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Research Article

Spatio-Spectral Analysis of Electromagnetic Wave Propagation

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Abstract: Numerical methods such as Finite Difference Time Domain Method (FDTD) solution of Electromagnetic Waves (EMW) are particularly useful for transient analysis of EMW propagation. However, transient analysis methods are not effective for analysis of spectral properties of wave propagation in anisotropic media. In order to consider frequency dependence of material parameters (ε_a-permittivity, μ_a-permeability and σ- conductivity), frequency-depended FD-FDTD methods ((FD)^{2}TD) are proposed. Besides, high computational complexity and stability problems of solution, relevance of solution is questionable. There is a need for straight forward analytical solution for perform spatio-spectral investigation of basic structures and the verification of simulation results. This study presents a solution of Maxwell equations for directional wave propagation in media where dielectric constant expressed with respect to wave frequency. This provides low complexity fundamental analyses of EMW propagation in homogeneous material.

Keywords: *Maxwell equation, Spatio-spectral analysis, Field phasors, EM wave propagation.*

Elektromanyetik Dalga Yayılımının Uzaysal-Spektral Analizi

Özet: Farklı elektrofiziksel özelliklere sahip lineer, homojen ve izotrop ortamlarda elektromanyetik dalga (EMW) yayılmasının transient analizi için zamanda sonlu farklar metodu (FDTD) gibi sayısal yöntemler önem arz etmektedir. Bununla birlikte, transient analiz yöntemleri, anizotropik ortamdaki dalga yayılımının spektral özelliklerinin analizi için pek etkili değildir. Bu nedenle, elektromanyetik dalganın yayıldığı bir ortamda, ortam parametrelerinin frekansa bağımlılığını (ε_a-elektrik geçirgenlik, μ_a-manyetik geçirgenlik ve σ-iletkenlik) dikkate almak için, frekans bağımlı FD-FDTD yöntemleri ((FD)^{2}TD) önerilmiştir. Ayrıca, problemin çözümü esnasında oluşan yüksek işlemsel karmaşıklık ve kararlılık problemleri, çözümden elde edilen sonuçların sorgulanmasını gerektirmektedir. Temel yapıların uzaysal-spektral incelemesi ve simülasyon sonuçlarının doğrulanması için ileri analitik çözüme ihtiyaç vardır. Bu çalışmada, dielektrik sabitinin dalga frekansına göre ifade edildiği ortamlarda yönlü dalga yayılımı için Maxwell denklemlerinden hareketle elde edilen çözüm verilmiştir. Elde edilen sonuçlar, düşük karmaşıklıktaki bir homojen yapıya sahip bir materyalde EM dalga yayılımının temel analizlerine olanak sağlar.

Anahtar kelimeler: *Maxwell denklemi, Uzaysal-spektral analiz, Alan fazörleri, EM dalga yayılımı.*

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1. Introduction

Since conductivity and dielectric constant of material in nature shows dependence on the frequency of waves, it is more convenient to express analytical solutions of electromagnetic wave propagation regarding frequency dependency of conductivity and dielectric constant. Although numerical transient EM wave simulations such as based on FDTD and FD-FDTD were effectively used for spatial-temporal analysis of wave propagation, it presents complication of high computational complexity, solution stability and verification problems. Transient simulation may not yield appropriate solution for spectral analysis.

Consideration of frequency dependence of electrical parameters makes modeling and analysis efforts of EM wave propagation in real materials more consistent. Particularly, for the analysis of wave propagation in anisotropic media is needed in fieldssuch as in communication [1], medical technologies [2-4], heating systems [5-8], energy conversion [9].

Electrical properties of biological tissues strongly depend on frequency of waves [10]. Experimental works shows that dielectric constant of tissues can decrease 2-3 times and conductivity of tissues can increase 2-3 times in 40-400 MHz [11].

This study presents a solution of Maxwell equations for one-directional wave propagation in homogenous medias where material conductivity is taken into account and the dielectric constant is assumed to be depended of frequency. Phasor solutions of the field components of EM waves is obtained and temporal evolution of field components are calculated for any point of material at nay frequency. This provides low complexity fundamental spatio-spectral analyses of EM propagation in materials.

2. Method

By considering electrical conductivity, Maxwell equations was given as

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \quad (1a)$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \quad (1b)$$

Where $\boldsymbol{B} = \mu_a \boldsymbol{H}$ and the current density is $\boldsymbol{J} = \sigma \boldsymbol{E}$ and σ is conductivity of medium. Electric displacement field at the right-hand side of equation (1a) was expressed in time domain in the form of $\boldsymbol{D} = \varepsilon_a \boldsymbol{E}$. In this case, equations (1a) and (2a) can be rearranged as

$$\nabla \times \boldsymbol{H} = \sigma \boldsymbol{E} + \frac{\partial \boldsymbol{D}}{\partial t} \quad (2a)$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -\mu_a \frac{\partial \boldsymbol{H}}{\partial t} \quad (2b)$$

Then, one can take curl of the both side of equation (2) and obtains,

$$\nabla \times \ (\nabla \times \boldsymbol{H}) = \nabla \times \left(\sigma \boldsymbol{E} + \frac{\partial \boldsymbol{D}}{\partial t}\right) \quad (3)$$

If $r\nabla \times \ (\nabla \times \boldsymbol{H}) = -\nabla^2 \boldsymbol{H}$ is considered, equation (3) can be expressed in time domain for a wave propagation along x axis by following equation,

$$\frac{d^2 \dot{H}}{dx^2} - \sigma \mu_a \frac{d\dot{H}}{dt} + \mu_a \varepsilon_a \frac{d^2 \dot{H}}{dt^2} = 0 \quad (4)$$

Then, equation (4) can be written in frequency domain as follows,

$$\frac{d^2 \dot{H}}{dx^2} - (\omega^2 \mu_a \varepsilon_a + j\omega\sigma\mu_a)\dot{H} = 0 \quad (5)$$

Let us assume a possible the solution of equation (5) in the form of

$$\dot{H} = Ae^{-bx} + Be^{bx} \quad (6)$$

In this case, we found $b^2 = (\omega^2 \mu_a \varepsilon_a + j\omega\sigma\mu_a)$. For $x \to \infty$, the field H should be limited. Hence $B = 0$. On the other hand, tangential component should be continuous, and the magnitude of H should be source magnitude H_0 at the position $x = 0$, So, one can take $A = H_0$ and obtains phasor of magnetic field ($\dot{H}$) with respect to wave frequency as,

$$\dot{H}(\omega, x) = \ H_0 \exp(-\sqrt{(\omega^2 \mu_a \varepsilon_a + j\omega\sigma\mu_a)}x) \quad (7)$$

For plane waves $\dot{E} = Z_c \dot{H}$ and characteristic impedance of dispersive medium is written as

$$\dot{z}_c = \frac{\omega\mu_a}{\gamma} = \sqrt{\frac{\mu_a}{\tilde{\varepsilon_a}}}.$$

Here, the complex dielectric constant with wave frequency dependency can be written as $\tilde{\varepsilon}_a = \varepsilon_a - j\frac{\sigma}{\omega}$.One can obtains the phasor electrical field E with respect to wave frequency as,

$$\dot{E}(\omega, x) = \ \sqrt{\frac{\mu_a}{(\varepsilon_a - j\frac{\sigma}{\omega})}} H_0 \exp(-\sqrt{(\omega^2 \mu_a \varepsilon_a + j\omega\sigma\mu_a)}x) \quad (8)$$

The instantaneous electric field is obtained by real part of products of phasor electric field amplitude and the time-dependence factor $\exp(j\omega t)$ as,

$$E(x, t) = Re\{\dot{E}(\omega, x) \exp(j\omega t)\} \quad (9)$$

This spatial-spectral solution is valid under monochromatic EM waves. The wide-band wave propagation can be analyzed by considering wide-band wave propagation as the superposition of multi frequency waves for linear medias. So, it can be written for wide-band wave propagation as

$$E(x, t) = \int_{\omega_{min}}^{\omega_{max}} Re\{\dot{E}(\omega, x) \exp(j\omega t)\} d\omega \quad (10)$$

3. An Example of Spatio-Spectral Analysis

This illustrative example presents a spatio-spectral analysis for a material with $\varepsilon_a = 1.2\varepsilon_0$, $\sigma = 0.64\ S/m$ and $\mu_a = \mu_0$. The material field is exited with $H_0 = 1$. Figure 1 shows spatio-spectral distribution of phasor field magnitudes. These results were obtained for angular frequency range of (0, 100.000) rad/s are material thickness with range of $(0,\ 5 \cdot 10^{-4})$ m. Figure 1 clearly shows that field penetration into material decrease as frequency of EM wave increases. Figure 2 shows this effect for various frequencies. As the EM frequency increases, magnitude of phasor fields decreases faster.

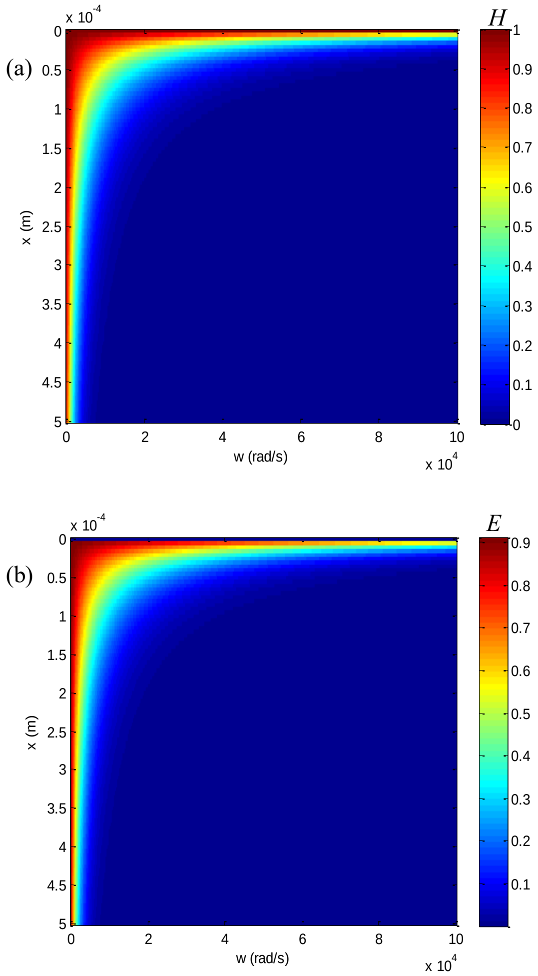

Figure 1. Spatio-Spectral distribution of magnitude of field phasors $\dot{H}$ in (a) and $\dot{E}$ in (b).

So, analyzing equation (8) it is not difficult to see that the amplitude of harmonic oscillations inside a well-conducting medium decreases exponentially with distance from the interface, while the phase varies linearly. The field and currents are concentrated in the layer immediately adjacent to the interface.

Temporal evolution of fields can be easily calculated by using equation (9) for any point of the material at any frequency. For instants, instantaneous electric field calculated for ω=99000 rad/s and $x = 9.9 \cdot 10^{-5}\ m$ was shown in Figure 3.

E(t) for ω=99000 rad/s and $x = 9.9 \cdot 10^{-5}\ m$.

4. Conclusions

For analytical analysis of EM wave propagation in materials, a spatio-spectral solution of Maxwell Equations was shown. The analysis considers the assumptions that the homogenous material has conductivity and dielectric constant is depended of frequency. The phasor solutions of field components can be used to obtain temporal evolution of field components at any point of material and EM wave frequency. These spatio-spectral analyses can provide fundamental analytical solutions for the estimation of wave propagation properties in structure and it can be used for the verification of frequency-depended FDFD simulations. Once, simulation results are in agreement with analytical results for homogenous materials, simulation of more complex structures can be carried out confidently.

Future study can be conducted to obtain spatio-spectral solutions for EM wave propagation in inhomogeneous materials.

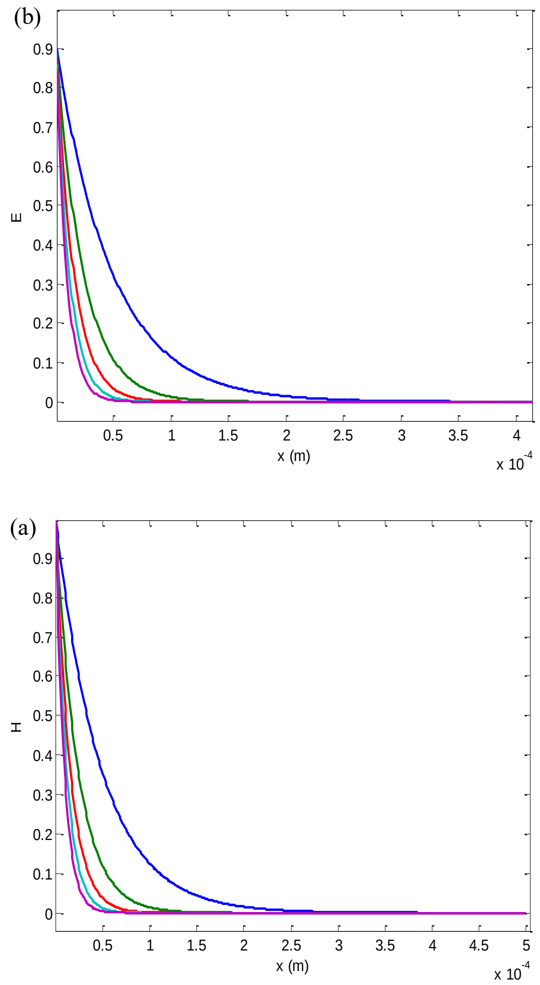

Figure 2. Spatial distribution magnitude of field phasors $\dot{H}$ in (a) and $\dot{E}$ in (b) for various frequency.

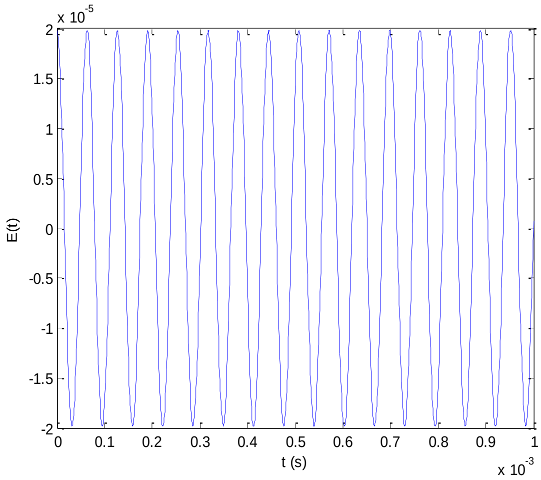

Figure 3. Temporal evolution of electric field component.

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