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POSITION VECTORS OF ADMISSIBLE CURVES IN 3-DIMENSIONAL PSEUDO-GALILEAN SPACE G_3^1

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Abstract. In this paper, position vectors of admissible curves in pseudo-Galilean space G_3^1 is studied in terms of Frenet equations. We compute the position vectors of admissible curves in pseudo-Galilean space G_3^1.Then we give some examples of position vectors for admissible curves.

1. Introduction

In the local differential geometry, curves are a geometric set of points, or locus. Intuitively, one can think a curve as the path traced out by a particle moving in Euclidean 3-space. So, to determine behaviour of the particle (or the curve, i.e.) we investigate position vectors of curves.

In the Euclidean space E^3, for each unit speed curve $\alpha : I \longrightarrow E^3$ with minimum four continuous derivatives, we can denote orthogonal unit vector fields t, n and b called, respectively, the tangent, the principal normal and the binormal vector fields. The planes spanned by $\{t, n\}$, $\{t, b\}$ and $\{n, b\}$ are called, respectively, osculating plane, rectifying plane and normal plane of the curve α. If position vector of $\alpha : I \subset \mathbb{R} \longrightarrow E^3$ always lie in its rectifying plane, the curves α are called rectifying curves. Similarly, the curves whose position vector α always lie in their osculating plane and their normal plane, are called ,respectively, osculating curves and normal curves. In [3] B.Y. Chen expressed characterization of rectifying curve . Then, the characterization of rectifying curves in Minkowski space is given in [6].

In the Euclidean space E^3 , the determination of parametric representation for position vector of arbitrary space with respect to intrinsic equations is still unknown [5,9]. Generally, to solve the above problem is difficult. But, the problem is solved some special case for example the event of a plane curve and a helix. Ali give some differential equation to solve the problem in the event of a general helix and slant helix in Minkowski 3- space [1,2]. Also, in Minkowski space position vectors of a spacelike W-curve is given in [8].

The aim of this study is to solve the problem for admissible curves in pseudo-Galilean 3-space G_3^1 . First of all, we define the position vector of an admissible

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curve according to the Frenet frame and then we obtain the position vector of an admissible curve according to standart frame in the way of curvature and torsion in pseudo-Galilean 3-space G_3^1 .

2. **Preliminaries**

The pseudo-Galilean geometry is one of the real Cayley-Klein geometries. As in [4] , pseudo-Galilean inner product can be written as

$$\langle v_1, v_2\rangle = \begin{cases} x_1x_2\ , & if \quad x_1 \neq 0 \ \vee \ x_2 \neq 0, \\ y_1y_2 - z_1z_2, & if \quad x_1 = 0 \ \wedge \ x_2 = 0, \end{cases}$$

where $v_1 = (x_1, y_1, z_1)$ and $v_2 = (x_2, y_2, z_2)$ and the pseudo-Galilean norm of the vector $v = (x, y, z)$ defined by

$$\|v\| = \begin{cases} x, & x \neq 0, \\ \sqrt{|y^2 - z^2|}, & x = 0. \end{cases}$$

A vector $v = (x, y, z)$ is in G_3^1 is said to be non-isotropic if $x \neq 0$, otherwise it is isotropic. All unit non-isotropic vectors are of the form $(1, y, z)$. There are four types of isotropic vectors : spacelike $\left(y^2 - z^2 > 0\right)$, timelike $\left(y^2 - z^2 < 0\right)$ and two types of lightlike $(y = \pm z)$ vectors. A non-lightlike isotropic vector is unit vector if $y^2 - z^2 = \pm 1$.

In pseudo-Galilean space a curve is given by

$$\alpha : I \longrightarrow G_3^1 \quad , \quad \alpha(t) = (x(t), y(t), z(t))$$

where $I \subseteq \mathbb{R}$ and $x(t), y(t), z(t) \in C^3$. A curve α given above is called an admissible curve if $x(t) \neq 0$.

The curves in pseudo-Galilean space are characterized as follows:

Type I.

An admissible curve $\alpha : I \subseteq \mathbb{R} \longrightarrow G_3^1$ can be parameterized by arc length $t = s$, given in coordinate form

$$\alpha(s) = (s, y(s), z(s)) . \tag{2.1}$$

Its curvature $\kappa(s)$ and torsion $\tau(s)$ are defined by

$$\kappa(s) = \sqrt{|y^2 - z^2|} \tag{2.2}$$

$$\tau(s) = \frac{\det(\alpha(s), \alpha(s), \alpha(s))}{\kappa^2(s)} \quad ,$$

The associated trihedron is given by

$$t(s) = \alpha(s) = (1, y(s), z(s)),$$

$$n(s) = \frac{1}{\kappa(s)}\alpha(s) = \frac{1}{\kappa(s)}(0, y(s), z(s)), \tag{2.3}$$

$$b(s) = \frac{1}{\kappa(s)}(0, z(s), y(s)) \quad .$$

The vectors $t(s)$, $n(s)$ and $b(s)$ are called the vectors of tangent, principal normal and binormal line of α , respectively. The curve α given by (2.1) is timelike, if $n(s)$

is spacelike vector. For derivatives of tangent vector $t(s)$, principal normal vector $n(s)$ and binormal vector $b(s)$, respectively, the following Frenet formulas hold

$$\begin{aligned} t(s) &= \kappa(s)n(s), \\ n(s) &= \tau(s)b(s), \\ b(s) &= \tau(s)n(s) \ . \end{aligned} \tag{2.4}$$

Type II.

An admissible curve $\beta : I \subseteq \mathbb{R} \longrightarrow G_3^1$ is given by $\beta(x) = (x, y(x), 0)$ and for this admissible curve, the curvature $\kappa(x)$ and the torsion $\tau(x)$ are defined by

$$\begin{aligned} \kappa(x) &= y(x), \\ \tau(x) &= \frac{a_2(x)}{a_3(x)}, \end{aligned} \tag{2.5}$$

where $a(x) = (0, a_2(x), a_3(x))$. The associated trihedron is given by

$$\begin{aligned} t(x) &= (1, y(x), 0), \\ n(x) &= (0, a_2(x), a_3(x)), \\ b(x) &= (0, a_3(x), a_2(x)). \end{aligned} \tag{2.6}$$

For tangent vector $t(x)$, principal normal vector $n(x)$ and binormal vector $b(x)$, the following Frenet formulas hold

$$\begin{aligned} t(x) &= \kappa(x)(\cosh\phi(x)n(x) - \sinh\phi(x)b(x)), \\ n(x) &= \tau(x)b(x), \\ b(x) &= \tau(x)n(x) \ . \end{aligned} \tag{2.7}$$

where ϕ is the angle between $a(x)$ and the plane $z = 0$ [4].

3. Position vectors of admissible curves in pseudo-Galilean space G_3^1

In this section, we give the position vectors of admissible curves according to Frenet frame in pseudo-Galilean space G_3^1 .

Theorem 3.1. Let $\alpha(x) = (x, y(x), z(x))$ be an admissible curve with curvature $\kappa(x)$ and torsion $\tau(x) \neq 0$ in G_3^1. Then its position vector is given by

$$\begin{aligned} \alpha(x) = \ & (x+c_1)t(x) + \left[c_2 - \frac{1}{2}(x+c_1)\kappa(x)e^{\tau(x)dx}dx\right]\left[e^{-\tau(x)dx}(n(x)+b(x))\right] \\ & + \left[c_3 + \frac{1}{2}(x+c_1)\kappa(x)e^{-\tau(x)dx}dx\right]\left[e^{\tau(x)dx}(b(x)-n(x)\right] \end{aligned} \tag{3.1}$$

where c_1 , c_2 and c_3 are arbitrary constants.

Proof. Let $\alpha(x) = (x, y(x), z(x))$ be an admissible curve in G_3^1. If $\lambda(x)$, $\mu(x)$ and $\gamma(x)$ are differentiable functions of $x\epsilon I \subset \mathbb{R}$, then we can write the position vector of α in the following form

$$\alpha(x) = \lambda(x)t(x) + \mu(x)n(x) + \gamma(x)b(x). \tag{3.2}$$

Differentiating the equation (3.2) with respect to x and considering the Frenet equations (2.4), we get

$$\lambda(x) - 1 = 0,$$

$$\lambda(x)\kappa(x)+\mu(x)+\gamma(x)\tau(x)=0, \tag{3.3}$$
$$\mu(x)\tau(x)+\gamma(x)=0 \quad .$$

Using the first equation of (3.3), we find

$$\lambda(x)=x+c_1, \tag{3.4}$$

where c_1 is an arbitrary constant. We can consider the variable $t=\tau(x)dx$. So, all functions of x will turn into the functions of t . The dot is used to denote the derivation with respect to t (prime is used to denote the derivative with respect to x) . We can write the third equation of (3.3) as follows

$$\mu(t)=-\dot{\gamma}(t) \tag{3.5}$$

Considering the equation (3.5) with the second equation of (3.3), we obtain

$$\ddot{\gamma}(t)-\gamma(t)=\frac{\lambda(t)\kappa(t)}{\tau(t)}. \tag{3.6}$$

Then the solution for the above equation is written

$$\gamma(t)=\left(c_2-\frac{1}{2}\frac{\lambda(t)\kappa(t)}{\tau(t)}e^{t}dt\right)e^{-t}+\left(c_3+\frac{1}{2}\frac{\lambda(t)\kappa(t)}{\tau(t)}e^{-t}dt\right)e^{t}, \tag{3.7}$$

where c_2 and c_3 are arbitrary constants. If we differentiate the equation (3.7) with respect to t and substituting this in the equation (3.5), we have

$$\mu(t)=\left(c_2-\frac{1}{2}\frac{\lambda(t)\kappa(t)}{\tau(t)}e^{t}dt\right)e^{-t}-\left(c_3+\frac{1}{2}\frac{\lambda(t)\kappa(t)}{\tau(t)}e^{-t}dt\right)e^{t}. \tag{3.8}$$

So, the equations (3.7) and (3.8) can be written

$$\gamma(x)=\left(c_2-\frac{1}{2}(x+c_1)\kappa(x)e^{\tau(x)dx}dx\right)e^{-\tau(x)dx} \tag{3.9}$$
$$+\left(c_3+\frac{1}{2}(x+c_1)\kappa(x)e^{-\tau(x)dx}dx\right)e^{\tau(x)dx},$$

$$\mu(x)=\left(c_2-\frac{1}{2}(x+c_1)\kappa(x)e^{\tau(x)dx}dx\right)e^{-\tau(x)dx} \tag{3.10}$$
$$-\left(c_3+\frac{1}{2}(x+c_1)\kappa(x)e^{-\tau(x)dx}dx\right)e^{\tau(x)dx}.$$

If we use the equations (3.4) ,(3.9) and (3.10) in (3.2) we obtain equation (3.1). □

Theorem 3.2. Let $\beta(x)=(x,y(x),0)$ be an admissible curve with constant ϕ angle and constant torsion $\tau(x)$ in G_3^1. Then its position vector is given by

$$\beta(x)=(x+c_1)t(x)+c_2e^{-\tau\coth\phi}n(x) \tag{3.11}$$

where c_1 , c_2 and c_3 are arbitrary constants.

Proof. Let $\beta(x)=(x,y(x),0)$ be an admissible curve in G_3^1. Then we write its position vector in the following form

$$\beta(x)=\lambda(x)t(x)+\mu(x)n(x) \tag{3.12}$$

where $\lambda(x)$ and $\mu(x)$ are differentiable functions of $x\epsilon I \subset \mathbb{R}$ .We can suppose τ, ϕ are constants. If we differentiate the above equation with respect to x and considering Frenet equations (2.7), we get

$$\begin{aligned} &\lambda(x) - 1 = 0, \\ &\lambda(x)\kappa \cosh\phi + \mu(x) = 0, \\ -&\lambda(x)\kappa \sinh\phi + \mu(x)\tau = 0. \end{aligned} \tag{3.13}$$

Using the first equation of (3.13), we find

$$\lambda(x) = x + c_1, \tag{3.14}$$

where c_1 is an arbitrary constant. If we use the second and third equation of (3.13), we have

$$\mu'(x) + \tau \coth\phi\mu = 0. \tag{3.15}$$

The general solution of these equation is

$$\mu(x) = c_2 e^{-\tau \coth\phi x} \tag{3.16}$$

Substituting equations (3.14), (3.16) to (3.12), we obtain equation (3.11) . □

4. Position vectors of admissible curves with respect to standart frame of G_3^1

Theorem 4.1. *Let $\alpha(x) = (x, y(x), z(x))$be an admissible curve with curvature $\kappa(x)$ and torsion $\tau(x)$ in the pseudo-Galilean space G_3^1.*

i) if α is an admissible curve with spacelike normal, then the position vector of α is given

(4.1)
$$\alpha(x) = \left(x, \int \left(\int \kappa(x)\cosh(\int \tau(x)dx)dx\right)dx, \left(\kappa(x)\sinh(\int \tau(x)dx)dx\right)dx\right).$$

ii) if α is an admissible curve with timelike normal, then the position vector of α is given

(4.2)
$$\alpha(x) = \left(x, \int \left(\int \kappa(x)\sinh(\int \tau(x)dx)dx\right)dx, \left(\kappa(x)\cosh(\int \tau(x)dx)dx\right)dx\right).$$

Proof. If $\alpha(x)$ is an admissible curve in G_3^1 , then from the second equation of (2.4) we obtain

$$b(x) = \frac{1}{\tau}n(x).$$

Using the third equation of (2.4) we have

$$\left(\frac{1}{\tau}n(x)\right) - \tau(x)n(x) = 0.$$

We can write the above equation by the form

$$\frac{d^2n}{dt^2} - n = 0, \tag{4.3}$$

where $t = \int \tau(x)dx$.

i) Let α be an admissible curve with spacelike normal. The principal normal vector can be written

$$n = (0, \cosh\theta(t), \sinh\theta(t)) \quad .$$

Considering the vector n in the equation (4.3) we have

$$\begin{aligned} \left(\dot{\theta}^2(t) - 1\right)\cosh\theta(t) + \ddot{\theta}(t)\sinh\theta(t) &= 0, \\ \left(\dot{\theta}^2(t) - 1\right)\sinh\theta(t) + \ddot{\theta}(t)\cosh\theta(t) &= 0. \end{aligned}$$

Using above equations we get

$$\dot{\theta}(t) = \pm 1 \quad , \quad \ddot{\theta}(t) = 0,$$

and from above equation we have $\theta(t) = \pm t = \pm \int \tau(x)dx$. We can take the positive sign for $\theta(t)$. Then the principal normal vector can be written

$$n(x) = \left(0, \cosh(\int \tau(x)dx)dx, \sinh(\int \tau(x)dx)dx\right).$$

Using the principal normal vector we have

$$t(x) = \int \kappa(x)\left(0, \cosh(\int \tau(x)dx), \sinh(\int \tau(x)dx)\right) + c,$$

where c is a constant vector. We can take $c = (1, 0, 0)$ because of the first component of tangent vector and then

$$t(x) = \left(1, \int \kappa(x)\cosh(\int \tau(x)dx)dx, \int \kappa(x)\sinh(\int \tau(x)dx)dx\right).$$

Using above equation we find

$$\alpha(x) = \int \left(1, \int \kappa(x)\cosh(\int \tau(x)dx)dx, \int \kappa(x)\sinh(\int \tau(x)dx)dx\right)dx$$

So the equation (4.1) is obtained.

ii) Let α be an admissible curve with timelike normal. The principal normal vector can be written

$$n = (0, \sinh(\theta(t)), \cosh(\theta(t))).$$

Considering n in the equation (4.3) we obtain

$$\begin{aligned} \left(\dot{\theta}^2(t) - 1\right)\sinh(\theta(t)) + \ddot{\theta}(t)\cosh(\theta(t)) &= 0, \\ \left(\dot{\theta}^2(t) - 1\right)\cosh(\theta(t)) + \ddot{\theta}(t)\sinh(\theta(t)) &= 0. \end{aligned}$$

Using above equations we get

$$\dot{\theta}(t) = \pm 1 \quad , \quad \ddot{\theta}(t) = 0,$$

and from above equation we have $\theta(t) = \pm t = \pm \int \tau(x)dx$. We can take the positive sign for $\theta(t)$. Then the principal normal vector can be written

$$n(x) = \left(0, \sinh\left(\int \tau(x)dx\right)dx, \cosh\left(\int \tau(x)dx\right)dx\right).$$

Using above equation we have

$$t(x)=\int\kappa(x)\left(0,\sinh\left(\int\tau(x)dx\right),\cosh\left(\int\tau(x)dx\right)\right)+c,$$

where c is a constant vector. We can take $c=(1,0,0)$ because of the first component of tangent vector and then

$$t(x)=\left(1,\int\kappa(x)\sinh\left(\int\tau(x)dx\right)dx,\int\kappa(x)\cosh\left(\int\tau(x)dx\right)dx\right).$$

Using above equation we obtain

$$\alpha(x)=\int\left(1,\int\kappa(x)\sinh\left(\int\tau(x)dx\right)dx,\int\kappa(x)\cosh\left(\int\tau(x)dx\right)dx\right)dx.$$

□

Theorem 4.2. *Let $\beta(x)=(x,y(x),0)$ be an admissible curve with curvature $\kappa(x)$ and torsion $\tau(x)$ in the pseudo-Galilean space G_3^1.*

i) if β be an admissible curve with spacelike normal, then the position vector of β is given

$$\beta(x)=\left(x,\int\left[\int\kappa(x)\left(\cosh\phi\cosh\left(\int\tau(x)dx\right)-\sinh\phi\sinh\left(\int\tau(x)dx\right)\right)dx\right]dx,\right.$$

$$\left.\int\left[\int\kappa(x)\left(\cosh\phi\sinh\left(\int\tau(x)dx\right)-\sinh\phi\cosh\left(\int\tau(x)dx\right)\right)dx\right]dx\right) \tag{4.4}$$

ii) if β be an admissible curve with timelike normal, then the position vector of β is given

$$\beta(x)=\left(x,\int\left[\int\kappa(x)\left(\cosh\phi\sinh\left(\int\tau(x)dx\right)-\sinh\phi\cosh\left(\int\tau(x)dx\right)\right)dx\right]dx,\right.$$

$$\left.\int\left[\int\kappa(x)\left(\cosh\phi\cosh\left(\int\tau(x)dx\right)-\sinh\phi\sinh\left(\int\tau(x)dx\right)\right)dx\right]dx\right) \tag{4.5}$$

Proof. **i)** Let β be an admissible curve with spacelike normal. If $\beta(x)$ is an admissible curve in G_3^1 , then the Frenet equations (2.7) are hold. From the second equation of (2.7), we have

$$b(x)=\frac{1}{\tau}n(x)\ .$$

Using the third equation of (2.7), we have

$$\left(\frac{1}{\tau}n(x)\right)-\tau(x)n(x)=0\quad .$$

So the above equation can be written

$$\frac{d^2n}{dt^2}-n=0, \tag{4.6}$$

where $t=\int\tau(x)dx$ .The principal normal vector can be written as follows

$$n=(0,\cosh\theta(t),\sinh\theta(t)).$$

If we use the vector n in the equation (4.6) we obtain

$$\begin{aligned}\left(\overset{.}{\theta}^{2}(t)-1\right)\cosh\theta(t)+\overset{..}{\theta}(t)\sinh\theta(t) &= 0.\\ \left(\overset{.}{\theta}^{2}(t)-1\right)\sinh\theta(t)+\overset{..}{\theta}(t)\cosh\theta(t) &= 0.\end{aligned}$$

Then we get

$$\overset{.}{\theta}(t)=\pm1 \quad , \quad \overset{..}{\theta}(t)=0,$$

and from above equation we have $\theta(t)=\pm t=\pm\int\tau(x)dx$. We can take the positive sign for $\theta(t)$. Then

$$n(x)=(0,\cosh\left(\int\tau(x)dx\right)dx,\sinh\left(\int\tau(x)dx\right)dx).$$

Since $\beta(x)$ is an admissible curve in G_3^1 , the Frenet equations (2.7) are hold. From the third equation (2.7), we have

$$n(x)=\frac{1}{\tau}b(x).$$

If we put the above equation in the second equation of (2.7) we obtain the differential equation with respect to principal normal vector n

$$\left(\frac{1}{\tau}b(x)\right)-\tau(x)b(x)=0.$$

The above equation can be written as follows

$$\frac{d^2b}{dt^2}-b=0, \tag{4.7}$$

where $t=\int\tau(x)dx$ .We can write the binormal vector in the following form

$$b=(0,\sinh\theta(t),\cosh\theta(t)).$$

Considering the second and the third components from the vector n in the equation (4.7) we obtain

$$\begin{aligned}\left(\overset{.}{\theta}^{2}(t)-1\right)\sinh\theta(t)+\overset{..}{\theta}(t)\cosh\theta(t) &= 0,\\ \left(\overset{.}{\theta}^{2}(t)-1\right)\cosh\theta(t)+\overset{..}{\theta}(t)\sinh\theta(t) &= 0.\end{aligned}$$

So, using the above equations we get

$$\overset{.}{\theta}(t)=\pm1 \quad , \quad \overset{..}{\theta}(t)=0,$$

and from above equation we have $\theta(t)=\pm t=\pm\int\tau(x)dx$. We can take the positive sign for $\theta(t)$. Then the principal normal vector is written as follows

$$b(x)=\left(0,\sinh\left(\int\tau(x)dx\right)dx,\cosh\left(\int\tau(x)dx\right)dx\right) \quad .$$

Using first equation of (2.7), we can write

$$\begin{aligned} t(x) &= \kappa(x)\cosh\phi\left(0,\cosh\left(\int\tau(x)dx\right)dx,\sinh\left(\int\tau(x)dx\right)dx\right) \\ &\quad -\kappa(x)\sinh\phi\left(0,\sinh\left(\int\tau(x)dx\right)dx,\cosh\left(\int\tau(x)dx\right)dx\right). \end{aligned}$$

If we integrate the above equation with respect to x , we have the equation (4.4).

ii) Let β be an admissible curve with timelike normal. If $\beta(x)$ is an admissible curve in G_3^1 , then the Frenet equations (2.7) are hold. From the second equation of (2.7), we obtain

$$b(x)=\frac{1}{\tau}n(x)\ .$$

Considering the above equation to the third equation of (2.7) we obtain

$$\left(\frac{1}{\tau}n(x)\right)-\tau(x)n(x)=0.$$

We can write the above equation in the following form

$$\frac{d^2n}{dt^2}-n=0, \tag{4.8}$$

where $t=\int\tau(x)dx$. The principal normal vector can be written

$$n=(0,\sinh\theta(t),\cosh\theta(t))\quad .$$

Using the equation (4.8) we have

$$\begin{aligned} \left(\dot{\theta}^2(t)-1\right)\sinh\theta(t)+\ddot{\theta}(t)\cosh\theta(t) &= 0, \\ \left(\dot{\theta}^2(t)-1\right)\cosh\theta(t)+\ddot{\theta}(t)\sinh\theta(t) &= 0. \end{aligned}$$

So,

$$\dot{\theta}(t)=\pm1\quad,\quad\ddot{\theta}(t)=0,$$

and from above equation we have $\theta(t)=\pm t=\pm\int\tau(x)dx$. We can take the positive sign for $\theta(t)$. Then

$$n(x)=\left(0,\sinh\left(\int\tau(x)dx\right)dx,\cosh\left(\int\tau(x)dx\right)dx\right).$$

Since $\beta(x)$ is an admissible curve in G_3^1 . From the third equation (2.7), we have

$$n(x)=\frac{1}{\tau}b(x).$$

Considering the second equation of (2.7) we have

$$\left(\frac{1}{\tau}b(x)\right)-\tau(x)b(x)=0.$$

The above equation can be written

$$\frac{d^2b}{dt^2}-b=0, \tag{4.9}$$

where $t=\int\tau(x)dx$ .Thus

$$b=(0,\cosh\theta(t),\sinh\theta(t))\quad .$$

Using the equation (4.9) we have

$$\begin{aligned}\left(\dot{\theta}^{2}(t)-1\right)\cosh\theta(t)+\ddot{\theta}(t)\sinh\theta(t) &= 0,\\ \left(\dot{\theta}^{2}(t)-1\right)\sinh\theta(t)+\ddot{\theta}(t)\cosh\theta(t) &= 0.\end{aligned}$$

Then

$$\dot{\theta}(t)=\pm 1 \quad , \quad \ddot{\theta}(t)=0,$$

and from above equation we have $\theta(t)=\pm t=\pm\int\tau(x)dx$. We can take the positive sign for $\theta(t)$. Then

$$b(x)=\left(0,\cosh\left(\int\tau(x)dx\right)dx,\sinh\left(\int\tau(x)dx\right)dx\right) .$$

Using first equation of (2.7), we can write

$$\begin{aligned}t(x) &= \kappa(x)\cosh\phi\left(0,\sinh\left(\int\tau(x)dx\right)dx,\cosh\left(\int\tau(x)dx\right)dx\right)\\ &\quad -\kappa(x)\sinh\phi\left(0,\cosh\left(\int\tau(x)dx\right)dx,\sinh\left(\int\tau(x)dx\right)dx\right).\end{aligned}$$

If we integrate the above equation with respect to x , we get the equation (4.5) . □

Example 4.1. Let α be a straight line with respect to the Frenet frame in G_3^1. If we take $\kappa(x)=0$ and consider this in the equation (4.1) and (4.4), then its position vector can be written

$$\alpha_1(x)=(x,c_1x+c_2,c_3x+c_4)$$

and

$$\alpha_1(x)=(x,c_1x-c_2x+c_3,c_4x+c_5x+c_6),$$

respectively, where c_i , $i=1,2,3,4,5,6$ are arbitrary constants.

Example 4.2. Let β be a planar curve with respect to the Frenet frame in G_3^1. If we take $\tau(x)=0$ and consider this in the equation (4.1) and (4.2), then its position vector can be written

$$\beta_3(x)=\left(x,\cosh\eta\int\left(\int\kappa(x)\,dx\right)dx,\sinh\eta\int\left(\int\kappa(x)\,dx\right)dx\right)$$

and

$$\beta_4(x)=(x,\sinh\eta\int\left(\int\kappa(x)\,dx\right)dx,\cosh\eta\int\left(\int\kappa(x)\,dx\right)dx),$$

respectively, where η is arbitrary constant. If we take $\tau(x)=0$ and consider this in the equation (4.4) and (4.5), then its position vector can be written

$$\begin{aligned}\beta_5(x) &= \left(x,\nu\int\left(\int\kappa(x)\cosh\phi(x)dx\right)dx-\delta\int\left(\int\kappa(x)\sinh\phi(x)dx\right)dx,\right.\\ &\quad \left.\delta\int\left(\int\kappa(x)\cosh\phi(x)dx\right)dx-\nu\int\left(\int\kappa(x)\sinh\phi(x)dx\right)dx\right)\end{aligned}$$

and

$$\begin{aligned}\beta_6(x) &= \left(x, \delta \int \left(\int \kappa\,(x) \cosh \phi(x) dx\right) dx - \nu \int \left(\int \kappa\,(x) \sinh \phi(x) dx\right) dx,\right.\\ &\quad \left.\nu \int \left(\int \kappa\,(x) \cosh \phi(x) dx\right) dx - \delta \int \left(\int \kappa\,(x) \sinh \phi(x) dx\right) dx\right),\end{aligned}$$

respectively, where η, ν and δ are arbitrary constants, $\cosh\,[\eta] = \nu$ and $\sinh\,[\eta] = \delta$.

Example 4.3. Let γ be an admissible curve with $\kappa(x) = const.$ and $\tau(x) = const.$in pseudo-Galilean space G_3^1 . If we take $\kappa(x)$ and $\tau(x)$ are constants and put it in the equation (4.1) and (4.2), we obtain

$$\gamma_1 = \left(x, \frac{\kappa}{\tau^2} \cosh\,(\tau x)\,, \frac{\kappa}{\tau^2} \sinh\,(\tau x)\right)$$

and

$$\gamma_2 = \left(x, \frac{\kappa}{\tau^2} \sinh\,(\tau x)\,, \frac{\kappa}{\tau^2} \cosh\,(\tau x)\right),$$

respectively.

If we take $\kappa(x)$ and $\tau(x)$ are constants and consider this in the equation (4.4) and (4.5), we get

$$\begin{aligned}\gamma_3 &= \left(x, \kappa \int \left(\int (\cosh \phi(x) \cosh\,(\tau x) - \sinh \phi(x) \sinh\,(\tau x))\, dx\right) dx,\right.\\ &\quad \left.\kappa \int \left(\int (\cosh \phi(x) \sinh\,(\tau x) - \sinh \phi(x) \cosh\,(\tau x))\, dx\right) dx)\right)\end{aligned}$$

and

$$\begin{aligned}\gamma_4 &= \left(x, \kappa \int \left(\int (\cosh \phi(x) \sinh\,(\tau x) - \sinh \phi(x) \cosh\,(\tau x))\, dx\right) dx\ ,\right.\\ &\quad \left.\kappa \int \left(\int (\cosh \phi(x) \cosh\,(\tau x) - \sinh \phi(x) \sinh\,(\tau x))\, dx\right) dx\right),\end{aligned}$$

respectively.

Example 4.4. Let ψbe an admissible general helix in pseudo-Galilean space G_3^1. Then if we take $\tau\,(x) = m\kappa\,(x)$, where m is arbitrary constant and consider this in the equation (4.1) and (4.2), we get

$$\psi_1 = \left(x, \int \left[\int \kappa(x) \left(\cosh\left(m \int \kappa(x) dx\right), \sinh\left(m \int \kappa(x) dx\right)\right) dx\right] dx\right)$$

and

$$\psi_2 = \left(x, \int \left[\int \kappa(x) \left(\sinh\left(m \int \kappa(x) dx\right), \cosh\left(m \int \kappa(x) dx\right)\right) dx\right] dx\right),$$

respectively.

If we take $\tau\,(x) = m\kappa\,(x)$, where m is arbitrary constant and consider this in the equation (4.4) and (4.5), we get

$$\begin{aligned}\psi_3 &= (x, \int \left[\int \kappa(x) \left(\cosh \phi \cosh\,[\xi(x)] - \sinh \theta \sinh\,[\xi(x)]\right) dx\right] dx,\\ &\quad \int \left[\int \kappa(x) \left(\cosh \phi \sinh\,[\xi(x)] - \sinh \theta \cosh\,[\xi(x)]\right) dx\right] dx)\end{aligned}$$

and

$$\begin{aligned}\psi_4 &= (x, \int \left[\int \kappa(x) \left(\cosh\phi \sinh\left[\xi(x)\right] - \sinh\theta \cosh\left[\xi(x)\right] \right) dx \right] dx, \\ &\int \left[\int \kappa(x) \left(\cosh\phi \cosh\left[\xi(x)\right] - \sinh\theta \sinh\left[\xi(x)\right] \right) dx \right] dx),\end{aligned}$$

respectively, where $m \int \kappa(x) dx = \xi(x)$.

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