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Research Article

Comparison of Parametric and Non-Parametric Estimation Methods in Linear Regression Model

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ABSTRACT

In this study, the aim was to review the methods of parametric and non-parametric analyses in simple linear regression model. The least squares estimator (LSE) in parametric analysis of the model, and Mood-Brown and Theil-Sen methods that estimates the parameters according to the median value in non-parametric analysis of the model are introduced. Also, various weights of Theil-Sen method are examined and estimators are discussed. In an attempt to show the need for non-parametric methods, results are evaluated based on real life data.

Keywords: Outlier, Least Squares, Mood-Brown Estimator, Theil-Sen Estimator, Median, Mean Absolute Deviation

Doğrusal Regresyon Modelinde Parametrik ve Parametrik Olmayan Tahmin Yöntemlerinin Karşılaştırması

ÖZ

Bu çalışmada, basit doğrusal regresyon modelinde parametrik ve parametrik olmayan analiz yöntemlerinin karşılaştırmalı olarak incelenmesi amaçlanmıştır. Modelin parametrik analizinde EKK tahmini, parametrik olmayan analizinde ise medyana göre parametre tahmini yapan Mood-Brown ve Theil-Sen yöntemleri tanıtılmıştır. Ayrıca Theil-Sen yöntemine ait çeşitli ağırlıklar incelenerek parametre tahmin ediciler tartışılmıştır. Parametrik olmayan yöntemlere olan ihtiyacı göstermek amacı ile sonuçlar gerçek yaşam verisi üzerinde değerlendirilmiştir.

Anahtar Kelimeler: Aykırı Değer, EKK, Mood-Brown Tahmini, Theil-Sen Tahmini, Medyan, Ortalama Mutlak Sapma

1. Introduction

Regression analysis examines the relation between two or more variables which have causality between them. Explanation of a dependent variable in the model by an independent variable is defined as simple linear regression. In simple linear regression, when assumptions are met, acquired estimates are, according to Gauss-Markov theorem, linear, unbiased estimators with least variance of their parameters. However, when assumptions are not met, acquired estimates lose the specifications which they should meet. In this situation, compatibility to real data is ensured through the use of non-parametric and robust regression methods. Theil (1950), one of the non-parametric regression methods, developed a method which finds the point estimation of β_1 curve coefficients. Theil, who indicates that Mood-Brown method which is intended to find curve is fast but not a much trusted method especially to find curve estimation, developed a method that is named after himself.

Mood and Graybill (1950), based on Mood-Brown hypothesis, developed a trial-and-error method which finds confidence interval for β_1 coefficient. In hypothesis testing related to β_0 and $\beta_1$in Mood-Brown method, he indicated that n_1 and n_2 distributed binomially with 0.5 parameter and based on this information developed the test criterion in Mood-Brown method.

Brown-Mood (1951), named after their names, developed a method which determines β_0 and β_1 coefficients. In non-parametric simple linear regression analysis, in estimation of the parameter according to median, examinations were made to test the hypothesis test $H_0: \beta_1 = \beta_{10}$ against its alternative $H_1: \beta_1 \neq \beta_{10}$.

Sen (1968) examined a rank score method which claims two or more curve parameter to be equal and tests null hypothesizes. Inspired from Kendall's Tau, he worked on simple and robust estimators of β_1.

Power and efficiency of Kendall's Tau test measurement criterion was examined. He described the point estimator as median of curve pairs population $(y_j - y_i)/(x_j - x_i)$ of $x_j \neq x_i$ and points. Sen examined estimators he introduced and made comparisons with least squares method which he named after himself and other non-parametric estimators.

2. Non-Parametric Regression Methods

Let simple linear regression model be defined as

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \tag{1}$$

Here y shows dependent variable; X_i shows independent variable. ε_i is the error term of regression model. β_0 and β_1 values respectively give breakpoints and curve of model. In order to estimate coefficients in regression model, as an alternative to least squares method, non-parametric or robust methods are used. These methods are used as alternative to least squares method when error term is not normally distributed and outliers affected the model (Candan, 1995).

2.1. Mood-Brown Method

To carry out this method for the estimation of parameters β_0 and β_1 for regression line given in equality (2.1), first Y values are separated into two groups as those with X values less than or equal to median values of Xs and those with X values greater than median values of Xs. Desired values of β_0 and β_1 is the estimation where median of deviations from regression line of both group is zero. The steps to obtain the parameters for β_0 and $\beta_1$is as follows:

1. Scatter plot is prepared for sample data.
2. A vertical line that passes through X values is drawn. If one or more points fall into the median line, this line is shifted to right or left as necessary, so the number of points on both sides of the median are as equal as possible.
3. In the second step, median values of X and Y is found for both groups. That is to say total 4 median is calculated.
4. In the first group, the point where medians of X and Y intersect is pointed out. Likewise the process is also carried out for the second group.
5. A line which connects two points determined in fourth step is drawn. This line is the first approximation to desired estimation of line.
6. If median of deviations from this line is not zero in both groups, position of this line is changed until deviations in every group is zero. If a better accuracy is desired, the iterative method which is suggested by Mood can be used (Daniel, 1990).
7. While the intersection between the line at the end and y gives coefficient $\hat{\beta}_0$, coefficient $\hat{\beta}_1$ is;

$$\hat{\beta}_1 = \frac{Y_1 - Y_2}{X_1 - X_2} \tag{2}$$

Here (X_1, Y_1) and (X_2, Y_2) are coordinates of any two point on the line (Kıroğlu, 2001).

2.2. Theil-Sen Method

Theil-Sen method is also expressed as Theil-Kendall or Theil method in literature (Zaman and Alakuş, 2016). In 1950, the method put forward by Theil is one of the methods researchers used mostly to find curve. Theil's (1950) method, which is used to estimate curve of a line, is based on calculation of median of observation pairs, (x_i, y_i) and (x_j, y_j) (Hussain and Sprent, 1983). The $(x_1, y_1), \ldots, (x_n, y_n)$ value we have consists of n observation pairs. x_i Values are known, different, and independent of each other and sorted as $x_1 < x_2 < \cdots < x_n$ (Yıldız and Topal, 2001). In Model (1), variance of e_i, consists of σ_e^2 and random errors as a result of a symmetric continuous distribution whose median is zero and originates from the same distribution (Rao and Gore, 1982). In Theil method, β_0 and β_1 should be estimated in such a way that median of the error term e_i must be zero (Maritz, 1979). Estimation of β_1, as $\hat{\beta}_1$, $i < j$ and $(x_i \neq x_j)$, is a weight median of all $N = \binom{n}{2}$ curve estimations of $S_{ij} = \frac{y_j - y_i}{x_j - x_i}$ (Daniel, 1995; Wang and Yu, 2004).

In other words, it is obtained as

$$\hat{\beta}_1 = median\left\{S_{ij}\right\} \tag{3}$$

and

$$\hat{\beta}_0 = median(Y) - \hat{\beta}_1 median(x_i) \tag{4}$$

(Hussain and Sprent, 1983).

There are two approaches for mutual estimation of slope and intercept parameters. These approaches are as follows; (Zaman, 2017).

Optimum estimation method values based on sign test

$d_i = y_i - \hat{\beta}_1 x_i$ values are calculated and median of these values is the estimation of β_0, $\hat{\beta}_0$. This approach does not require assumption of symmetrically distributed d_i. It is better suited especially for extreme data $\left(\hat{\beta}_0 = \text{median}(d_i)\right)$

Hodges-Lehmann Method

Let's define $d_i = y_i - \hat{\beta}_1 x_i$ variable. This approach requires the assumption where the d_i are distributed symmetrically around β_0. According to Hodges-Lehmann method, $\hat{\beta}_0$ is arithmetic mean of the d_i. This modification may not be viable for extremely pointed data. $\left(\hat{\beta}_0 = \text{mean}(d_i)\right)$ (D'Abrera and Lehmann, 1975).

$w_{ij1} = (x_j - x_i)$ and $w_{ij2} = (x_j - x_i)^2$ as two different weights, the weighted curve parameter estimators in Theil Method is given with (5) and (6). As is seen, estimators are weighted mean and median of the S_{ij}.

$$\hat{\beta}_{1wij(1,2)(mean)} = \frac{\sum_{i<j} w_{ij(1,2)} S_{ij}}{\sum_{i<j} w_{ij(1,2)}} \tag{5}$$

and

$$\hat{\beta}_{1wij(1,2)(med)} = \frac{med_{i<j}\left(w_{ij(1,2)} S_{ij}\right)}{med_{i<j}\left(w_{ij(1,2)}\right)} \tag{6}$$

and estimator of curve parameter can be given as

$$\hat{\beta}_0 = median(Y) - \hat{\beta}_{1wij(mean,med)} median(x_i) \tag{7}$$

(Sievers, 1978 and Scholz, 1978). Also Randles and Wolfe (1979) suggested unweighted means of S_{ij} as estimator of $\hat{\beta}_1$. In this situation, estimator of curve parameter is given with equation of

$$\hat{\beta}_{1sij(mean)} = \frac{\sum_{i<j} S_{ij}}{N} \tag{8}$$

(Toka et. al, 2011).

3. Numerical Illustration

In the study, in an attempt to demonstrate the need for non-parametric analysis method in simple linear regression model, Pilot-Plant (Daniel and Wood, 1980) data in Table 1. is examined. Here, dependent variable is amount of acid determined by titration and independent variable is amount of organic acid determined by sampling and weighing. Previously, this data was used with different purposes in Yale and Farsythe (1976). Data is as given in Table 3.1. Also, scatter graph of the data is displayed in Figure 1. As can be seen in the figure, there is a strong linear correlation between exponent variable and dependent variable (Zaman, 2017).

Observation	Sampling	Titration
1	123	76
2	109	70
3	62	55
4	104	71
5	57	55 (5.5)
6	37	48
7	44	50
8	100	66
9	16	41
10	28	43
11	138	82
12	105	68
13	159	88
14	75	58
15	88	64
16	164	88
17	169	89
18	167	88
19	149	84
20	167	88

Table 1. Pilot-Plant Data

Let us assume that a wrong entry was made for one value. Let observation y on 5^{th} row be 5.5 instead of 55. This mistake results in a deviated value in direction of y. Now, let us create models for the methods we will use.

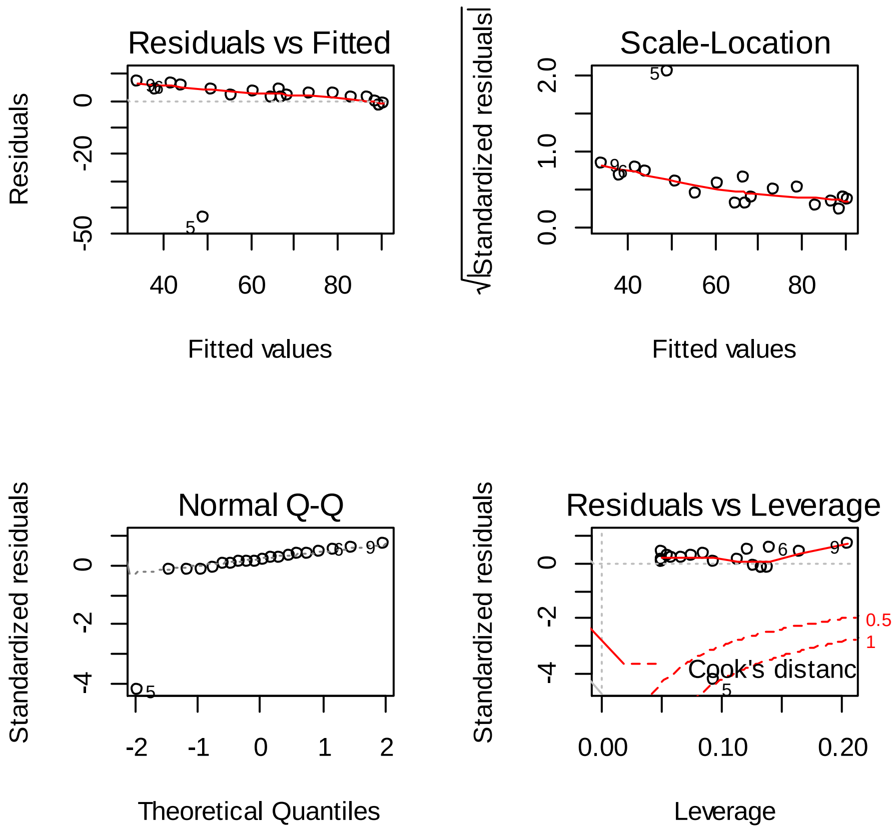

Figure 1. Graphs of the error terms found in the practice

When Figure 1 is examined, the top-left graph shows the graph of estimations of residuals opposite to $\hat{Y}$ values. It must be distributed randomly around horizontal line which represents the errors around zero. In other words, there should not be a clear trend in the distribution of points. Bottom-left graph is the standard $Q-Q$ graph which shows the residuals distributed normally. Top-right graph shows the graph of estimation values of $\hat{Y}$ and square root of standardized residuals. Again, these points should not have a clear trend. Lastly, bottom-right graph shows each point leverage power which is an important measurement to evaluate regression results. Also, in regression, Cook's distance which is another important measurement of each observation is demonstrated. If the distance is greater than 1, it means there is a questionable and possible outlier or weak model. When 4 graphs are examined, it is seen that 5th observation deviated from regression line.

Regression Model Estimation for Theil-1 Method

Now, we will compute the curve coefficient of $\hat{\beta}_1$ to define the correlation between these two variables. For this goal, $\binom{19}{2} = 171$ times S_{ij} is computed when the Theil method is used. Observation value of x in a normal data and the corresponding y values are averaged, and $(x_i \neq x_j)$ assumption is made in terms of the computed curve for Theil method.

$$\hat{\beta}_1 = median\{S_{ij}\} = 0.326087$$

and is

$$\hat{\beta}_0 = median(Y) - \hat{\beta}_1 median(x_i) = 69 - 0.326087 * 104.5 = 34.92391304$$

The regression estimation equation for Theil-1 method is found as $\hat{Y} = 34.924 + 0.326x_i$.

Regression model computed according to Theil-1 method is found to be significant in 5% significance level, and the arithmetic mean of absolute deviations from the estimation values of bound variations is

$$MAD = \frac{\sum_{i=1}\left|Y_i - \hat{Y}_i\right|}{n} = 3.392$$

Regression Model Estimation for Mood-Brown Method

The data is first put in order according to x values. And then the ordinal median value is found to be 104,5. According to this median value, the data is divided into two as those lower than this value and those higher than this value. The median values for those in Group 1 is computed as 59,5 and 52,5 for x and y respectively. The Median values for those in Group 2 is computed as 154 and 86 for x and y respectively.

Considering these values, it is;

$$\hat{\beta}_1 = \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{84 - 52.5}{149 - 59.5} = 0.351955$$

$$\hat{\beta}_0 = Y_1 - \hat{\beta}_1 X_1 = 52.5 - 0.351955 * 59.5 = 31.5587$$

And thus, the model estimation equation is

$$\hat{Y} = 31.559 + 0.352x_i$$

Regression model computed according to Mood-Brown method is found to be significant in 5% significance level, and the arithmetic mean of absolute deviations from the estimation values of bound variations is

$$MAD = \frac{\sum_{i=1}\left|Y_i - \hat{Y}_i\right|}{n} = 4.953$$

Regression Model Estimation for Least Squares Method

The estimation equation based on the observation pairs in Table 1 is

$$\hat{Y} = 28.193 + 0.368x_i$$

For the application of least squares method to be possible, the error terms ε_i should meet the normal distribution conditions whose independent mean with the same distribution is zero, and variance is σ^2. To check whether normal distribution assumption is met, $Q - Q$ graph of the error terms in Figure 1 can be examined. As seen in Figure 1, the error terms do not have normal distribution because the boxes are not on the line. Thereby, the use of non-parametric regression techniques ensures more reliable results.

Regression model computed according to least squares method is found to be significant in 5% significance level, and the arithmetic mean of absolute deviations from the estimation values of bound variations is

$$MAD = \frac{\sum_{i=1}\left|Y_i - \hat{Y}_i\right|}{n} = 4.899$$

Regression model estimation for Optimum Type Theil Method

The estimation equation with this method is

$$\hat{Y} = 34.652 + 0.326x_i$$

Regression model computed according to Optimum type Theil method is found to be significant in 5% significance level, and the arithmetic mean of absolute deviations from the estimation values of bound variations is

$$MAD = \frac{\sum_{i=1}\left|Y_i - \hat{Y}_i\right|}{n} = 3.378$$

Regression model estimation for Hodges-Lehmann Type Theil Method

The model equation acquired with this method is

$$\hat{Y} = 32.522 + 0.326x_i$$

Regression model computed according to Hodges-Lehmann type Theil method is found to be significant in 5% significance level, and the arithmetic mean of absolute deviations from the estimation values of bound variations is

$$MAD = \frac{\sum_{i=1}\left|Y_i - \hat{Y}_i\right|}{n} = 4.560$$

Regression model estimation for Weighted Theil-1 Method

Let $w_{ij1} = (x_j - x_i)$. Here, the value S_{ij} is equal to w_{ij1} That is, $\binom{19}{2} = 171$ is estimated as much as w_{ij1} Observation value of x and the corresponding y value is averaged and assumption of $(x_i \neq x_j)$ is made in the sense of the computed curve. For this weight, two regression model estimation is made according to both mean and median value.

The results based on mean is computed as;

$$\hat{\beta}_{1wij1(ort)} = \frac{\sum_{i<j} w_{ij1} S_{ij}}{\sum_{i<j} w_{ij1}} = \frac{2095}{5182} = 0.40428$$

$$\hat{\beta}_{0wij1(ort)} = 69 - 0.40428 * 104.5 = 26.7523$$

and the regression estimation equation based on these results is;

$$\hat{Y} = 26.752 + 0.404x_i$$

Regression model computed according to Weighted Theil-1 (Mean) is found to be significant in 5% significance level, and the arithmetic mean of absolute deviations from the estimation values of bound variations is

$$MAD = \frac{\sum_{i=1}\left|Y_i - \hat{Y}_i\right|}{n} = 5.823$$

The results based on median is computed as;

The weight is $w_{ij1} = (x_j - x_i)$

$$\hat{\beta}_{1wij1(med)} = \frac{med_{i<j}(w_{ij1} S_{ij})}{med_{i<j}(w_{ij1})} = \frac{13}{38} = 0.3421$$

$$\hat{\beta}_{0wij1(ort)} = 69 - 0.3421 * 104.5 = 33.25$$

And so, the regression estimation equation is as follows:

$$\hat{Y} = 33.25 + 0.342x_i$$

Regression model computed according to Weighted Theil-1 (Median) is found to be significant in 5% significance level, and the arithmetic mean of absolute deviations from the estimation values of bound variations is

$$MAD = \frac{\sum_{i=1}|Y_i - \hat{Y}_i|}{n} = 3.783$$

Regression model estimation for Weighted Theil-2 Method

Assuming $w_{ij2} = (x_j - x_i)^2$ now we will examine the regression models estimated according to both the mean and the median.

First, the mean is;

$$\hat{\beta}_{1wij2(ort)} = \frac{\sum_{i<j} w_{ij2} S_{ij}}{\sum_{i<j} w_{ij2}} = \frac{314950.5}{849834} = 0.3706$$

$$\hat{\beta}_{0wij2(ort)} = 69 - 0.3706 * 104.5 = 30.2720$$

The regression estimation equation for the Weighted Theil-2 (Mean) is

$$\hat{Y} = 30.272 + 0.371x_i$$

Regression model computed according to Weighted Theil-2 (Mean) is found to be significant in 5% significance level, and the arithmetic mean of absolute deviations from the estimation values of bound variations is

$$MAD = \frac{\sum_{i=1}|Y_i - \hat{Y}_i|}{n} = 4.374$$

And now, the median is;

$$\hat{\beta}_{1wij2(med)} = \frac{med_{i<j}(w_{ij2} S_{ij})}{med_{i<j}(w_{ij2})} = \frac{1071}{3025} = 0.35404$$

$$\hat{\beta}_{0wij2(ort)} = 69 - 0.35404 * 104.5 = 32.0018$$

The regression estimation equation for the Weighted Theil-2(Median) is as follows.

$$\hat{Y} = 32.002 + 0.354x_i$$

Regression model computed according to Weighted Theil-2 (Median) is found to be significant in 5% significance level, and the arithmetic mean of absolute deviations from the estimation values of bound variations is

$$MAD = \frac{\sum_{i=1}|Y_i - \hat{Y}_i|}{n} = 4.106$$

And for the Theil-2 method;

$$\hat{\beta}_{1sij(ort)} = \frac{\sum_{i<j} S_{ij}}{N} = 0.3759$$

$$\hat{\beta}_{0sij(ort)} = 69 - 0.3759 * 104.5 = 29.71549$$

And for the model;

$$\hat{Y} = 29.716 + 0.376x_i$$

Regression model computed according to Theil-2 method is found to be significant in 5% significance level, and the arithmetic mean of absolute deviations from the estimation values of bound variations is

$$MAD = \frac{\sum_{i=1}|Y_i - \hat{Y}_i|}{n} = 4.774$$

If we were to summarize the results in the table below after all the algorithms are applied, we can see which regression equation gives the best result for this application.

Method	Model	MAD
Least Squares	$\hat{Y} = 28.193 + 0.368x_i$	4.899
Mood-Brown	$\hat{Y} = 31.559 + 0.352x_i$	4.953
Theil-1	$\hat{Y} = 34.924 + 0.326x_i$	3.392
Theil-Opt	$\hat{Y} = 34.652 + 0.326x_i$	**3.378**
Theil-Hod.	$\hat{Y} = 32.522 + 0.326x_i$	4.560
Weighted Theil-1 (Mean)	$\hat{Y} = 26.752 + 0.404x_i$	5.823
Weighted Theil-1 (Median)	$\hat{Y} = 33.25 + 0.342x_i$	3.783
Weighted Theil-2 (Mean)	$\hat{Y} = 30.272 + 0.371x_i$	4.374
Weighted Theil-2 (Median)	$\hat{Y} = 32.002 + 0.354x_i$	4.106
Theil-2	$\hat{Y} = 29.716 + 0.376x_i$	4.774

Table 2. MAD values of estimators

When the Result Table is examined, the smallest mean of absolute deviations of the obtained estimate values through the applied regression methods is the Optimum type Theil method. As we have already pointed out in the theory of the study, when there are extreme values in the data, the Optimum type Theil method is supposed to come out better overall. The application carried out in this context supports this result. Theil-1 method and Weighted Theil-1 method based on median follows the Optimum type Theil method.

4. Conclusions and Discussion

We emphasized on the necessity to use alternative regression techniques when the distribution of error terms is not normal and that the least squares method is affected by the outliers within the observation values. As is known, when we are working with actual data, the biggest problem is that the distribution of data does not match that of the normal distribution. For this reason there is a need for robust regression methods. Because one of the assumptions of the least squares method is that the distribution of error terms is normal.

In the application section of the study, non-parametric simple linear methods are compared. After the acquisition of the results, the average absolute deviation value is computed. When the Figure 3.1 is examined, in the presence of extreme values in average absolute deviation values, the best results were gained with Optimum type Theil method. After that comes Theil-1, Weighted Theil-1 based on median, Weighted Theil-2 based on median, Weighted Theil-2 based on mean, Hodges-Lehmann type Theil, Theil-2, Least Squares Method, Mood-Brown method, and Weighted Theil-1 based on mean respectively.

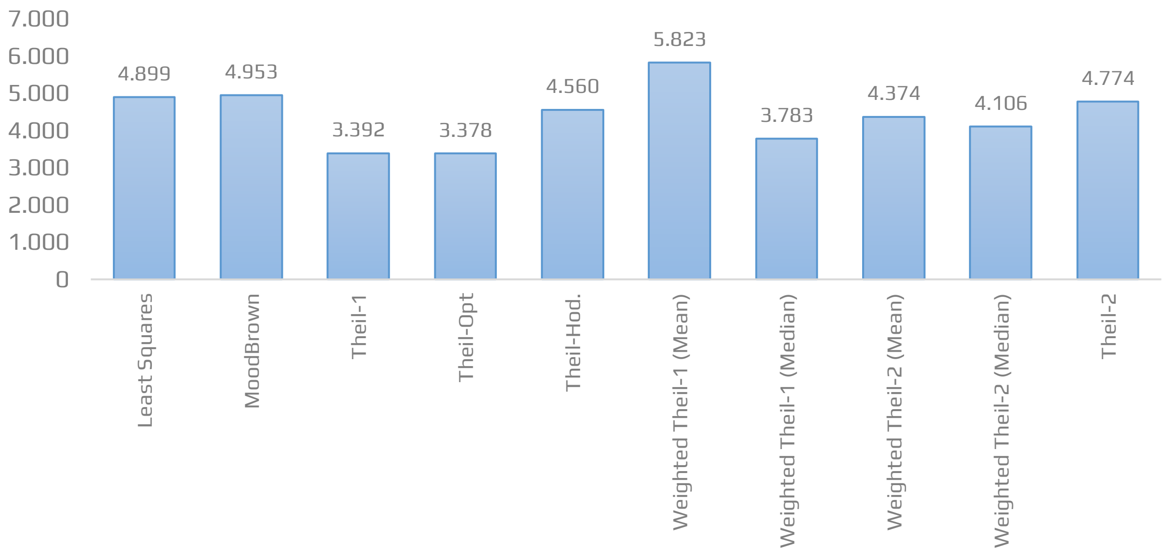

Figure 4. The status of the methods according to Mean Absolute Deviation

In light of all this information, when the mean absolute deviation is taken as a reference point for the application study, Optimum type Theil, Theil-1, Weighted Theil-1 based on median, Weighted Theil-2 based on median, Weighted Theil-2 based on mean, Hodges-Lehmann type Theil, and Theil-2 methods appeared to give better results than the Least Squares method. These results apply to this application. In different applications different results may be obtained. As for the suggestions, the resampling methods of bootstrap and jackknife may be backcrossed to these existing methods and the results may be compared again.

The fact that the non-parametric linear methods are less restrictive when compared to parametric linear estimation methods and the assumption that the sample population comes from only identified distribution in these methods contribute to wide use and spread of non-parametric methods. In this case, another topic that was impactful is the use of non-parametric methods in cases when the parametric methods are valid also. Non-parametric linear regression methods can be applied whenever parametric linear regression methods are valid. There is no restriction for this situation.

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