJD_xD_y(f(x,y)) &= 6x^4+\frac{24}{5}x^5+\frac{3(9n-20)}{2}x^6.\\
S_x^3Q_{-2}JD_x^3D_y^3(f(x,y)) &= 48x^2+32x^3+\frac{729}{64}(9n-20)x^4.
\end{aligned}$$

Using Table 1.we have,

(1) $M_1(L(S(L_n))) = (D_x+D_y)(f(x,y))|_{x=y=1} = 54n-76.$
(2) $M_2(L(S(L_n))) = (D_xD_y)(f(x,y))|_{x=y=1} = 81n-132.$
(3) $M_2^m(L(S(L_n))) = (S_xS_y)(f(x,y))|_{x=y=1} = \frac{18n-1}{18}.$
(4) $R_\alpha(L(S(L_n))) = (D_x^\alpha D_y^\alpha)(f(x,y))|_{x=y=1} = 6(4)^\alpha+4(6)^\alpha+(9n-20)(9)^\alpha.$
(5) $RR_\alpha(L(S(L_n))) = (S_x^\alpha S_y^\alpha)(f(x,y))|_{x=y=1} = \frac{6}{4^\alpha}+\frac{4}{6^\alpha}+\frac{9n-20}{9^\alpha}.$
(6) $SDD(L(S(L_n))) = (D_xS_y+S_xD_y)(f(x,y))|_{x=y=1} = 18n-\frac{58}{3}.$
(7) $H(L(S(L_n))) = 2(S_x)(f(x,y))|_{x=1} = 3n-\frac{31}{15}.$
(8) $I(L(S(L_n))) = S_xJD_xD_y(f(x,y))|_{x=1} = \frac{27n+48}{10}$
(9) $A(L(S(L_n))) = S_x^3Q_{-2}JD_x^3D_y^3(f(x,y))|_{x=1} = \frac{6561n-9460}{64}.$

This completes the proof. □

3.2. **Computational aspects of the Line graph of subdivision of Tadpole graph.** We compute the $M$-polynomial of the line graph of subdivision graph of the Tadpole graph in the following theorem.

**Theorem 3.** *Let $L(S(T_{n,k}))$ be the line graph of subdivision graph of the Tadpole graph. Then we have,*

$$M(L(S(T_{n,k}))) = xy^2+2(n+k-3)x^2y^2+3x^2y^3+3x^3y^3.$$

*Proof.* The graph of $L(S(T_{n,k})$ has $2(n+k)$ vertices, having three partitions of vertices,

$$|V_{\{1\}}| = |\{u\in V(L(S(L_n))) : d_u = 1\}| = 1.$$

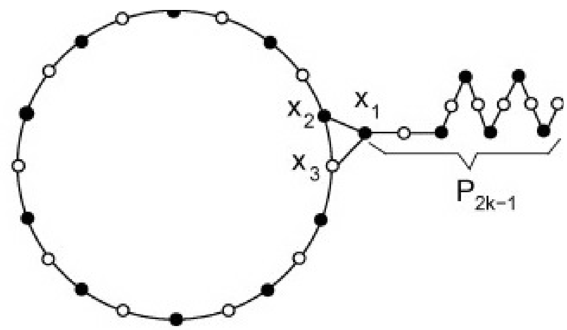


FIGURE 2. The line graph of subdivision graph of the Tadpole graph.

$$|V_{\{2\}}| = |\{u \in V(L(S(L_n))) : d_u = 2\}| = 2n + 2k - 4.$$
$$|V_{\{3\}}| = |\{u \in V(L(S(L_n))) : d_u = 3\}| = 3.$$

This shows that $L(S(T_{n,k})$ has 9n-10 edges. The edge set of $L(S(T_{n,k}))$ has partitions as follows,

$$|E_{\{1,2\}}| = |\{uv \in E(L(S(T_{n,k}))) : d_u = 1, d_v = 2\}| = 1.$$
$$|E_{\{2,2\}}| = |\{uv \in E(L(S(T_{n,k}))) : d_u = 2, d_v = 2\}| = 2(n + k - 3).$$
$$|E_{\{2,3\}}| = |\{uv \in E(L(S(T_{n,k}))) : d_u = 2, d_v = 3\}| = 3.$$
$$|E_{\{3,3\}}| = |\{uv \in E(L(S(T_{n,k}))) : d_u = 3, d_v = 3\}| = 3.$$

From the definition, the $M$-polynomial of $L(S(T_{n,k}))$ is obtained bellow,

$$\begin{aligned}
M(L(S(T_{n,k}))) &= \sum_{i\leqslant j} m_{ij}(L(S(T_{n,k})))x^i y^j \\
&= \sum_{1\leqslant 2} m_{ij}(L(S(T_{n,k})))x^1 y^2 + \sum_{2\leqslant 2} m_{ij}(L(S(T_{n,k})))x^2 y^2 + \\
&\quad + \sum_{2\leqslant 3} m_{ij}(L(S(T_{n,k})))x^2 y^3 \\
&\quad + \sum_{3\leqslant 3} m_{ij}(L(S(T_{n,k})))x^3 y^3 \\
&= |E_{\{1,2\}}|xy^2 + |E_{\{2,2\}}|x^2y^2 + |E_{\{2,3\}}|x^2y^3 + |E_{\{3,3\}}|x^3y^3 \\
&= xy^2 + 2(n + k - 3)x^2y^2 + 3x^2y^3 + 3x^3y^3.
\end{aligned}$$

This completes the proof. □

Now using this $M$-polynomial, we calculate some degree based topological index of the Line graph of subdivision of the Tadpole graph in the following theorem.

**Theorem 4.** *Let $L(S(T_{n,k}))$ be the line graph of subdivision of the Ladder graph. Then we have,*

1. $M_1(L(S(T_{n,k}))) = 8n + 8k + 12$.
2. $M_2(L(S(T_{n,k}))) = 8n + 8k + 23$.
3. $M_2^m(L(S(T_{n,k}))) = \frac{3n+3k-1}{6}$.
4. $R_\alpha(L(S(T_{n,k}))) = 2^\alpha + (n+k-3)2^{2\alpha+1} + 3(6^\alpha) + 3^{2\alpha+1}$.
5. $RR_\alpha(L(S(T_{n,k}))) = \frac{1}{2^\alpha} + \frac{n+k-3}{2^{2\alpha-1}} + \frac{3}{6^\alpha} + \frac{1}{3^{2\alpha-1}}$.
6. $SDD(L(S(T_{n,k}))) = 4n + 4k - 1$.
7. $H(L(S(T_{n,k}))) = n + k - \frac{2}{15}$.
8. $I(L(S(T_{n,k}))) = 2n + 2k - \frac{17}{30}$.
9. $A(L(S(T_{n,k}))) = 16n + 16k + \frac{1163}{64}$.

*Proof.* Let $M(L(S(T_{n,k})); x, y) = f(x, y) = xy^2 + 2(n+k-3)x^2y^2 + 3x^2y^3 + 3x^3y^3$. Then we have,

$$
\begin{aligned}
D_x(f(x,y)) &= xy^2 + 4(n+k-3)x^2y^2 + 6x^2y^3 + 9x^3y^3.\\
D_y(f(x,y)) &= 2xy^2 + 4(n+k-3)x^2y^2 + 9x^2y^3 + 9x^3y^3.\\
D_xD_y(f(x,y)) &= 2xy^2 + 8(n+k-3)x^2y^2 + 18x^2y^3 + 27x^3y^3\\
S_x(f(x,y)) &= xy^2 + (n+k-3)x^2y^2 + \frac{3}{2}x^2y^3 + x^3y^3.\\
S_y(f(x,y)) &= \frac{xy^2}{2} + (n+k-3)x^2y^2 + x^2y^3 + x^3y^3\\
S_xS_y(f(x,y)) &= \frac{xy^2}{2} + \frac{(n+k-3)}{2}x^2y^2 + \frac{x^2y^3}{2} + \frac{x^3y^3}{3}.\\
D_x^\alpha D_y^\alpha(f(x,y)) &= 2^\alpha xy^2 + (n+k-3)(2^{2\alpha+1})x^2y^2 + 3(6^\alpha)x^2y^3 + 3^{2\alpha+1}x^3y^3.\\
S_x^\alpha S_y^\alpha(f(x,y)) &= \frac{xy^2}{2^\alpha} + \frac{(n+k-3)}{2^{2\alpha-1}}x^2y^2 + \frac{3}{6^\alpha}x^2y^3 + \frac{1}{3^{2\alpha-1}}x^3y^3.\\
D_xS_y(f(x,y)) &= \frac{xy^2}{2} + 2(n+k-3)x^2y^2 + 2x^2y^3 + 3x^3y^3.\\
S_xD_y(f(x,y)) &= 2xy^2 + 2(n+k-3)x^2y^2 + \frac{9}{2}x^2y^3 + 3x^3y^3.\\
S_xJ(f(x,y)) &= \frac{1}{3}x^3 + \frac{n+k-3}{2}x^4 + \frac{3}{5}x^5 + \frac{1}{2}x^6.\\
S_xJD_xD_y(f(x,y)) &= \frac{2}{3}x^3 + 2(n+k-3)x^4 + \frac{18}{5}x^5 + \frac{27}{6}x^6.\\
S_x^3Q_{-2}JD_x^3D_y^3(f(x,y)) &= 8x + 16(n+k-3)x^2 + 24x^3 + \frac{2187}{64}x^4.
\end{aligned}
$$

Using table 1.we have,

(1) $M_1(L(S(T_{n,k}))) = (D_x + D_y)(f(x,y))|_{x=y=1} = 8n + 8k + 12$.
(2) $M_2(L(S(T_{n,k}))) = (D_xD_y)(f(x,y))|_{x=y=1} = 8n + 8k + 23$.
(3) $M_2^m(L(S(T_{n,k}))) = (S_xS_y)(f(x,y))|_{x=y=1} = \frac{3n+3k-1}{6}$.

(4) $R_\alpha(L(S(T_{n,k}))) = (D_x^\alpha D_y^\alpha)(f(x,y))|_{x=y=1} = 2^\alpha + (n+k-3)2^{2\alpha+1} + 3(6^\alpha) + 3^{2\alpha+1}$.
(5) $RR_\alpha(L(S(T_{n,k}))) = (S_x^\alpha S_y^\alpha)(f(x,y))|_{x=y=1} = \frac{1}{2^\alpha} + \frac{n+k-3}{2^{2\alpha-1}} + \frac{3}{6^\alpha} + \frac{1}{3^{2\alpha-1}}$.
(6) $SDD(L(S(T_{n,k}))) = (D_xS_y + S_xD_y)(f(x,y))|_{x=y=1} = 4n + 4k - 1$.
(7) $H(L(S(T_{n,k}))) = 2(S_x)(f(x,y))|_{x=1} = n + k - \frac{2}{15}$.
(8) $I(L(S(T_{n,k}))) = S_xJD_xD_y(f(x,y))|_{x=1} = 2n + 2k - \frac{17}{30}$.
(9) $A(L(S(T_{n,k}))) = S_x^3Q_{-2}JD_x^3D_y^3(f(x,y))|_{x=1} = 16n + 16k + \frac{1163}{64}$.

This completes the proof. □

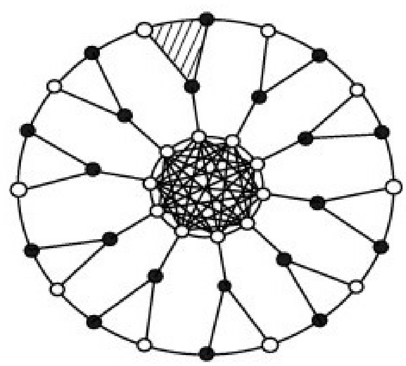

FIGURE 3. The line graph of subdivision graph of the wheel graph.

3.3. **Computational aspects of the Line graph of subdivision of Wheel graph.** We compute the $M$-polynomial of the line graph of subdivision graph of the Wheel graph in the following theorem.

**Theorem 5.** *Let $L(S(W_{n+1}))$ be the line graph of subdivision graph of the Wheel graph. Then we have,*

$$M(L(S(W_{n+1}))) = 4nx^3y^3 + nx^3y^n + \frac{n(n-1)}{2}x^ny^n + 3x^3y^3.$$

*Proof.* It is easy to see that the order of the graph $L(S(W_{n+1})$ is $4n$ out of which $3n$ vertices are of degree 3 and $n$ vertices are of degree $n$. By handshaking lemma, the size of the graph $L(S(W_{n+1}))$ is $\frac{n^2+9n}{2}$. The edge set of $L(S(W_{n+1}))$ can be partitioned as,

$$|E_{\{3,3\}}| = |\{uv \in E(L(S(W_{n+1}))) : d_u = 3, d_v = 3\}| = 4n.$$
$$|E_{\{n,3\}}| = |\{uv \in E(L(S(W_{n+1}))) : d_u = n, d_v = 3\}| = n.$$

$$|E_{\{n,n\}}| = |\{uv \in E(L(S(W_{n+1}))) : d_u = n, d_v = n\}| = \frac{n(n-1)}{2}.$$

From the definition, the $M$-polynomial of $L(S(W_{n+1}))$ is obtained bellow,

$$\begin{aligned} M(L(S(W_{n+1}))) &= \sum_{i\leqslant j} m_{ij}(L(S(W_{n+1})))x^i y^j \\ &= \sum_{3\leqslant 3} m_{ij}(L(S(W_{n+1})))x^3 y^3 + \sum_{3\leqslant n} m_{ij}(L(S(W_{n+1})))x^3 y^n + \\ &\quad + \sum_{n\leqslant n} m_{ij}(L(S(W_{n+1})))x^n y^n \\ &= |E_{\{3,3\}}|x^3y^3 + |E_{\{3,n\}}|x^3y^n + |E_{\{n,n\}}|x^ny^n \\ &= 4nx^3y^3 + nx^3y^n + \frac{n(n-1)}{2}x^ny^n + 3x^3y^3. \end{aligned}$$

This completes the proof. □

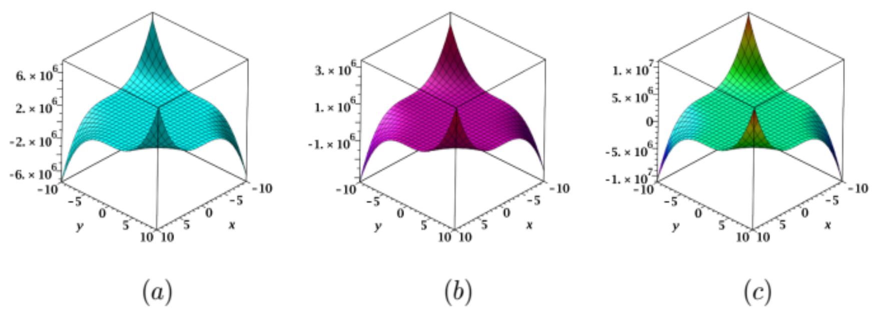


FIGURE 4. The M-polynomal for line graph of subdivision graph of (a) Ladder graph, (b) Tadpole graph, and (c) Wheel graph.

Now using this $M$-polynomial, we calculate some degree based topological index of the Line graph of subdivision of the Wheel graph in the following theorem.

**Theorem 6.** *Let $L(S(W_{n+1}))$ be the line graph of subdivision of the Wheel graph. Then we have,*

1. $M_1(L(S(W_{n+1}))) = n^3 + 27n$.
2. $M_2(L(S(W_{n+1}))) = \frac{n(n^3-n^2+6n+72)}{2}$.
3. $M_2^m(L(S(W_{n+1}))) = \frac{8n^2+15n-9}{18n}$.
4. $R_\alpha(L(S(W_{n+1}))) = 4n(3^{2\alpha}) + n(3n)^\alpha + \frac{n^{2\alpha+1}(n-1)}{2}$.
5. $RR_\alpha(L(S(W_{n+1}))) = \frac{4n}{3^{2\alpha}} + \frac{n}{3n^\alpha} + \frac{n(n-1)}{2(n^{2\alpha})}$.

6. $SDD(L(S(W_{n+1}))) = \frac{4n^2+21n+9}{3}$.
7. $H(L(S(W_{n+1}))) = \frac{11n^2+42n-9}{6(n+3)}$.
8. $I(L(S(W_{n+1}))) = \frac{n^4+2n^3+33n^2+72n}{4(n+3)}$.
9. $A(L(S(W_{n+1}))) = \frac{n^7}{16(n-1)^2} + \frac{27n^4}{(n+1)^3} + \frac{729n}{16}$.

*Proof.* Let $M(L(S(W_{n+1})); x, y) = f(x, y) = 4nx^3y^3 + nx^3y^n + \frac{n(n-1)}{2}x^ny^n + 3x^3y^3$. Then we have,

$$\begin{aligned}
D_x(f(x,y)) &= 12nx^3y^3 + 3nx^3y^n + \frac{n^2(n-1)}{2}x^ny^n.\\
D_y(f(x,y)) &= 12nx^3y^3 + n^2x^3y^n + \frac{n^2(n-1)}{2}x^ny^n.\\
D_xD_y(f(x,y)) &= 36nx^3y^3 + 3n^2x^3y^n + \frac{n^3(n-1)}{2}x^ny^n\\
S_x(f(x,y)) &= \frac{4n}{3}x^3y^3 + \frac{n}{3}x^3y^n + \frac{n-1}{2}x^ny^n.\\
S_y(f(x,y)) &= \frac{4n}{3}x^3y^3 + x^3y^n + \frac{n-1}{2}x^ny^n\\
S_xS_y(f(x,y)) &= \frac{4n}{3}x^3y^3 + \frac{x^3y^n}{3} + \frac{n-1}{2n}x^ny^n.\\
D_x^\alpha D_y^\alpha(f(x,y)) &= 4n(3^{2\alpha})x^3y^3 + n(3n)^\alpha x^3y^n + \frac{n^{2\alpha+1}(n-1)}{2}x^ny^n.\\
S_x^\alpha S_y^\alpha(f(x,y)) &= \frac{4n}{3^{2\alpha}}x^3y^3 + \frac{n}{(3n)^\alpha}x^3y^n + \frac{n(n-1)}{2n^{2\alpha}}x^ny^n.\\
D_xS_y(f(x,y)) &= 4nx^3y^3 + 3x^3y^n + \frac{n(n-1)}{2}x^ny^n.\\
S_xD_y(f(x,y)) &= 4nx^3y^3 + \frac{n^2}{3}x^3y^n + \frac{n(n-1)}{2}x^ny^n.\\
S_xJ(f(x,y)) &= \frac{2n}{3}x^6 + \frac{n}{n+3}x^{n+3} + \frac{n-1}{4}x^{2n}.\\
S_xJD_xD_y(f(x,y)) &= 6nx^6 + \frac{3n^2}{n+3}x^{n+3} + \frac{n^2(n-1)}{4}x^{2n}.\\
S_x^3Q_{-2}JD_x^3D_y^3(f(x,y)) &= \frac{729n}{16}x^4 + \frac{27n^4}{(n+1)^3}x^{n+1} + \frac{n^7}{16(n-1)^2}x^{2n-2}.
\end{aligned}$$

Using table 1.we have,

(1) $M_1(L(S(W_{n+1}))) = (D_x + D_y)(f(x,y))|_{x=y=1} = n^3 + 27n$.
(2) $M_2(L(S(W_{n+1}))) = (D_xD_y)(f(x,y))|_{x=y=1} = \frac{n(n^3-n^2+6n+72)}{2}$.
(3) $M_2^m(L(S(W_{n+1}))) = (S_xS_y)(f(x,y))|_{x=y=1} = \frac{8n^2+15n-9}{18n}$.
(4) $R_\alpha(L(S(W_{n+1}))) = (D_x^\alpha D_y^\alpha)(f(x,y))|_{x=y=1} = 4n(3^{2\alpha}) + n(3n)^\alpha + \frac{n^{2\alpha+1}(n-1)}{2}$.

(5) $RR_{\alpha}(L(S(W_{n+1}))) = (S_x^{\alpha} S_y^{\alpha})(f(x,y))|_{x=y=1} = \frac{4n}{3^{2\alpha}} + \frac{n}{3n^{\alpha}} + \frac{n(n-1)}{2(n^{2\alpha})}$.

(6) $SDD(L(S(W_{n+1}))) = (D_x S_y + S_x D_y)(f(x,y))|_{x=y=1} = \frac{4n^2+21n+9}{3}$.

(7) $H(L(S(W_{n+1}))) = 2(S_x)(f(x,y))|_{x=1} = \frac{11n^2+42n-9}{6(n+3)}$.

(8) $I(L(S(W_{n+1}))) = S_x J D_x D_y(f(x,y))|_{x=1} = \frac{n^4+2n^3+33n^2+72n}{4(n+3)}$.

(9) $A(L(S(W_{n+1}))) = S_x^3 Q_{-2} J D_x^3 D_y^3(f(x,y))|_{x=1} = \frac{n^7}{16(n-1)^2} + \frac{27n^4}{(n+1)^3} + \frac{729n}{16}$.

This completes the proof. □

## 4. Conclusion

We obtained many topological indices for line graph of Ladder graph, Tadpole graph, Wheel graph using subdivision concept. Firstly, we commuted M-polynomial of these graphs and later recovered many degree-based topological indices applying it. These results can play an important role in Electrical, Electronics, Wireless communication, Cryptography etc.

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