



# JER

Osmangazi Journal of Educational Research © OJER

Volume 5, Number 2, Autumn 2018

## Research Article

**To cite this article:** Doğan-Coşkun, S. (2018). Are pre-service elementary teachers able to pose problems for the subtraction of fractions? *Osmangazi Journal of Educational Research*, 5(2), 94-105. Retrieved from <http://ojer.ogu.edu.tr/vol5no1/Number2Autumn/OJER-V5-N2-3.pdf>

Submitted: 28.08.2018

Revised: 17.09.2018

Accepted: 26.10.2018

# Are Pre-service Elementary Teachers Able to Pose Problems for the Subtraction of Fractions?

Sümeýra Dođan Coşkun<sup>1</sup>

*Eskisehir Osmangazi University, Eskisehir, TURKEY*

## Abstract

The purpose of this study is to examine whether or not pre-service elementary teachers are able to pose appropriate problems for the subtraction of fractions, if not, to determine the types of errors made in their posed problems. A qualitative research method was used in this study and the data were collected from 83 pre-service elementary teachers in the spring 2017 academic semester of a public university in Turkey using a Problem Posing Test. The test consisted of four items related to the subtraction of fractions given in number sentences. Findings showed that most of the pre-service elementary teachers could not appropriately pose problems and made distinct types of errors. It was found that the two most common errors were failing to include subtraction in the question root and expressing the subtrahend fraction as a certain amount of the minuend fraction. Teacher educators can integrate problem posing activities in their courses to give pre-service teachers opportunities to pose problems.

## Keywords

Error types, fractions, problem posing, subtraction.

The understanding of fractions both conceptually and procedurally is critical for students' mathematical proficiency since the topic of fractions is closely related to the other content domains of mathematics such as algebra and measurement (Bailey, Hoard, Nugent, & Geary, 2012; Ministry of National Education [MoNE], 2018). In spite of its importance and strong relevance, students have some difficulties when they are learning fractions (Charalambous & Pitta-Pantazi, 2006; Pantziara & Philippou, 2012). Researchers explain that one of the reasons for these difficulties results from not being able to recognize the differences between whole numbers and rational numbers. Hence, they do not accept fractions as a single number, instead, they see them as two discrete numbers separated by a horizontal bar (Ni & Zhou, 2005). Students having troubles due to their lack of

<sup>1</sup> Corresponding author: Dr. Res. Asst., Eskisehir Osmangazi University Elementary and Early Childhood Education Department,

knowledge of properties of fractions, try to subtract fractions separately such as  $\frac{4}{6}$  for  $\frac{5}{8} - \frac{1}{2}$  (Newton, Willard, & Teufel, 2014). Another reason for students' difficulties is that "fractions comprise a multifaceted construct" (Charalambous, 2007; Lamon, 1999). That is, fractions consist of sub-constructs namely part-whole, measure, quotient, operator, and ratio (Kieren, 1993). However, most of the textbooks focus on only part-whole sub-construct of fractions and define fractions as "a part of a whole" (Carraher, 1996). If students are not able to know the differences or similarities among these sub-constructs, then they would not make sense of the fractions conceptually and hence would have difficulties and misconceptions. Comprehending these sub-constructs is crucial both to apply the necessary procedures to solve the problems related to the subtraction of fractions and to understand the underlying reasons behind these procedures (Hecht, 1998).

The problems for the subtraction of fractions may have the meanings of separating and comparing (Cathcart, Pothier, Vance, & Bezuk, 2003). Knowing the difference between these meanings may not be necessary for students; however, it is important for teachers to help struggling students. At this point, problem posing activities focusing on the different meanings of the subtraction of fractions may help students notice these differences. Furthermore, teachers can help students be aware of how the topic of fractions is related to the real world by means of these activities (Abu-Elwan, 2002). Apart from showing the connections between the fractions and the real world, problem posing activities improve students' reasoning and problem solving skills (Crespo & Sinclair, 2008; Kilic, 2017; NCTM, 2000). To effectively carry out these activities and hence to make positive contributions to students' engagement in mathematics, first of all, teachers need to be able to pose appropriate problems.

### **Problem Posing**

Starting from 1987, researchers have used different terms such as problem formulation, problem generation, or problem generation in reference to the term of problem posing (Brown & Walter, 1993; Kilpatrick, 1987; Silver, 1994). For example, Silver (1994) uses the term of problem generation which refers to "both the generation of new problems and the reformulation of given problems that can take place before, during, or after the act of problem solving" (p. 19). In 1989 National Council of Teachers of Mathematics (NCTM) highlighted the importance of integrating problem posing activities in mathematics by stating that students "should also have some experience recognizing and formulating their own problems, an activity that is at the heart of doing mathematics (p. 138). Similarly, in 1991 the NCTM stated that "students should be given opportunities to formulate problems from given situations and create new problems by modifying the conditions of a given problem" (p. 95). While students pose their problems, they would depend on their previous knowledge which would provide opportunities for themselves to make connections among different mathematical ideas (Silver, 1994). In another study, Silver and Cai (2005) mention that students find chances to participate in "authentic mathematical activity" if they are allowed to pose their problems (p. 129). Problem posing activities also enable students to have "some ownership of the mathematics" (Barlow & Cates, 2006, p. 69). Apart from the advantages of integrating problem posing activities for students into the classroom, researchers have noted these activities can be useful to teachers in assessing students' understanding of mathematics (Barlow & Cates, 2006). Similarly; Silver, Kilpatrick, and Schlesinger (1990) assert that teachers can notice their students' misconceptions or difficulties by means of problem posing activities. That is,

teachers should not only see the problem posing as a goal of the instruction, but also see it as a way of the instruction (Kilpatrick, 1987).

Researchers have accepted and used problem posing activities as one of the important ways to improve mathematics instruction since 1989 (Brown & Walter, 2005; Crespo & Sinclair, 2008). Then, different researchers developed frameworks considering the stages of problem posing activities (Silver, 1994), processes of problem posing activities (Christou, Mousoulides, Pittalis, Pitta-Pantazi, & Sriraman, 2005), and classifying problem posing activities into situations of problems posed (Stoyanova & Ellerton, 1996). Silver (1994), who suggests the first of these frameworks, accepts that problem posing can take place prior to problem solving in addition to during and after problem solving. If a problem solver is generating a new problem from a given problem, it will be an example of prior problem posing. On the other hand, if the problem solver is generating a new problem while engaging in the given problem, then it will be an example of during problem posing. Finally, if the problem solver examines the given problem to generate other related problems or to extend the problem using his/her mathematical knowledge, then it will be referred to as after problem posing.

Another framework used to classify problem posing situations is introduced by Christou and his colleagues (2005) and depends on the thinking processes used by students in order to pose problems. Their framework consists of “editing quantitative information, selecting quantitative information, comprehending and organizing quantitative information, and translating quantitative information from one form to another” (Christou et al., 2005, p. 151). Editing quantitative information is related to situations in which students are asked to pose a problem without any limitations. On the other hand, the process of selecting quantitative information asks students to pose a problem for a specific answer. The process of selecting quantitative information is more difficult for students compared to the process of editing quantitative information as they must be able to understand both the context of a problem and the relationships among information given in the problem. Comprehending and organizing quantitative information refers to situations that require students to pose problems for given equations. In the process of translating quantitative information, students are given graphs, charts, or tables and are asked to pose a problem using the information presented in these representations.

Another framework developed by Stoyanova and Ellerton (1996) distinguishes problem posing situations as free, semi-structured and structured. Considering their explanation, while free problem posing situation refers to a situation in which students are asked to pose problems without any restrictions, a semi-structured problem posing situation refers to a situation in which students are given a problem, a picture, or a diagram and asked to pose a similar problem to the given problem or asked to pose a problem using the picture or diagram. Finally, structured problem posing situations refers to a situation in which students are asked to reformulate or manipulate the given problem.

By combining the frameworks of Christou and his colleagues (2005) and Stoyanova and Ellerton (1996), Kilic (2013b) suggests another problem posing framework. Similar to Stoyanova and Ellerton’s framework (1996), Kilic (2013b) distinguishes problem posing situations as free, semi-structured and structured. However, Kilic (2013b) also elaborates the free problem posing situations into two kind of situations which are posing a difficult problem and posing problem for a specific topic. While semi-structured problem posing situation is related to editing and translating situations, structured problem posing situations is related to comprehending and selecting. Considering the above explained frameworks, Stoyanova and Ellerton’s model (1996) is a better fit for this study because it enables to examine pre-service elementary teachers’ semi-structured problems posed

for the subtraction of fractions. Specifically, the present study attempts to answer the following research questions:

- What are the meanings, including separating and comparing, emphasized in the problems posed by pre-service elementary teachers for the subtraction of fractions?
- What are the different types of errors in the problems posed by pre-service elementary teachers for the subtraction of fractions?

### Method

As the purpose is to examine whether or not pre-service elementary teachers are able to pose appropriate problems for the subtraction of fractions, if not, to determine the types of errors made in their posed problems, this study was based on a qualitative research method.

### Participants

A convenience sampling technique was used including 83 pre-service elementary teachers enrolled in the course of The Methods of Teaching Mathematics in the spring 2017 semester 2017 of a public university in Turkey. This course is a 14-week course designed to help pre-service elementary teachers deepen their knowledge of mathematics by focusing on instructional methods and strategies for teaching fundamental mathematical concepts of elementary school mathematics. Specifically, this course is designed to focus on fractions, geometry, measurement, and data analysis. That is, it is expected that pre-service elementary teachers would be better prepared to pedagogically teach the above-mentioned topics. Pre-service elementary teachers who agreed to be volunteers of the study was assigned a code, namely, PT1, PT2, PT3, ... PT83.

### Data Collection

A Problem Posing Test (PPT), including four items related to the subtraction of fractions given in number sentences, was developed by the author of this study. The pre-service elementary teachers were administered the PPT and asked to pose story problems whose solutions required to use these number sentences. Specifically, each item was comprised of two fractions with a proper fraction and a mixed fraction (see Table 1).

Table 1.

*Items and the Characteristics of the Items in the PPT*

Items	Characteristics
Write a story problem for $\frac{5}{6} - \frac{1}{6} = ?$	Subtraction of proper fractions where the difference is a proper fraction
Write a story problem for $\frac{3}{4} - \frac{1}{8} = ?$	Subtraction of proper fractions where the difference is a proper fraction
Write a story problem for $1\frac{5}{6} - \frac{1}{6} = ?$	Subtraction of a proper fraction from a mixed fraction where the difference is a mixed fraction

*(continued)*

Table 1. (continued)

---

Write a story problem for $1\frac{3}{4} - \frac{1}{8} = ?$	Subtraction of a proper fraction from a mixed fraction where the difference is a mixed fraction
--	---

---

Before conducting the study, the pre-service elementary teachers were provided with an information sheet explaining the purpose of this study, mentioning that their answers would be used for research purposes only, and would be kept confidential. Furthermore, they were provided 45 minutes to complete the Problem Posing Test.

### Data Analysis

To examine whether or not the pre-service elementary teachers are able to pose problems for the subtraction of fractions, an analysis was conducted in three steps: categorizing their answers as problem, not-a-problem, or unable to pose a problem, identifying the meanings emphasized in their problems, and detecting the errors made in their problems using the categorical analysis. For the study purposes, after investigating pre-service elementary teachers' problems for the second item in the PPT, errors determined in previous studies were used as a guide to determine their errors in the posed problems (Ma, 1999; McAllister & Beaver, 2012). Specifically, 10 different kinds of errors in pre-service elementary teachers' story problems for  $\frac{3}{4} - \frac{1}{8} = ?$  were determined. The first one, (SE1) refers to expressing the subtrahend fraction over the remainder of the whole. While the second kind of error, (SE2), occurs when pre-service elementary teachers fail to establish a part-whole relationship, the third one (SE3) occurs when they attribute natural number meaning to the result of the operation. The fourth one, (SE4) refers to confusing units. Attributing natural number meaning to the fractions is another kind of error, (SE5). When pre-service elementary teachers fail to express the operation in the question root, then the error type is coded as (SE6). Another type of error, (SE7), is related to attributing a value to the whole. Another type of error, (SE8), occurs when pre-service elementary teachers express the subtrahend fraction as a certain amount of minuend fraction. Making logical errors are coded as SE9 and expressing the fractions over the different wholes are coded as SE10. Throughout the analysis process, the pre-service elementary teachers' problems were also examined by the second coder to reduce the researcher bias and to ensure the credibility of the study. The author and the second coder came together and discussed if any inconsistencies existed, after analyzing the data separately. Furthermore, to minimize the inferences that the author might make, direct quotations were used to report the findings of this study. Specifically, pre-service elementary teachers' problems were presented in English and then in Turkish given in figures throughout the findings part of this study.

### Findings

After the findings related to the meanings emphasized by the pre-service elementary teachers in their posed problems were presented, the type of errors made in these problems was explored.

### The Meanings of the Problems Posed for the Subtraction of Fractions

To identify the meanings focused on in pre-service elementary teachers' story problems for the second item in the PPT, their posed problems were analyzed regardless of whether the problems were correct or not, and the meanings of them are presented in Table 2.

Table 2.

*The Distribution of Meanings Focused on in the Second Item of the PPT*

	Problem			Unable to Pose
	Separate	Compare	Others	
Write a story problem for $\frac{3}{4} - \frac{1}{8} = ?$	38	6	5	34

Table 2 shows the distribution of meanings focused on in the pre-service elementary teachers' problems in addressing fraction subtraction. According to the table, for the subtraction of fractions, the separate problems involving separation of quantities is predominant. Figure 1 shows an example of appropriate posing of separation problem. For Figure 1, the pre-service elementary teacher PT<sub>5</sub> wrote: " $\frac{3}{4}$  of a jug is filled with water.  $\frac{1}{8}$  of this jug is poured into a glass. How much water is left in the jug?"

$\frac{3}{4}$  ü su ile dolu olan bir sürahinin  $\frac{1}{8}$  i bir bardağa boşaltılıyor, sürahinin ne kadar su ile dolu kalır ?

Figure 1. Appropriate posing: Separate

Furthermore, as can be seen from the table, most of the pre-service elementary teachers could not pose a subtraction problem for the given mathematical expression. While five of the pre-service elementary teachers posed a problem requiring an operation other than subtraction, six of them posed a problem focusing on the compare meaning of subtraction. One of these problems is as follows: "Ayşe buys two same size cakes (strawberry and chocolate) for her guests as a dessert to serve with tea. While Ayşe divides the strawberry cake into 4 equal pieces and serves three pieces of them to some of the guests, she divides the chocolate cake into eight pieces and serves one piece of them to the remaining guests. How much more do the guests who are served with the strawberry cake eat compared to the guests who are served with the chocolate one?" corresponds to the comparing meaning of the subtraction.

Ayşe misafirlerine çayla birlikte tatlı olarak ikram etmek üzere aynı büyüklükte 2 tane pasta (çilekli ve çikolatalı) alır. Çilekli pastayı 4 eşit parçaya bölüp 3 parçasını bazı misafirlerine ikram ederken, çikolatalı pastasını 8 parçaya bölüp 1 parçasını kalan misafirlere ikram eder. Çilekli pasta ikram edilen misafirler çikolatalı pasta ikram edilen misafirlere ne kadar fazla yemiştir ?

Figure 2. Appropriate posing: Comparing

### The Error Types in the Problems Posed for the Subtraction of Fractions

To identify what kind of errors made by the pre-service elementary teachers in their problems, problems with error(s) were identified and the frequencies of these errors are given below:

Table 3.

*The distribution of Answers Given to the Second item of the PPT*

	Errorless	With Error	Unable to Pose
Write a story problem for $\frac{3}{4} - \frac{1}{8} = ?$	15	34	34

As can be seen from the table, only 15 of 83 pre-service elementary teachers successfully posed a problem for the second item of the PPT, and 34 pre-service elementary teachers posed problems with at least one error or could not pose a problem. Analysis of the problems with errors demonstrated that the pre-service elementary teachers made four types of errors: (a) expressing the subtrahend fraction over the remainder of whole (SE1), (b) attributing natural number meaning to the result of the operation (SE3), (d) failure in expressing the operation in the question root (SE6), (f) expressing the subtrahend fraction as a certain amount of minuend fraction (SE8). The distribution of these error types is given in Table 4.

Table 4.

*The Distribution of Error Types for the Second Item of the PPT*

	SE1	SE2	SE3	SE4	SE5	SE6	SE7	SE8	SE9	SE10
Write a story problem for $\frac{3}{4} - \frac{1}{8} = ?$	1	0	8	0	0	21	0	16	0	0

As it is seen from the table when the pre-service elementary teachers were asked to pose a story problem for  $\frac{3}{4} - \frac{1}{8} = ?$ , most of them posed a problem with an SE6 error, and all of these pre-service elementary teachers used the multiplication operation in their posed problems. The example given below shows an instance of a pre-service elementary teacher's failure in expressing the subtraction in the question root.

“We have  $\frac{3}{4}$  of a bread for our breakfast. I ate  $\frac{1}{8}$  of the bread that we have. How much of the whole bread did I eat?”

*Zahvaltı için ayırdığımız  $\frac{3}{4}$  ekmeğin  $\frac{1}{8}$  ini yedim. Bir Ekmeğin ne kadarını yedim?*

Figure 3. A problem with SE6

Sixteen pre-service elementary teachers made an error of type SE8, which results from expressing the subtrahend fraction as a certain amount of the minuend fraction. When the problems with SE8 were examined, it was seen that the pre-service elementary teachers understood that the minuend fraction represented a certain amount of a whole; however, they could not discover that the subtrahend fraction represents a certain amount of the same whole as well. A typical example of these problems was “Ali bought a cake and separated  $\frac{3}{4}$  of the cake for himself. Then, Ali gave  $\frac{1}{8}$  of the cake that had reserved for himself to Ayse. What fraction of the cake is now left for Ali?” PT<sub>8</sub>'s statement, “gave  $\frac{1}{8}$  of the cake that had reserved for himself”, refers to expressing the subtrahend fraction [ $\frac{1}{8}$ ] as a certain amount of the minuend fraction [ $\frac{3}{4}$ ] in the above problem and this expression requires the multiplication of  $\frac{1}{8}$  and  $\frac{3}{4}$  rather than the subtraction of  $\frac{1}{8}$  from  $\frac{3}{4}$ . That is, the necessary mathematical expression for PT<sub>8</sub>'s problem should be  $\frac{3}{4} - (\frac{1}{8} \times \frac{3}{4})$ ; hence the problem also contains another type of error, SE6, which is related to failure to include the subtraction operation in the question root.

Ali bir pasta almış ve pastanın  $\frac{3}{4}$  ünü kendisi için ayırmıştır. Daha sonra Ali ayırdığı bu pastanın  $\frac{1}{8}$  ini Ayşe'ye vermiştir. Aliye pastanın ne kadarı kalmıştır?

Figure 4. A problem with SE8

Eight pre-service elementary teachers attributed natural number meaning to the result of the operation. Of those pre-service elementary teachers who made the error SE3 failed to articulate the question root of the problem (Figure 5).

“Ali bought a cake and divided it into 8 equal slices. While Ali ate  $\frac{3}{4}$  of the cake, Ali's sister ate  $\frac{1}{8}$  of it. How many more slices did Ali eat compared to Ali's sister?”

Ali bir pasta almıştır ve 8 eşit parçaya bölmüştür. Ali pastanın  $\frac{3}{4}$  ünü yerken kardeşi ise  $\frac{1}{8}$  ini yemiştir. Ali kardeşi ile karşılaştırıldığında kaç tane parça fazla yemiştir?

Figure 5. A problem with SE3

The last type of error observed in the pre-service elementary teachers' problems is SE1. The example below demonstrates how a pre-service elementary teacher, PT<sub>36</sub>, made the error SE1 by expressing the subtrahend fraction over the remainder of the whole.

“While I read  $\frac{3}{4}$  of a book today, I am planning to read  $\frac{1}{8}$  of the remaining part of the book tomorrow. How much more book will I have read today than tomorrow?”



Bir kitabın  $\frac{3}{4}$  üzü bugün okurken kalan bölümün  $\frac{1}{8}$  ini yarın okunacağı planlıyorum. Buna göre bugün yarınca ne kadar fazla kitap okunmuş olacaktır?

Figure 6. A problem with SE1

Since PT<sub>36</sub> used the term “remaining”, the subtrahend fraction, which is  $\frac{1}{8}$  in the above problem, does not represent a certain amount of a whole; instead, it represents a certain amount of  $\frac{1}{4}$ . Therefore, the mathematical expression for the problem should be  $\frac{3}{4} - (\frac{1}{8} \times \frac{1}{4}) = ?$  rather than  $\frac{3}{4} - \frac{1}{8} = ?$

### Discussion and Conclusion

The purpose of this study was to examine if pre-service elementary teachers were able to pose appropriate problems for the subtraction of fractions, if not, then, to determine the common errors made in their posed problems. When their problems were examined considering the meanings emphasized in their problems, it was seen that most of them focused on the separate meaning of the subtraction operation. However, more than half of the pre-service elementary teachers (68/83) could not able to pose an appropriate problem. Specifically, 34 pre-service elementary teachers could not even pose a problem for the given expression.

Similar to the studies in national and international contexts, pre-service teachers have difficulties in problem posing activities for the topic of fractions (Kar & Isik, 2014; Kilic, 2013a; Luo, 2009; Rizvi 2004; Toluk-Ucar, 2009). The failure of these pre-service elementary teachers is critical as it affects students’ understanding of fractions (Crespo & Sinclair, 2008). Furthermore, MoNE (2018) emphasizes that students are able to solve and pose problems which calculating with fractions. Of the problems posed for the given expression, more than half of them had errors, and the two most common errors were failing to include subtraction in the question root, SE6, and expressing the subtrahend fraction as a certain amount of the minuend fraction, SE8. McAllister and Beaver (2012) explain that although pre-service elementary teachers are asked to pose a problem for “a – b”, they tend to pose a problem for “a – (a×b)” (p. 94). Similar to the explanation above, most of the pre-service elementary teachers in this study posed a problem for  $\frac{3}{4} - (\frac{1}{8} \times \frac{3}{4})$  instead of  $(\frac{3}{4} - \frac{1}{8})$ . Actually, SE6 and SE8 are closely related to each other as the pre-service elementary teachers used multiplication to express the subtrahend fraction as a certain amount of the minuend fraction and hence failed to express subtraction in the question root.

One reason for this difficulty in including subtraction in the question root seems to be related to not being able to refer both the subtrahend and minuend fractions to a single whole. Actually, a similar result was found in Kar and Isik’s (2014) study in which the students were asked to pose a problem for a number sentence related to the fractions, they were not able to include the subtraction operation in the question root of their problems. The pre-service elementary teachers’ difficulties may stem from the fact that they “have never been asked to pose problems” (Rizvi, 2004, p. 15). Therefore, teacher educators can integrate problem posing activities in their courses to give pre-service teachers opportunities to pose problems. Johnston and Ahtee (2006) emphasize that teacher

education programs need to prepare pre-service teachers to their future teaching profession by making them be aware of students' errors. The error types determined in this study could also be used as a guide by teacher educators to inform pre-service elementary teachers about possible errors. Furthermore, the underlying reasons for pre-service elementary teachers' errors can be discussed throughout the methods of teaching mathematics courses. Since, the focus of this study, the subtraction of fractions, is a small part of the curriculum, further studies can be conducted whether or not pre-service elementary teachers' answers are similar for the other operations of fractions.

### References

- Abu-Elwan, R. (2002). Effectiveness of problem posing strategies on prospective mathematics teachers' problem solving performance. *Journal of Science and Mathematics Education*, 25(1), 56-69.
- Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology*, 113(3), 447-455.
- Barlow, A. T., & Cates, J. M. (2006). The impact of problem posing on elementary teachers' beliefs about mathematics and mathematics teaching. *School Science and Mathematics*, 106(2), 64-73.
- Brown, S. I., & Walter, M. I. (1993). *Problem posing: Reflection and application*. Hillsdale, NJ: Erlbaum.
- Brown, S. I., & Walter, M. I. (2005). *The art of problem posing*. Mahwah, NJ: Lawrence Erlbaum.
- Carraher, D. W. (1996). Learning about fractions. In L. P. Steffe and P. Nesher, (Eds.), *Theories of Mathematical Learning* (pp. 241-266). Mahwah, NJ: Lawrence Erlbaum.
- Cathcart, W. G., Pothier, V. M., Vance, T. H., & Bezuk, N. S. (2003). *Learning mathematics in elementary and middle schools*. (3rd Ed.) River, N.J: Merrill/Prentice Hall.
- Charalambous, C. Y. (2007). Developing and testing a scale for measuring students' understanding of fractions. *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education 2*, 105-112.
- Charalambous, C. Y., & Pitta-Pantazi, D. (2006). Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics*, 64(3), 293-316.
- Christou, C., Mousoulides, N., Pittalis, M., Pitta-Pantazi, D., & Sriraman, B. (2005). An empirical taxonomy of problem posing process. *ZDM-The International Journal on Mathematics Education*, 37(3), 149-158.
- Crespo, S., & Sinclair, N. (2008). What makes a problem mathematically interesting? Inviting prospective teachers to pose better problems. *Journal of Mathematics Teacher Education*, 11(5), 395-415.
- Hecht, S. A. (1998). Toward an information-processing account of individual differences in fraction skills. *Journal of Educational Psychology*, 90(3), 545-559.

- Johnston, J., & Ahtee, M. (2006). Comparing primary student teachers' attitudes, subject knowledge and pedagogical content knowledge needs in a physics activity. *Teaching and Teacher Education*, 22(4), 503-512.
- Kar, T., & Isik, C. (2014). Analyzing problems posed by seventh grade middle school students for subtraction operation with fractions. *Elementary Education Online*, 13(4), 1223-1239.
- Kieren, T. E. (1993). Rational and fractional numbers: From quotient fields to recursive understanding. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational Numbers: An Integration of Research* (pp. 49–84). Hillsdale, NJ: Lawrence Erlbaum.
- Kilic, C. (2013a). Pre-service primary teachers' free problem-posing performances in the context of fractions: An example from Turkey. *The Asia-Pacific Education Researcher*, 22(4), 677-686.
- Kilic, Ç. (2013b). Sınıf öğretmeni adaylarının farklı problem kurma durumlarında sergilemiş oldukları performansın belirlenmesi [Determining the performances of pre-service primary school teachers in problem posing situations]. *Educational Sciences: Theory & Practice*, 13(2), 1195–1211.
- Kilpatrick, J. (1987). Problem formulating: where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123-147). Hillsdale, NJ: Lawrence Erlbaum.
- Lamon, S. J. (1999). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. Mahwah, NJ: Lawrence Erlbaum.
- Luo, F. (2009). Evaluating the effectiveness and insights of preservice elementary teachers' abilities to construct word problems for fraction multiplication. *Journal of Mathematics Education*, 2(1), 83–98.
- Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum.
- McAllister, C. J., & Beaver, C. (2012). Identification of error types in preservice teachers' attempts to create fraction story problems for specified operations. *School Science and Mathematics*, 112(2), 88-98.
- Ministry of National Education [MoNE]. (2018). *Mathematics program for elementary and middle school*. Ankara: Devlet Kitapları Müdürlüğü.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.
- Newton, K. J., Willard, C., & Teufel, C. (2014). An examination of the ways that students with learning disabilities solve fraction computation problems. *The Elementary School Journal*, 115(1), 1–21.

- Ni, Y., & Zhou, Y. D. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist, 40*(1), 27–52.
- Pantziara, M., & Philippou, G. (2012). Levels of students’ “conception” of fractions. *Educational Studies in Mathematics, 79*(1), 61-83.
- Rizvi, N. F. (2004). Prospective teachers’ ability to pose word problems. *International Journal for Mathematics Teaching and Learning, 12*, 1-22.
- Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19-28.
- Silver, E. A., & Cai, J. (2005). Assessing students’ mathematical problem posing. *Teaching Children Mathematics, 12*(3), 129- 135.
- Silver, E. A., Kilpatrick, J., & Schlesinger, B. (1990). *Thinking through mathematics: Fostering inquiry and communication in mathematics classrooms*. New York: The College Board.
- Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students’ problem posing in school mathematics. In P. Clarkson, (Ed.), *Technology in mathematics education* (pp.518-525). Melbourne: Mathematics Education Research Group of Australasia.
- Toluk-Ucar, Z. (2009). Developing pre-service teachers understanding of fractions through problem posing. *Teaching and Teacher Education, 25*(1), 166-175.

### **Biographical Statement**

**Sümeýra Dođan Coşkun** graduated from Middle East Technical University in 2006, received her M.S. degree in Elementary Mathematics and Science Education in 2009 from Middle East Technical University, and her Ph.D. degree in Elementary Teacher Education in 2017 from Gazi University. She currently works as a research assistant in the Department of Elementary and Early Childhood Education at Eskisehir Osmangazi University. During her graduate studies, Sümeýra Dođan Coşkun worked as an elementary mathematics teacher in the Ministry of National Education for eight years. Build upon her experience as a teacher, she is interested in the improvement of mathematics teaching and learning at the elementary school level. Specifically, her research interests include designing and studying ways to help pre- and in-service teachers develop mathematical knowledge in teaching.