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The Forgotten Topological Index of Double Corona of Graphs Related to the Different Subdivision Graphs

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Abstract: Graph theory plays an important role in the chemical sciences to model chemical compounds graphically. A chemical graph is a representation of molecular structure whose vertices are considered as atoms of the molecule and edges are the bonds between them. A topological index is a numeric quantity associated with chemical compounds to predict various physicochemical properties and biological activities theoretically. The sum of the cube of degrees of all the vertices for a graph is called the "forgotten topological index" or F-index of that graph. In this study, we compute the F-index of double corona graphs related to the different subdivision graphs.

1. Introduction

A chemical graph is a model through a simple graph $G = (V(G), E(G))$ related to the structure of the compound where $V(G)$ and $E(G)$ denote the vertex and edge set of G. For a vertex a in G, the degree of a is defined as the number of those vertices in G which are connected to a and is denoted by $d_G(a)$. For a graph the topological index is a numeric quantity derived from that graph by mathematically and is remain same under graph isomorphisms or briefly it is a molecular descriptor correlated with the chemical compounds to predict some physicochemical properties and biological activities theoretically. There are many applications of topological indices in chemical graph theory for isomer discrimination, distinguished the physico-chemical properties of chemical compounds, QSAR/QSPR investigations, in pharmaceutical drug design and much more. The Zagreb indices were introduced by Gutman and Trinajestić [1] to study the total π-electron energy(ε) of carbon atoms in 1972, and these are denoted by

$$M_1(G) = \sum_{a\in V(G)} d_G(a)^2 = \sum_{ab\in E(G)} [d_G(a) + d_G(b)]$$

and

$$M_2(G) = \sum_{ab\in E(G)} d_G(a)d_G(b).$$

For some recent study about these indices, we refer our reader to [2–4]. The "forgotten topological index" is introduced as the general version of second Zagreb index in [1] where Zagreb indices are introduced and are denoted by

$$F(G) = \sum_{a\in V(G)} d_G(a)^3 = \sum_{ab\in E(G)} [d_G(a)^2 + d_G(b)^2].$$

We refer our reader to [5–8] for some recent study about this index. In 2013, Ranjini et al. first introduced the redefined Zagreb index in [9] and is denoted by

$$ReZM(G) = \sum_{ab\in E(G)} d_G(a)d_G(b)[d_G(a) + d_G(b)].$$

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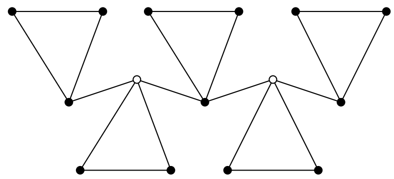


$P_3^S \otimes (P_2^\bullet, P_2^o)$
Fig. 1. The example of $P_3^S \otimes (P_2^\bullet, P_2^o)$ graph.

We refer our reader to [10–12], for some recent study about this index. Shirrdel et al. first study the hyper Zagreb index in a paper [13], where they compute the hyper Zagreb index of some known graphs in 2013 and this index is denoted by

$$HM(G) = \sum_{ab \in E(G)} [d_G(a) + d_G(b)]^2.$$

For further study about this index we refer our reader to [14, 15]. Another version of first Zagreb index is defined as

$$M_4(G) = \sum_{a \in V(G)} d_G(a)^4 = \sum_{ab \in E(G)} [d_G(a)^3 + d_G(b)^3].$$

In recent time there are various new graph operations can be introduced by some researchers and they help to modelling molecular structures of the chemical compound.
The sub division graph of a graph G is obtained by inserting an extra vertex into each edge of G and is denoted by $S(G)$.
The graph R(G) is obtained from G by inserting an additional vertex into each edge of G and joining each additional vertex to the end vertices of the corresponding edge of G.
Q(G) is a graph derived from G by adding a new vertex to each edge of G, then joining with edges those pairs of new vertices on adjacent edges of G.
The total graph T(G) is derived from G by inserting an new vertex to each edge of G, then joining each new vertex to the end vertices of the corresponding edge and joining with edges those pairs of new vertices on adjacent edges of G.
In this paper, we study the "forgotten topological index" for double corona of graphs based on different subdivision graphs and hence first we define them. Let us suppose that $\mathscr{F} = \{S, R, Q, T\}$.

Definition 1.1. Let G_1 and G_2 be two simple graphs with n_1, n_2 number of vertices and m_1, m_2 number of edges. The corona of G_1 and G_2 is obtain by taking one copy of G_1 and n_1 copies of G_2 and then connecting each i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2, where $i = 1, 2, ..., n_1$ and is denoted by $G_1 \circ G_2$.

Definition 1.2 ([16])**.** Suppose that G be a graph of n number of vertices and m number of edges. The $\mathscr{F}$-double corona graph of G, G_1 and G_2, is denoted by $\{G^{\mathscr{F}} \otimes (G_1^\bullet, G_2^o)\}$, and derived by taking one copy of $\mathscr{F}(G)$, n copies of G_1 and m copies of G_2, then joining i^{th} old-vertex of $\mathscr{F}(G)$ to every vertex in the i^{th} copy of G_1 and the j^{th} new-vertex of $\mathscr{F}(G)$ to every vertex in the j^{th} copy of G_2. Now if we replace $\mathscr{F}$ by S, R, Q, T then we get S-double corona, R-double corona, Q-double corona, and T-double corona respectively.

2. Main Results:

In this section we compute the "forgotten topological index" of $\mathscr{F}$-double corona of graphs based on different subdivision graphs. First we start with S-double corona graphs. The figure of S-double corona shown in Figure 1.

Lemma 2.1. *All the vertex degrees of* $(G^S \otimes (G_1^\bullet, G_2^o))$ *are given by*

$$d_{G^S \otimes (G_1^\bullet, G_2^o)}(a) = \begin{cases} 2d_G(a_i) + n_1, \ if \ a_i \in V(G), \ i = 1, 2, ..., n \\ 2 + n_2, \ if \ a_i b_j = w \in I(G), \ i, j = 1, 2, .., n, \ i \neq j \\ d_{G_1}(a_i^j) + 1, \ if \ a_i^j \in V(G_1), \ i = 1, 2, .., n_1 \ and \ j = 1, 2, .., n \\ d_{G_2}(a_i^j) + 1, \ if \ a_i^j \in V(G_2), \ i = 1, 2, .., n_2 \ and \ j = 1, 2, .., m. \end{cases}$$

where $I(G)$ denotes the set of all new vertices which are inserted in G.

Theorem 2.2. *For three connected graphs G, G_1 and G_2, we get*

$$\begin{aligned} F(G^S \otimes (G_1^\bullet, G_2^o)) &= F(G)+nF(G_1)+mF(G_2)+3n_1M_1(G)+3nM_1(G_1) \\ &+3mM_1(G_2)+nn_1(1+n_1^2)+m[n_2+(n_2+2)^3] \\ &+6(mm_2+mn_1^2+m_1n). \end{aligned} \tag{1}$$

Proof. By the definition of F-index and Lemma 2.1, we have

$$\begin{aligned} F(G^S \otimes (G_1^\bullet, G_2^o)) &= \sum_{a\in(G^S\otimes(G_1^\bullet,G_2^o))} d_{G^S\otimes(G_1^\bullet,G_2^o)}(a)^3 \\ &= \sum_{a\in V(G)} d_{G^S\otimes(G_1^\bullet,G_2^o)}(a)^3 + \sum_{a\in I(G)} d_{G^S\otimes(G_1^\bullet,G_2^o)}(a)^3 \\ &\quad +n\sum_{a\in V(G_1)} d_{G^S\otimes(G_1^\bullet,G_2^o)}(a)^3 + m\sum_{a\in V(G_2)} d_{G\vee^S(G_1^\bullet,G_2^o)}(v)^3 \\ &= \sum_{a\in V(G)} (d_G(a)+n_1)^3 + \sum_{a\in I(G)} (2+n_2)^3 \\ &\quad +n\sum_{a\in V(G_1)} (d_{G_1}(a)+1)^3 + m\sum_{a\in V(G_2)} (d_{G_2}(a)+1)^3 \\ &= \sum_{a\in V(G)} [d_G(a)^3+3n_1d_G(a)^2+3n_1^2d_G(a)+n_1^3]+m(2+n_2)^3 \\ &\quad +n\sum_{a\in V(G_1)} [d_{G_1}(a)^3+3d_{G_1}(a)^2+3d_{G_1}(a)+1] \\ &\quad +m\sum_{a\in V(G_2)} [d_{G_2}(a)^3+3d_{G_2}(a)^2+3d_{G_2}(a)+1] \\ &= \sum_{a\in V(G)} d_G(a)^3+3n_1\sum_{a\in V(G)} d_G(a)^2+3n_1^2\sum_{a\in V(G)} d_G(a) \\ &\quad +nn_1^3+m(2+n_2)^3+n\sum_{a\in V(G_1)} d_{G_1}(a)^3+3n\sum_{a\in V(G_1)} d_{G_1}(a)^2 \\ &\quad +3n\sum_{a\in V(G_1)} d_{G_1}(a)+n_1n+m\sum_{a\in V(G_2)} d_{G_2}(a)^3 \\ &\quad +3m\sum_{a\in V(G_2)} d_{G_2}(a)^2+3m\sum_{a\in V(G_2)} d_{G_2}(a)+n_2m \\ &= F(G)+3n_1M_1(G)+6mn_1^2+nn_1^3+m(2+n_2)^3 \\ &\quad +nF(G_1)+3nM_1(G_1)+6nm_1+n_1n+mF(G_2) \\ &\quad +3mM_1(G_2)+6mm_2+n_2m. \end{aligned}$$

Hence the proof is done. □

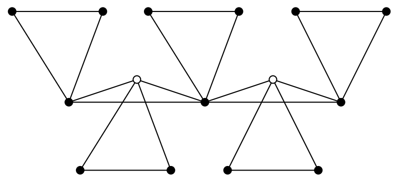
Example 2.3. Using the Equation 1, we get

$$\begin{aligned} (i)\ F(P_l^S \otimes (C_m^\bullet, C_n^o)) &= lm(1+m^2)+(l-1)\{(n+2)^3+6m^2+n\}+38lm \\ &\quad +26ln+8l-18m-26n-14,\ l\geq 2,\ m,n\geq 3, \\ (ii)\ F(C_l^S \otimes (P_m^\bullet, C_n^o)) &= lm\{(1+m^2)+6m\}+l(n+2)^3+38lm \\ &\quad +27ln-30l,\ l,n\geq 3,\ m\geq 2. \end{aligned}$$

Now, we consider R-double corona graphs and obtain "forgotten topological index" of this graph operation. The figure of $R(G)$-double corona is shown in Figure 2. The degree of all the vertices of R-double corona showing in the following lemma:

Lemma 2.4. *All the vertex degrees of $(G^R \otimes (G_1^\bullet, G_2^o))$ are given by*

$$d_{G^R\otimes(G_1^\bullet,G_2^o)}(a) = \begin{cases} 2d_G(a_i)+n_1,\ if\ a_i\in V(G),\ i=1,2,...,n \\ 2+n_2,\ if\ a_ib_j=w\in I(G),\ i,j=1,2,..,n,\ i\neq j \\ d_{G_1}(a_i^j)+1,\ if\ a_i^j\in V(G_1),\ i=1,2,..,n_1\ and\ j=1,2,..,n \\ d_{G_2}(a_i^j)+1,\ if\ a_i^j\in V(G_2),\ i=1,2,..,n_2\ and\ j=1,2,..,m. \end{cases}$$



$P_3^R \otimes (P_2^\bullet, P_2^o)$

Fig. 2. The example of $P_3^R \otimes (P_2^\bullet, P_2^o)$ graph.

Theorem 2.5. *If G, G_1 and G_2 are three connected graphs, then we get*

$$\begin{aligned} F(G^R \otimes (G_1^\bullet, G_2^o)) &= 8F(G) + nF(G_1) + mF(G_2) + 12n_1M_1(G) + 3nM_1(G_1) \\ &\quad + 3mM_1(G_2) + nn_1(1+n_1^2) + m[n_2 + (n_2+2)^3] \\ &\quad + 6(mm_2 + 2mn_1^2 + m_1 n). \end{aligned} \tag{2}$$

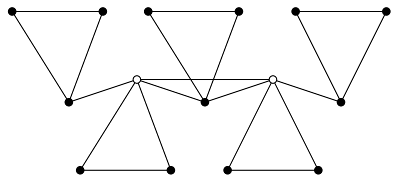
Proof. By the definition of "forgotten topological index" and Lemma 2.4, we have

$$\begin{aligned} F(G^R \otimes (G_1^\bullet, G_2^o)) &= \sum_{a \in (G^R \otimes (G_1^\bullet, G_2^o))} d_{G^R \otimes (G_1^\bullet, G_2^o)}(a)^3 \\ &= \sum_{a \in V(G)} d_{G^R \otimes (G_1^\bullet, G_2^o)}(a)^3 + \sum_{a \in I(G)} d_{G^R \otimes (G_1^\bullet, G_2^o)}(a)^3 \\ &\quad + \sum_{a \in V(G_1)} d_{G^R \otimes (G_1^\bullet, G_2^o)}(a)^3 + \sum_{a \in V(G_2)} d_{G^R \otimes (G_1^\bullet, G_2^o)}(a)^3 \\ &= \sum_{a \in V(G)} (2d_G(a) + n_1)^3 + \sum_{a \in I(G)} (2+n_2)^3 \\ &\quad + n \sum_{a \in V(G_1)} (d_{G_1}(a)+1)^3 + m \sum_{a \in V(G_2)} (d_{G_2}(a)+1)^3 \\ &= \sum_{a \in V(G)} [8d_G(a)^3 + 12n_1 d_G(a)^2 + 6n_1^2 d_G(a) + n_1^3] + m(2+n_2)^3 \\ &\quad + n \sum_{a \in V(G_1)} [d_{G_1}(a)^3 + 3d_{G_1}(a)^2 + 3d_{G_1}(a) + 1] \\ &\quad + m \sum_{a \in V(G_2)} [d_{G_2}(a)^3 + 3d_{G_2}(a)^2 + 3d_{G_2}(a) + 1] \\ &= 8 \sum_{a \in V(G)} d_G(a)^3 + 12n_1 \sum_{a \in V(G)} d_G(a)^2 + 6n_1^2 \sum_{a \in V(G)} d_G(a) \\ &\quad + nn_1^3 + m(2+n_2)^3 + n \sum_{a \in V(G_1)} d_{G_1}(a)^3 + 3n \sum_{a \in V(G_1)} d_{G_1}(a)^2 \\ &\quad + 3n \sum_{a \in V(G_1)} d_{G_1}(a) + n_1 n + m \sum_{a \in V(G_2)} d_{G_2}(a)^3 \\ &\quad + 3m \sum_{a \in V(G_2)} d_{G_2}(a)^2 + 3m \sum_{a \in V(G_2)} d_{G_2}(a) + n_2 m \\ &= 8F(G) + 12n_1M_1(G) + 12mn_1^2 + nn_1^3 + m(2+n_2)^3 \\ &\quad + nF(G_1) + 3nM_1(G_1) + 6nm_1 + n_1 n + mF(G_2) \\ &\quad + 3mM_1(G_2) + 6mm_2 + n_2 m. \end{aligned}$$

Hence the proof is over. □

Example 2.6. From Equation 2, we get

$$\begin{aligned} (i)\ F(P_l^R \otimes (C_m^\bullet, C_n^o)) &= (l-1)\{(n+2)^3 + 12m^2\} + lm(1+m^2) + 74lm + 27ln \\ &\quad + 64l - 72m - 27n - 112,\ l \geq 2,\ m,n \geq 3, \\ (ii)\ F(C_l^R \otimes (P_m^\bullet, C_n^o)) &= lm(m^2 + 12m + 1) + l(n+2)^3 + 74lm + 27ln \\ &\quad + 26l,\ l,n \geq 3,\ m \geq 2. \end{aligned}$$



$P_3^Q \otimes (P_2^{\bullet}, P_2^{o})$

Fig. 3. The example of $P_3^Q \otimes (P_2^{\bullet}, P_2^{o})$ graph.

Here, we consider Q-double corona graphs and compute the "forgotten topological index" of this graph operation. The $Q(G)$-double corona of G, G_1 and G_2 is shown in Figure 3. We define the degree of all the vertices of Q-double corona graphs in the following lemma:

Lemma 2.7. *The degree of all the vertices of* $(G^Q \otimes (G_1^{\bullet}, G_2^{o}))$ *are given by*

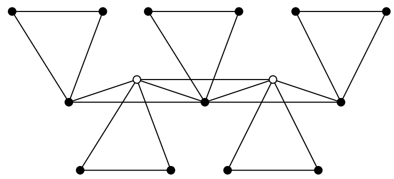
$$d_{G^Q \otimes (G_1^{\bullet}, G_2^{o})}(a) = \begin{cases} d_G(a_i) + n_1, \ if \ a_i \in V(G), \ i = 1, 2, ..., n \\ d_G(a_i) + d_G(b_j) + n_2, \ if \ a_i b_j = w \in I(G), \ i, j = 1, 2, .., n, \ i \neq j \\ d_{G_1}(a_i^j) + 1, \ if \ a_i^j \in V(G_1), \ i = 1, 2, .., n_1 \ and \ j = 1, 2, .., n \\ d_{G_2}(a_i^j) + 1, \ if \ a_i^j \in V(G_2), \ i = 1, 2, .., n_2 \ and \ j = 1, 2, .., m. \end{cases}$$

Theorem 2.8. *For three connected graphs* G*,* G_1 *and* G_2*, we get*

$$\begin{aligned} F(G^Q \otimes (G_1^{\bullet}, G_2^{o})) &= F(G) + nF(G_1) + mF(G_2) + 3(n_1 + n_2^2)M_1(G) + 3nM_1(G_1) \\ &\quad + 3mM_1(G_2) + 3n_2 HM(G) + 3ReZM(G) + M_4(G) \\ &\quad + mn_2(1 + n_2^2) + nn_1(1 + n_1^2) + 6(mm_2 + m_1 n + mn_1^2). \end{aligned} \tag{3}$$

Proof. From Lemma 2.7 and applying the definition of "forgotten topological index", we have

$$\begin{aligned} F(G^Q \otimes (G_1^{\bullet}, G_2^{o})) &= \sum_{a \in (G^Q \otimes (G_1^{\bullet}, G_2^{o}))} d_{G^Q \otimes (G_1^{\bullet}, G_2^{o})}(a)^3 \\ &= \sum_{a \in V(G)} d_{G^Q \otimes (G_1^{\bullet}, G_2^{o})}(a)^3 + \sum_{a \in I(G)} d_{G^Q \otimes (G_1^{\bullet}, G_2^{o})}(a)^3 \\ &\quad + n \sum_{a \in V(G_1)} d_{G^Q \otimes (G_1^{\bullet}, G_2^{o})}(a)^3 + m \sum_{a \in V(G_2)} d_{G^Q \otimes (G_1^{\bullet}, G_2^{o})}(a)^3 \\ &= \sum_{a \in V(G)} (d_G(a) + n_1)^3 + \sum_{ab \in E(G)} (d_G(a) + d_G(b) + n_2)^3 \\ &\quad + n \sum_{a \in V(G_1)} (d_{G_1}(a) + 1)^3 + m \sum_{a \in V(G_2)} (d_{G_2}(a) + 1)^3 \\ &= \sum_{a \in V(G)} [d_G(a)^3 + 3n_1 d_G(a)^2 + 3n_1^2 d_G(a) + n_1^3] \\ &\quad + \sum_{ab \in E(G)} [(d_G(a) + d_G(b))^3 + 3n_2(d_G(a) + d_G(b))^2 \\ &\quad + 3n_2^2(d_G(a) + d_G(b)) + n_2^3] \\ &\quad + n \sum_{a \in V(G_1)} [d_{G_1}(a)^3 + 3d_{G_1}(a)^2 + 3d_{G_1}(a) + 1] \\ &\quad + m \sum_{a \in V(G_2)} [d_{G_2}(a)^3 + 3d_{G_2}(a)^2 + 3d_{G_2}(a) + 1] \end{aligned}$$



$P_3^T \otimes (P_2^{\bullet}, P_2^{o})$

Fig. 4. The example of $P_3^T \otimes (P_2^{\bullet}, P_2^{o})$ graph.

$$\begin{aligned}
&= \sum_{a\in V(G)} d_G(a)^3 + 3n_1 \sum_{a\in V(G)} d_G(a)^2 + 3n_1^2 \sum_{a\in V(G)} d_G(a) \\
&\quad + nn_1^3 + \sum_{ab\in E(G)} (d_G(a)^3 + d_G(b)^3) \\
&\quad + 3\sum_{ab\in E(G)} d_G(a)d_G(b)(d_G(a)+d_G(b)) \\
&\quad + 3n_2 \sum_{ab\in E(G)} (d_G(a)+d_G(b))^2 \\
&\quad + 3n_2^2 \sum_{ab\in E(G)} (d_G(a)+d_G(b)) + mn_2^3 \\
&\quad + n\sum_{a\in V(G_1)} d_{G_1}(a)^3 + 3n \sum_{a\in V(G_1)} d_{G_1}(a)^2 \\
&\quad + 3n\sum_{a\in V(G_1)} d_{G_1}(a) + n_1 n + m \sum_{a\in V(G_2)} d_{G_2}(a)^3 \\
&\quad + 3m \sum_{a\in V(G_2)} d_{G_2}(a)^2 + 3m \sum_{a\in V(G_2)} d_{G_2}(a) + n_2 m \\
&= F(G) + 3n_1 M_1(G) + 6mn_1^2 + nn_1^3 + M_4(G) + 3ReZM(G) \\
&\quad + 3n_2 HM(G) + 3n_2^2 M_1(G) + mn_2^3 + nF(G_1) + 3nM_1(G_1) \\
&\quad + 6nm_1 + n_1 n + mF(G_2) + 3mM_1(G_2) + 6mm_2 + mn_2,
\end{aligned}$$

which is the desired result as in Theorem 2.8. □

Example 2.9. From Equation 3, we get

$$\begin{aligned}
(i)\ F(P_l^Q \otimes (C_m^{\bullet}, C_n^{o})) &= n(l-1)(n^2+1) + lm(m^2+1) + 6l(m^2+2n^2) \\
&\quad -6(m^2+3n^2) + 38lm + 74ln + 72l - 18m \\
&\quad -116n - 152,\ \ l,m,n \geq 3, \\
(ii)\ F(C_l^Q \otimes (P_m^{\bullet}, C_n^{o})) &= 6l(m^2+2n^2) + l(m^3+n^3) + 39lm + 75ln \\
&\quad +34l,\ l,n \geq 3, m \geq 2.
\end{aligned}$$

Finally, we drive the "forgotten topological index" of T-double corona graphs. The example of T-double corona graphs of P_3, P_2 and P_2 is shown in Figure 4.

Lemma 2.10. *The degree of all the vertices of $(G^T \otimes (G_1^{\bullet}, G_2^{o}))$ are given by*

$$d_{G^T\otimes(G_1^{\bullet},G_2^{o})}(a) = \begin{cases} 2d_G(a_i) + n_1,\ if\ a_i \in V(G),\ i = 1,2,...,n \\ d_G(a_i) + d_G(b_j) + n_2,\ if\ a_ib_j = w \in I(G),\ i,j = 1,2,..,n,\ i \neq j \\ d_{G_1}(a_i^j) + 1,\ if\ a_i^j \in V(G_1),\ i = 1,2,..,n_1\ and\ j = 1,2,..,n \\ d_{G_2}(a_i^j) + 1,\ if\ a_i^j \in V(G_2),\ i = 1,2,..,n_2\ and\ j = 1,2,..,m. \end{cases}$$

Theorem 2.11. *If G, G_1 and G_2 are three connected graphs, then*

$$\begin{aligned}
F(G^T \otimes (G_1^{\bullet}, G_2^{o})) &= 8F(G) + nF(G_1) + mF(G_2) + 3(n_1+n_2^2)M_1(G) + 3nM_1(G_1) \\
&\quad + 3mM_1(G_2) + 3n_2 HM(G) + 3ReZM(G) + M_4(G) \\
&\quad + mn_2(1+n_2^2) + nn_1(1+n_1^2) + 6(mm_2 + m_1 n + mn_1^2). \qquad (4)
\end{aligned}$$

Proof. By the definition of F-index and using Lemma 2.10, we have

$$\begin{aligned}
F(G^T\otimes(G_1^\bullet,G_2^o)) &= \sum_{a\in(G^T\otimes(G_1^\bullet,G_2^o))} d_{G^T\otimes(G_1^\bullet,G_2^o)}(a)^3\\
&= \sum_{a\in V(G)} d_{G^T\otimes(G_1^\bullet,G_2^o)}(a)^3+\sum_{a\in I(G)} d_{G^T\otimes(G_1^\bullet,G_2^o)}(a)^3\\
&\quad +n\sum_{a\in V(G_1)} d_{G^T\otimes(G_1^\bullet,G_2^o)}(a)^3+\sum_{a\in V(G_2)} d_{G^T\otimes(G_1^\bullet,G_2^o)}(a)^3\\
&= \sum_{a\in V(G)}(2d_G(a)+n_1)^3+\sum_{ab\in E(G)}(d_G(a)+d_G(b)+n_2)^3\\
&\quad +n\sum_{a\in V(G_1)}(d_{G_1}(a)+1)^3+m\sum_{a\in V(G_2)}(d_{G_2}(a)+1)^3\\
&= \sum_{a\in V(G)}[8d_G(a)^3+12n_1d_G(a)^2+6n_1^2d_G(a)+n_1^3]\\
&\quad +\sum_{ab\in E(G)}[(d_G(a)+d_G(b))^3+3n_2(d_G(a)+d_G(b))^2\\
&\quad +3n_2^2(d_G(a)+d_G(b))+n_2^3]\\
&\quad +n\sum_{a\in V(G_1)}[d_{G_1}(a)^3+3d_{G_1}(a)^2+3d_{G_1}(a)+1]\\
&\quad +m\sum_{a\in V(G_2)}[d_{G_2}(a)^3+3d_{G_2}(a)^2+3d_{G_2}(a)+1]\\
&= 8\sum_{a\in V(G)} d_G(a)^3+12n_1\sum_{a\in V(G)} d_G(a)^2+6n_1^2\sum_{a\in V(G)} d_G(a)\\
&\quad +nn_1^3+\sum_{ab\in E(G)}(d_G(a)^3+d_G(b)^3)\\
&\quad +3\sum_{ab\in E(G)} d_G(a)d_G(b)(d_G(a)+d_G(b))\\
&\quad +3n_2\sum_{ab\in E(G)}(d_G(a)+d_G(b))^2\\
&\quad +3n_2^2\sum_{ab\in E(G)}(d_G(a)+d_G(b))+mn_2^3\\
&\quad +n\sum_{a\in V(G_1)} d_{G_1}(a)^3+3n\sum_{a\in V(G_1)} d_{G_1}(a)^2\\
&\quad +3n\sum_{a\in V(G_1)} d_{G_1}(a)+n_1n+m\sum_{a\in V(G_2)} d_{G_2}(a)^3\\
&\quad +3m\sum_{a\in V(G_2)} d_{G_2}(a)^2+3m\sum_{a\in V(G_2)} d_{G_2}(a)+n_2m\\
&= 8F(G)+12n_1M_1(G)+12mn_1^2+nn_1^3+M_4(G)+3ReZM(G)\\
&\quad +3n_2HM(G)+3n_2^2M_1(G)+mn_2^3+nF(G_1)+3nM_1(G_1)\\
&\quad +6nm_1+n_1n+mF(G_2)+3mM_1(G_2)+6mm_2+n_2m.
\end{aligned}$$

□

Hence, the theorem.

Example 2.12. Using our derived result as in Equation 4, we get

$$\begin{aligned}
(i)\ F(P_l^T\otimes(C_m^\bullet,C_n^o)) &= n(l-1)(n^2+1)+lm(m^2+1)+12l(m^2+n^2)\\
&\quad -3(4m^2+6n^2)+74lm+74ln+128l-72m\\
&\quad -116n-250,\ l,m,n\geq 3,\\
(ii)\ F(C_l^T\otimes(P_m^\bullet,C_n^o)) &= ln(n^2+1)+lm(m^2+1)+12l(m^2+n^2)+74lm\\
&\quad +74ln+90l,\ l,n\geq 3,\ m\geq 2.
\end{aligned}$$

3. Conclusions

In this work, we study the double corona of graphs and found some closed expressions of “forgotten topological index" of this graph operation related to the different subdivision of graphs and using our results we compute “forgotten topological index" of some known graphs. In future study others topological indices to compute in this graph operations.

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