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From Congruent Angles to Congruent Triangles: The Role of Dragging, Grid and Angle Tools of a Dynamic Geometry System*

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Abstract

This study aims to establish a conceptual relationship between the angles of two parallel lines intersected by a transversal and the congruent triangles formed through the points on the parallel lines and the transversal. At this point, the study considers semiotic potential of dragging, grid and angle tools of a dynamic geometry system. The study was designed according to qualitative paradigm, and the collected data was analyzed through the techniques used in the same perspective. Within the scope of the study, an instructional task was designed by employing the tools used in a dynamic geometry system and its functions. This task was expected to enable the participants to make a successful conceptual bridging by using their already existing background knowledge. In addition, two sessions of 25-minute clinical interviews were conducted with two students – one from 7th and one from 8th grade – who were selected according to the principles of purposeful sampling method. The findings obtained from qualitative data analysis show that the designed task can be used as a tool for students to figure out conceptual relationships between congruent angles and congruent triangles. The results clearly revealed that the students went through different cognitive processes while using the dragging tool. Generally speaking, the findings are consistent with the findings of similar studies in the literature, and some suggestions were proposed under the light of these findings.

Keywords

Congruent Angles, Congruent Triangles, Dynamic Geometry System, Semiotic Potential.

Euclid's Elements are known to be one of the earliest works that display a careful hierarchical structure built on logic, verification and proof. The geometrical concepts dealt with in this work are built up *in relation* to each other. Thus, emphasizing such

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making connection point, National Council of Mathematics Teachers in the USA in 2000 defined certain process standards in mathematical education, one of which was making connection between different concepts and different representation. Accordingly, it was recommended that teaching environments should be designed in a way to enable geometry learners to make conceptual connections (Van de Walle, Karp, & Bay-Williams, 2010). It is also stated that the concepts in textbooks and curriculums are to be presented hierarchically because they allow learners to move and interplay in various contexts and internalize different concepts and representations as a whole accordingly.

For the development of ability of making connection, teaching-learning environments should enable students a context for transition from their previous phenomenological experiences to formal mathematics (i.e. from their informal experiences to formal mathematics) (Freudenthal, 1983). By doing so, it might be possible to help learners to develop mathematical meanings and ideas through making connection existing concept images (Tall & Vinner, 1981). Addressing the importance of making connection, Narlı (2016), points four interrelated factors as making connection types (p. 235–241):

- Bridging between new and existing information,
- Relating mathematical concepts to each other,
- Relating concepts to daily life experiences,
- Making connection within different disciplines.

Common point in the interrelated factors above is transforming and extending students' existing knowledge into new mathematical meanings by considering students' pre-knowledge and possible misconceptions. What about the design of teaching learning environments and as well as the teacher's role? In this work, we refer to *dynamic geometry environments* to make connection between students' pre-knowledge and new meanings as will be elaborated in methods section.

One of the topics covered in geometry in lower secondary schools is the angles formed by two parallel lines intersected by a transversal. It is known that learners face epistemological difficulties while learning about this topic, make some mistakes and have a number of misconceptions (Yılmaz & Nasibov, 2012). At this point, Baykul (2014) suggests the use of computers while teaching the angles formed by two parallel lines intersected by a transversal so that they can make effective reasoning and will not experience misconceptions. This study further aims to help students to make an epistemological connection between corresponding angles and the congruent triangles formed on a geometric structure. It is believed that students can internalize the concepts related to the angles formed by two parallel lines and intersected by a transversal.

Since it is thought that *dynamic variation* could be heuristic tool for students to relate congruent triangles to corresponding alternate interior and alternate exterior angles, the study focuses on a specific Dynamic Geometric Environment (DGE), in particular GeoGebra. This environment creates a geometric manipulation context, which is virtual but based on theoretical backgrounds of Euclidean Geometry. The basic factor leading to dynamic variety in a DGE is *dragging tool*, which enables learners to move the drawn and formed objects and create an environment where they can make reasoning, generalizations and establish conjectures and validate them (Leung, 2008; Leung, Baccaglini-Frank, & Mariotti, 2013; Lopez-Real & Leung, 2006). In addition, DGE context allows students to make mathematics, make hypotheses and test them by themselves. To illustrate, GeoGebra includes various tools and functions, and also has Algebra window which has been designed according to other synchronous windows. In addition to dragging tool, we focus on grid function and Angle tool and consider the notion of *semiotic potential* (Bartolini Bussi & Mariotti, 2008). Integrating grid and angle tools of GeoGebra (as will be explained later) to dragging context, we designed an environment, where students could make connection between existing and new mathematical notions. In sum, the aim of this work, to design and test an environment that allows students to conceptually relate corresponding alternate interior and alternate exterior angles to the congruent triangles.

Methods

This study adopted a qualitative paradigm in order to explore the role of dragging, grid and angle tools while bridging between congruent angles to congruent triangles. In this sense, task-based clinical interviews (Maher & Sigley, 2014) were conducted. The participants of the study were selected by using purposeful sampling method, which is one of the non-random sampling methods. Accordingly, A, one student from 7^{th} grade (male, 13 year-old) and B, one student from 8^{th} grade (male, 14 year-old) were chosen by considering the following criterions: mathematical performance, DGE experience and self-expression skills. Each application of the task with A and B lasted approximately 25 minutes. The designed instructional task was used as a tool for data collection. The application process was audio-recorded and screenshots were taken when necessary. All the data collected were analyzed qualitatively in three different phases expressed by (Leung, 2011): "Explore", "Reconstruct" and "Explain". Leung does not suggest these three phases as an analytical lens, but the researchers of this paper discussed their usability in the context of the study. Here, "explore" phase refers to the phase when learners discover the invariants in DGE and manipulate dependent and independent objects through dragging test. "Reconstruct" phase includes drawing and formation processes realized by using various tools and their functions of the dynamic geometry system. Finally, "explain" phase is about mathematical discourse regarding the situation in hand. Although these three phases are not linear, they can be observed in dialectical relationships intermittently or directly.

The Task and Design Process

As part of the design process, the researchers planned to consider specific tools of a dynamic geometry system. GeoGebra was chosen for the purposes of the study because it is free, easily accessible and have Turkish language support. In addition, it is useful for instructional design focusing on relating congruent angles to congruent triangles since it includes certain tools allowing users to draw parallel lines and a transversal, as well as polygons on a plane, mark the existing angles and apply transformations. This task was designed because of dragging and angle tools and grid function of GeoGebra are thought to make connection between congruent angles and congruent triangles; that is, they have *semiotic potential* (Bartolini Bussi & Mariotti, 2008). The semiotic potential of these tools can be summarized as follows:

- *Dragging Tool:* This tool is the basic component of DGE that allows users of dynamic geometry system move the geometrical objects and manipulate them. Since it is possible to manipulate the shapes and the points to give them new shapes and location by using this tool, it is thought to be a useful tool for students to discover the relationship among mathematical situations presented to them and make generalizations accordingly. Dragging tool is also believed to have semiotic potential for exploring parallel lines and intersected by a transversal, exploring a number of cases of triangles, analyzing and comparing the situations.

— *Grid:* This tool builds up parallel lines on axes and can play an effective role in figuring out certain situations such as whether the shapes and drawings made by using dragging tool are similar or not, or when they are symmetrical. The Lower Secondary School Mathematics Curriculum published by MEB (Ministry of National Education) (2013) and the course book used include activities focusing on using unit squares while determining congruent angles. Thus, it is believed that grid function of GeoGebra used in this task is in parallel with phenomenological experiences of students.

— *Angle Tool*: It is a tool used to measure the degree of angles. It allows users to monitor the changing angles when they are manipulated with the dragging tool. It was included in this study because it is believed that it can be used to check whether intuitively determined congruent angles are really congruent or not.

In conclusion, in the context of GeoGebra, a specific environment was designed for the participants. It should be noted that the students already knew about congruent angles and parallel lines but were not competent with the use of GeoGebra software. The steps mentioned below were followed, and Figure 1 was obtained accordingly.

i. Two parallel lines were drawn: *g* line that passes through D and C points; and *h* line that passes through E and G points. Later, an *f* line was drawn which passes through A and B points and intersection with these lines.

ii. DEB triangle was formed by joining D, E and B points on *g* and *h* lines.

iii. The middle point of A and B, which is the intersection point of *g* and *h* lines with *f* line, was determined as F.

iv. The triangle formed in step "ii" was reflected according to F point, forming the ADE triangle.

v. DEB angle was measured.

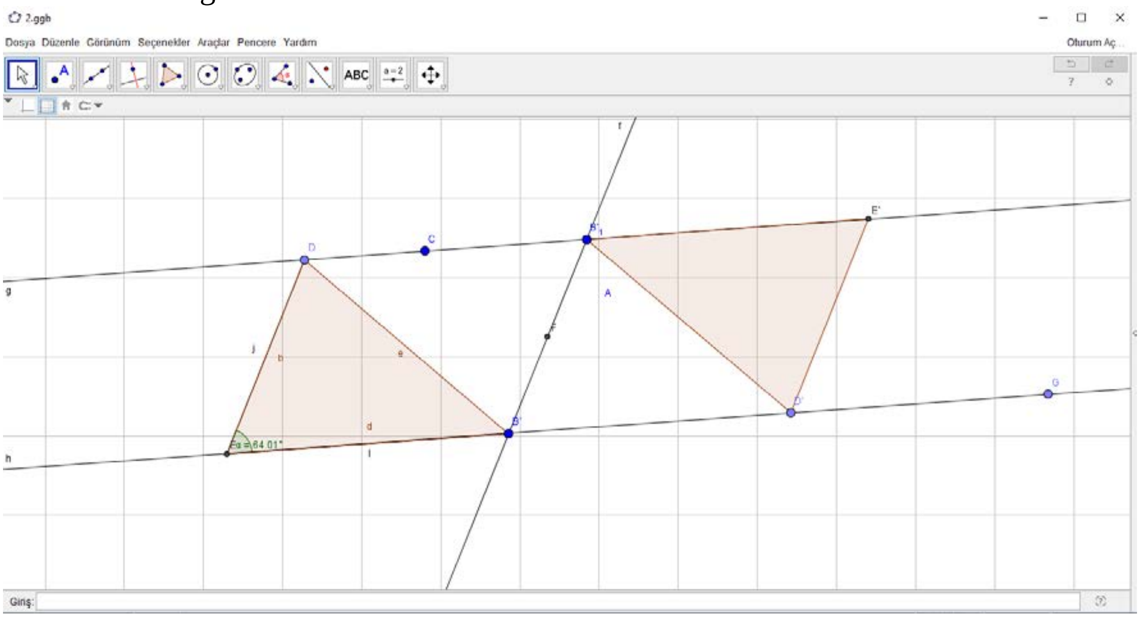

Figure 1. GeoGebra interface to be presented to the students

The task was carried out individually with the participants, and a computer with GeoGebra software installed was used. When students were shown the GeoGebra screen displayed in Figure 1, they were asked the following question: "*Are there any angles congruent to DEB angle in this geometrical structure? Explore it by using draggable points. Find congruent angles, if there are any.*

Apriori Analysis

According to the Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008), teachers, like an orchestra chef, manage the students' learning process, an evolution from their informal inferences acquired through their previous instructional practices into new mathematical generalizations. The teacher of this research, in particular the first author is aware of the dialectic relationship between congruent angles and congruent triangles, and their interaction with congruency concept.

Here, the students have basic level Euclidean Geometry background and they are expected to determine congruent angles formed on two parallel lines and intersected by a transversal by manipulating the drag gable points (i). In the next step, it is necessary to notice that the angles of the given triangles are congruent (ii) and to feel intuitively that the triangles are (still) congruent after the manipulations (iii). Finally, it is necessary to match the lengths of lines forming the triangles through the semiotic potential of grid tool but under the supervision of the teacher (iv), and to make angle-side correlations where angles and the opposite side relationships are determined (v), and to find out whether these triangles are congruent or similar based on the inferences made above (vi). It is possible that the students with advanced visual-spatial skills can also notice that these triangles are symmetrical.

Findings

In this section, we present the findings within two sub-sections focusing on each participant. We provide a number of (mathematically rich) transcripts and their analysis for each case.

The Case of A

Table 1 below displays the findings obtained from the interviews conducted with student A, the analyses regarding the "explore" phase – the first step- as well as the analyses describing the relationships between what the student did and said (T: Teacher, A: Student).

Table 1
The Explore Phase of A

Interview Transcript	Analysis
T: Can you tell us what do you see in this structure?	
A: Triangle, four triangles, an angle of 119^0 *marked points, lines: f, g and h lines*	*The student focuses on polygons rather than the whole shape.*
T: First of all, we will deal with the angles. Can you move the points in the structure by using the mouse?	
A: (The student starts dragging the points) B point is not draggable…	*The student starts to differentiate between the draggable point and non-draggable point.*
T: Continue to try. Would you bring the formed shape into its original state?	
A: C points does not work. Let's try from point B. Now, it is ok.	*The student makes hypotheses about the structure and applies accordingly.*

In "explore" phase, the student started to examine the pieces that form the whole structure by using the dragging tool. When he became familiar with the structure and dynamic geometry environment, he realized the differences between the original objects and the formed ones and make simple dragging practices and test his hypotheses through trial-error method. Later, the student passed to "reconstruct" phase. Table 2 displays an excerpt and analyses obtained in this phase.

Table 2
The Reconstruct Phase for A

Interview Transcript	**Analysis**
T: There is an angle of 18^0 in the triangle. Do you think there is an angle congruent to that angle in this shape?	
A: It might be the other angle of the triangle. It might be an isosceles triangle but there is no information that shows that it is an isosceles triangle. We had already learned about corresponding angles. These two angles might be congruent.	*The student realizes his mistakes after a simple proof procedure. He transfers his previous knowledge into the dynamic geometry environment.*
T: You can check by using your angle tool…	
A: The angle we measured is 35^0, so it is not congruent then. Let's try another angle. The degree of this angle is the same. This is the corresponding angle then.	*The student realizes his mistake and makes a new hypothesis and confirms it.*
T: Can you check how the angles change when you drag the points?	
A: When it becomes bigger, it becomes bigger and when it gets smaller, it gets smaller. Is it because it is corresponding angle?...	*The student figures out that when the points are dragged, correspondence is not disturbed and the congruence of the angles will not change accordingly.*

In this phase, the student went into a more profound reasoning. He explored the congruency of congruent angles – which was once understood intuitively- by using angle tool. The relationship among the angles (interior, opposite and corresponding etc.) were determined, and the momentary change of the angles was tracked through dragging tool. It was observed that the congruency of the angles did not change. As the next step, the student started to focus on "explain" phase, in which he made mathematical explanations. Table 3 displays the related findings and analyses for this phase.

Table 3
The Explain Phase for A

Interview Transcript	**Analysis**
T: What can you say about "h" and "g" lines?	
A: They are parallel and one of them cuts. They go to infinity and do not intersect… (Using zoom in and zoom out tools, he shows that they do not intersect)…	*The student knows well about the features of parallel lines and proves the situation visually.*
T: Is there a relationship between angles and parallelism? Would the angles be the same, if they were not parallel? Can you bring "g" and "h" lines closer?	*The student knows that only the angles formed by two parallel lines intersected by a transversal.*
A: They overlapped. Because they are parallel. If they were not parallel, they would not overlap.	
T: I am measuring another angle for you. It is about 88^0, Do you think there is an angle congruent to this	

one?	
A: This one? (the congruent angle in the reflection triangle) Let’s measure it... Yes, it is congruent.	*The student can easily match the congruent pieces formed on two parallel lines intersected by a transversal.*
T: Let’s compare all the angles of triangles. There is an angle of 48^0. Do you think there is a relationship between these triangles?	
A: We should find the same angle there too. Here we found (the angle corresponding with the congruent angle in the other triangle)	*The student intuitively figures out the congruency relationship between the two angles. He looks for evidence for his hypothesis in order to explain this relationship.*
T: What do you see between the two triangles?	
A: Not a rectangle. If it were a rectangle, all its angles would be 90^0. So, it is a parallelogram then…	*He makes inferences by using the parallelism of the lines and the congruency of the angles.*
T: Can you compare the angles of the triangles?	
A: All the angles are congruent, and the triangles too…	*The congruency of the triangles makes the student think about congruency concept.*
T: Do you think there is a line segment in the other triangle that is congruent to [BE] line segment in the first triangle?	
A: Yes, there is. The sides of parallelogram between the two triangles are congruent.	*The student makes all the matches for the sides by making use of the relationships in the whole shape.*
T: Can you examine the angles that look these sides in these two triangles?	
A: They are corresponding. Both are congruent (he tries several drags) The congruency of the angles do not change.	*The student finds the correlations among the sides and the angles and realizes that there are congruent sides opposite the congruent angles.*
T: Can you summarize what you have learned so far?	
A: We can never change the triangles because the lines are parallel. We have learned about congruent angles. This triangle is congruent with that one…	*By using his knowledge about congruent angles, the student concludes that the triangles are congruent.*

The student first figured out parallelism relationship between the lines, and later he supported his hypothesis through the argument that the distance between the lines did not change. In addition, student A found the interior angles of the triangles and their matches in the other triangle and realized that the triangles had the same interior angles, which made the student think that the angles were congruent. Also, he found that the lengths of the triangles were the same by using his knowledge about parallelograms, whose sides are also the sides of congruent triangles. Finally, the task was fulfilled when the sides and the angles of these sides were matched. Thus, it was concluded that the triangles are congruent.

The Case of B

Similar to student A, the phases that student B followed were also described in the forthcoming statements. Table 4 below presents the data about the "explore" phase and the related analysis.

Table 4
The Explore Phase for B

Interview Transcript	Analysis
T: Can you explain what do you see in this structure? *B: Two triangles, rectangle, no no square, no no it does not look like a square. Let's call it a quadrangle. Can it be parallelogram, sir? There is also an angle. Just a moment, there is another parallelogram outside as well.*	*The student is wrong at first, but later he explains the situation in detail through inductive reasoning. Still later, he focuses on the whole shape and realizes small and big parallel sides.*
T: You can drag the points in this structure (makes a sort of dragging practice). Can you drag the points? *B: This point is not draggable and this one moves only on the line.*	*Just like student A, he understands the distinction between the draggable and non-draggable points.*
T: Yes, there are draggable and un-draggable points in this structure. *A: The triangles changed and quadrangle also got distorted.* T: Can you bring the shape into its original form? *A: Let's drag it like that then (he is dragging the point). Yes I think, I did it…*	*Following a short period of thinking, he again tries to differentiate points and figures.*

At this phase, the students was observed to make some dragging practices to warm up for the task and to be familiar with the geometry environment. Since he identified parallelograms more quickly and accurately than student A, we can conclude that student B has better background than student A. In addition, he made accurate hypotheses by making fewer mistakes in trial and error practices. It can be said that student B entered a new process by using DGE tools and observations. Table 5 below displays the data regarding "reconstruct" phase for student B.

Table 5
The Reconstruct Phase for B

Interview Transcript	Analysis
T: Can you name the lines you see (on the screen)? *B: "f", "g" and "h" lines…* T: By dragging "g" and "h" points, can you show them as if they were one single line? *B: They are overlapping, Now it is ok.*	*The student overlaps the lines by using his knowledge regarding parallelism.*
T: Is it always possible to overlap two lines like this? What do you think the reason is?	

B: Not possible, I think…	*Although the student cannot give a clear explanation for the situation, he is aware that the lines that are parallel cannot intersect.*
T: I measured another angle for you and it is 81^0. Do you think there is another angle with the same degree?	
B: I think so. "D" angle. They both look like the same triangle; but in reverse shape. There is a parallel side, and mutual angles were congruent.	*Although it was not directly stated in the previous step that the lines are parallel, the student finds congruent angles through a different approach by using the angles of parallelogram. Also, congruent triangles are noticed intuitively.*
T: Can you check your claim by using the angle tool? (He explains how to use angle tool)	
B: Yes, both of them are $80^{0.}$ So, it is a parallelogram.	*The student confirms his hypothesis.*
T: If it were not a parallelogram, would these two angles be congruent?	
B: No, they wouldn't. Look, these lines are cutting lines, and the angle is not congruent. These angles are congruent, too. (He finds all the angles of 81^0 in the shape)	*When the student notices parallelism, a domino effect occurs and he finds all corresponding angles.*

In the "reconstruct" phase, the student discovered certain relationships such as congruent angles and parallelism more quickly than student A by using the advantage of having more domain-specific knowledge, and identifies the triangles intuitively. However, he tries to elaborate his arguments through dragging test experience. Similar to this, he explains mathematically what happens on the screen in explain phase. Table 6 present the data about this phase.

Table 6.

The Explain Phase for B

Interview Transcript	**Analysis**
T: Let's measure other angles of the triangle too? Can you find their match?	
B: Can it be the angle formed on "E"? Let me measure. Yes, it is the same. Is the triangle reversed? (He continues to drag the points). Well, this angle is $49^{0,}$ also the one at the opposite side. I think the triangle is reversed.	*The student actively uses the angle tool and checks the similarity by manipulating the points. He uses the term "reversed" for the triangles that are reflected of each other, which shows his awareness about the formation relationship between the triangles.*
T: We measured all the angles of the triangle on the left.	
Two angles of the triangles must be congruent with the third one. So these triangles are congruent, similar.	*By using the information stating that "if two angles of any two triangles are congruent, the third angle is congruent", the student figures out that the third angles are congruent, too. The student said: "They are congruent, similar", which implies that, however, he cannot distinguish these two concepts.*
T: Are congruency and similarity the same	

things? *We say similar when all the angles have the same degree. Congruency means "being exactly identical". I think lengths must be the same too. The sides of parallelogram are the sides of the triangle at the same time. The lengths are also the same then. So, the triangles are congruent too.*	*After clarifying congruency and similarity concepts a little bit, these concepts are clearly explained and it is concluded that the triangles are congruent.*

Since the eighth grade student is more knowledgeable enough to explain congruency and similarity conditions, he made a quicker transition to congruency concept by using parallelism relationships. The task carried out with student B reached its goal, and student B learned congruent triangles by using his previous knowledge about congruent angles.

Conclusions

In this study, an instructional task was designed in order to allow students to relate congruent angles to the congruency of triangles formed on a structure where congruent angles exist. Here, DGE was used as a tool for the didactical goal. The results of the study are consistent with the related literature. It is known that specific tools and functions of DGE – particularly the dragging tool- allow students to make effective reasoning and inferences as well as theoretical generalizations (Baccaglini-Frank & Mariotti, 2010; Mariotti, 2013, 2014). In this study, grid function was also activated in addition to dragging function and the use of angle tool was also integrated into the task. As a result, a transition from congruent angle to congruent triangle concept was achieved accordingly.

The findings here confirmed the semiotic potential of DGE, where students expand their knowledge although they have different background information. Thus, they were able to bridge between *congruent angles and congruent triangles.* In other words, making connection was achieved by using DGE as a tool. The most effective role was played by the dragging tool, which is the essence of DGE. Similarly, grid function can be said to allow students to easily realize that the triangles are congruent. However, the limitation of the study is that the data was collected only from two students.

MEB (2013) often suggests the integration of technology into lower secondary school mathematics teaching. Therefore, students should use certain tools and their functions individually or in groups because, according to theory of instrumental genesis (Drijvers, 2003; Trouche, 2004), the techniques used by students while using software is the product of a longitudinal process blended with mathematical knowledge. Thus, teachers can design similar tasks in the classrooms that might be carried out in groups by using tablets and PCs.

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