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ON THE GAUSSIAN CURVATURE FOR A HOLOMORPHIC HERMETIAN SUBMANIFOLD OF REAL$\hat{\imath}$REAL VECTOR SPACES

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Abstract. At the beginning of this paper we introduce a new class of vector spaces called real$\hat{\imath}$real vector spaces. Based on the holomorphic coordinate transformations we focus in the main part of the paper on the Hermitian submanifolds of this class of ambient spaces and study their principal characteristics as are the first and second fundamental forms and the Gaussian curvature. It is shown that such submanifolds must be flat.

1. Introduction

For an ordered pair of points $(X,Y) = (x^\nu, y^\nu)$ of the n-dimensional real vector space $\mathbb{R}^n$ with an orthonormal basis $\mathbf{e}_\nu$ $(\nu = 1, 2, ..., n)$ let $(\mathbf{r}_X, \mathbf{r}_Y)$ be an ordered pair of position vectors $\mathbf{r}_X = x^\nu \mathbf{e}_\nu$ and $\mathbf{r}_Y = y^\nu \mathbf{e}_\nu$ of X and Y, respectively. Here, we have used the *Einstein* summation convention: If an index appears repeated, once up and once down, then summation over that index is implied.

Since $\partial_{x^\nu}\mathbf{r}_X = \partial_{y^\nu}\mathbf{r}_Y = \mathbf{e}_\nu$ it follows that

$$ds^2 = ds_X^2 = ds_Y^2,$$

where $ds_X^2 = d\mathbf{r}_X \cdot d\mathbf{r}_X = dx^\nu dx_\nu$ and $ds_Y^2 = d\mathbf{r}_Y \cdot d\mathbf{r}_Y = dy^\nu dy_\nu$.

Denote by $\hat{\imath}\mathbb{R}^n$, where $\hat{\imath}$ is the imaginary unit, the n-dimensional $\hat{\imath}$real vector space [4] with an orthonormal basis $\hat{\mathbf{e}}_\nu = \hat{\imath}\mathbf{e}_\nu$. Note that this vector space is also defined over the field of real numbers $\mathbb{R}$. The union of $\mathbf{e}_\nu$ and $\hat{\mathbf{e}}_\nu$ is a set of $2n$ linearly independent vectors over $\mathbb{R}$ but not over the field of complex numbers $\mathbb{C}$ (over $\mathbb{C}$ this union is a linearly independent set of n vectors). Accordingly, if $\hat{\mathbf{r}}_Y = y^\nu \hat{\mathbf{e}}_\nu$ is the position vector of a point Y in $\hat{\imath}\mathbb{R}^n$, then $(\mathbf{r}_X, \hat{\mathbf{r}}_Y)$ is an ordered pair of position vectors of points X and Y, respectively, in the $2n$-dimensional vector space over $\mathbb{R}$ formed by the direct sum of the vector spaces $\mathbb{R}^n$ and $\hat{\imath}\mathbb{R}^n$. In symbols, $\mathbb{R}^n \oplus \hat{\imath}\mathbb{R}^n$. It is obvious that this vector space is isomorphic to the n-dimensional complex vector space $\mathbb{C}^n$ [3]. By analogy with the algebraic form of complex numbers we

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can decompose $\mathbb{C}^n$ into a real and $\hat{\imath}$real part, so that the position vector $\mathbf{r}_Z = z^\nu \mathbf{e}_\nu$ of any point $Z = (z^\nu)$ in $\mathbb{C}^n$, where $z^\nu = x^\nu + \hat{\imath} y^\nu$, is determined by $(\mathbf{r}_X, \hat{\mathbf{r}}_Y)$ in a such a way that $\mathbf{r}_Z = \mathbf{r}_X + \hat{\mathbf{r}}_Y = x^\nu \mathbf{e}_\nu + y^\nu \hat{\mathbf{e}}_\nu$. The complex vector space $\bar{\mathbb{C}}^n = \mathbb{R}^n \oplus (-\hat{\imath})\, \mathbb{R}^n$ represented by $(\mathbf{r}_X, -\hat{\mathbf{r}}_Y)$, more precisely by the position vector $\mathbf{r}_{\bar{Z}} = \bar{z}^\nu \mathbf{e}_\nu$, where $\bar{z}^\nu = x^\nu - \hat{\imath} y^\nu$ is the complex conjugate of z^ν, is the complex conjugate vector space of $\mathbb{C}^n$. The quadratic differential form $d\sigma^2 = ds_X^2 + ds_Y^2$ is the fundamental metric form of $\mathbb{C}^n$ and $\bar{\mathbb{C}}^n$. In addition, $\mathbf{r}_X = (\mathbf{r}_Z + \mathbf{r}_{\bar{Z}})/2$ and $\hat{\mathbf{r}}_Y = (\mathbf{r}_Z - \mathbf{r}_{\bar{Z}})/2$, so that

$$\partial_{x^\nu} \mathbf{r}_X = \frac{\partial_{x^\nu} (\mathbf{r}_Z + \mathbf{r}_{\bar{Z}})}{2} \text{ and } \partial_{y^\nu} \hat{\mathbf{r}}_Y = \frac{\partial_{y^\nu} (\mathbf{r}_Z - \mathbf{r}_{\bar{Z}})}{2}. \tag{1.1}$$

Therefore,

$$\partial_{x^\nu} \mathbf{r}_X - \hat{\imath} \partial_{y^\nu} \hat{\mathbf{r}}_Y = \frac{(\partial_{x^\nu} - \hat{\imath} \partial_{y^\nu}) \mathbf{r}_Z + (\partial_{x^\nu} + \hat{\imath} \partial_{y^\nu}) \mathbf{r}_{\bar{Z}}}{2} = \partial_{z^\nu} \mathbf{r}_Z + \partial_{\bar{z}^\nu} \mathbf{r}_{\bar{Z}}, \tag{1.2}$$

where $\partial_{z^\nu} := (\partial_{x^\nu} - \hat{\imath} \partial_{y^\nu})/2$ and $\partial_{\bar{z}^\nu} := (\partial_{x^\nu} + \hat{\imath} \partial_{y^\nu})/2$. Note that both $\mathbb{C}^n$ and $\bar{\mathbb{C}}^n$ have the same orthonormal basis. By contrast, consider the case in which the union of two orthonormal bases $\mathbf{e}_\nu$ and $\bar{\mathbf{e}}_\nu$ $(\nu = 1, 2, ..., n)$ for $\mathbb{C}^n$ and $\bar{\mathbb{C}}^n$, respectively, is a set of $2n$ mutually orthogonal linearly independent vectors over $\mathbb{C}$. Then, $\mathbb{C}^n \oplus \bar{\mathbb{C}}^n$ is a direct sum of the two n-dimensional complex vector spaces, whose position vectors are $\mathbf{r}_Z = z^\nu \mathbf{e}_\nu$ and $\mathbf{r}_{\bar{Z}} = \bar{z}^\nu \bar{\mathbf{e}}_\nu$, respectively. It suggests us the following very important problem. Namely, this $2n$-dimensional vector space is not closed under scalar multiplication by elements of $\mathbb{C}$ and so it is not complex. Hence, a question that arises is what type of vector spaces this space can be. As we shall see, in what follows, the answer to this question is closely related to the so-called class of real$\hat{\imath}$real vector spaces [4].

2. Real$\hat{\imath}$real vector spaces

Let the union of two orthonormal bases $\bar{\mathbf{e}}_\nu^\star$ $(\nu = 1, 2, ..., n)$ and $\hat{\mathbf{e}}_{\tilde{\nu}}^\star = \hat{\imath} \mathbf{e}_{\tilde{\nu}}^\star$ $(\tilde{\nu} = 1, 2, ..., m)$ for the n-dimensional real vector space $\mathbb{R}^n$ and m-dimensional $\hat{\imath}$real vector space $\hat{\imath}\mathbb{R}^m$, respectively, be a set of $n+m$ mutually orthogonal linearly independent vectors over $\mathbb{R}$. Then, $(\mathbf{r}_X, \hat{\mathbf{r}}_Y)$ is an ordered pair of the position vectors of points X and Y, respectively, in the $n+m$-dimensional real$\hat{\imath}$real vector space $\hat{\mathbb{R}}^{n+m}$ defined over the field of real numbers $\mathbb{R}$ and formed from the direct sum of $\mathbb{R}^n$ and $\hat{\imath}\mathbb{R}^m$. In symbols, $\hat{\mathbb{R}}^{n+m} = \mathbb{R}^n \oplus \hat{\imath}\mathbb{R}^m$. In case $m = n$ we may create coordinate transformations $x^\nu (z^\nu, \bar{z}^\nu) = (z^\nu + \bar{z}^\nu)/2$ and $y^\nu (z^\nu, \bar{z}^\nu) = -\hat{\imath} (z^\nu - \bar{z}^\nu)/2$, whose *Jacobian* is $\hat{\imath}/2$. Clearly, there is no doubt that there exists an isomorphism between the $2n$-dimensional vector spaces represented by ordered pairs of position vectors $(x^\nu \bar{\mathbf{e}}_\nu^\star, y^\nu \hat{\mathbf{e}}_\nu^\star)$ and $(z^\nu \mathbf{e}_\nu, \bar{z}^\nu \bar{\mathbf{e}}_\nu)$, where

$$\mathbf{e}_\nu = \partial_{z^\nu} (x^\nu \bar{\mathbf{e}}_\nu^\star + y^\nu \hat{\mathbf{e}}_\nu^\star) = \frac{1}{2} (\bar{\mathbf{e}}_\nu^\star - \hat{\imath} \hat{\mathbf{e}}_\nu^\star) \text{ and} \tag{2.1}$$

$$\bar{\mathbf{e}}_\nu = \partial_{\bar{z}^\nu} (x^\nu \bar{\mathbf{e}}_\nu^\star + y^\nu \hat{\mathbf{e}}_\nu^\star) = \frac{1}{2} (\bar{\mathbf{e}}_\nu^\star + \hat{\imath} \hat{\mathbf{e}}_\nu^\star).$$

So, in this case, an ordered pair of points $(X, Y) = (x^\nu, y^\nu)$, each of which belongs to only one of the two n-dimensional real vector spaces, represented by the position vectors $\mathbf{r}_X = x^\nu \bar{\mathbf{e}}_\nu^\star$ and $\hat{\mathbf{r}}_Y = y^\nu \hat{\mathbf{e}}_\nu^\star$, respectively, and which form, in a form of the direct sum $\mathbb{R}^n \oplus \hat{\imath}\mathbb{R}^n$, the real$\hat{\imath}$real vector space $\hat{\mathbb{R}}^{2n}$, represented by the ordered pair of the position vectors $(\mathbf{r}_X, \hat{\mathbf{r}}_Y)$, maps to an ordered pair of points $\left(Z, \bar{Z}\right) = (z^\nu, \bar{z}^\nu)$, each of which belongs to only one of the two n-dimensional complex vector spaces $\mathbb{C}^n$

and $\bar{\mathbb{C}}^n$, represented by the position vectors $\mathbf{r}_Z = z^\nu \mathbf{e}_\nu$ and $\mathbf{r}_{\bar{Z}} = \bar{z}^\nu \bar{\mathbf{e}}_\nu$, respectively, and wich also form, in a form of the direct sum $\mathbb{C}^n \oplus \bar{\mathbb{C}}^n$, the same real$\hat{\imath}$real vector space $\hat{\mathbb{R}}^{2n}$, but represented now by the ordered pair of the position vectors $(\mathbf{r}_Z,\mathbf{r}_{\bar{Z}})$. In addition, $\partial_{x^\nu}\mathbf{r}_X = \partial_{z^\nu}\mathbf{r}_Z + \partial_{\bar{z}^\nu}\mathbf{r}_{\bar{Z}}$ and $\partial_{y^\nu}\hat{\mathbf{r}}_Y = \hat{\imath}(\partial_{z^\nu}\mathbf{r}_Z - \partial_{\bar{z}^\nu}\mathbf{r}_{\bar{Z}})$. Finally, since $\mathbf{r}^*_Z = \bar{z}^\nu \mathbf{e}_\nu$ and $\mathbf{r}^*_{\bar{Z}} = z^\nu \bar{\mathbf{e}}_\nu$, it follows that $d\sigma^2 = dz^\nu d\bar{z}_\nu = dx^\nu dx_\nu + dy^\nu dy_\nu$, where $d\sigma^2 = d(\mathbf{r}_Z + \mathbf{r}_{\bar{Z}}) \cdot d(\mathbf{r}^*_Z + \mathbf{r}^*_{\bar{Z}}) = |d(\mathbf{r}_Z + \mathbf{r}_{\bar{Z}})|^2$.

Clearly, it is very important to distinguish between $\mathbb{C}^{2n}$ and $\hat{\mathbb{R}}^{2n}$. Accordingly, the so-called "complex plane" is not a flat two-dimensional complex vector space, but a flat two-dimensional real$\hat{\imath}$real vector space, so that it must bear the name real$\hat{\imath}$real plane.

2.1. **A holomorphic *Hermitian* submanifold of $\hat{\mathbb{R}}^{2n}$.** Let $\hat{\mathbb{R}}^{2n} = \mathbb{R}^n \oplus \hat{\imath}\mathbb{R}^n$ be a $2n$-dimensional real$\hat{\imath}$real vector space. To consider a map from the ambient space $\hat{\mathbb{R}}^{2n}$ to a $2m$-dimensional *Hermitian* manifold $M^{\hat{\mathbb{R}}^{2n}}$, embedded in $\hat{\mathbb{R}}^{2n}$, we shall create the coordinate transformations $z^\nu\left(z^i, \bar{z}^i\right) = x^\nu\left(x^i, y^i\right) + \hat{\imath} y^\nu\left(x^i, y^i\right)$ and $\bar{z}^\nu\left(z^i, \bar{z}^i\right) = x^\nu\left(x^i, y^i\right) - \hat{\imath} y^\nu\left(x^i, y^i\right)$, where $z^i = x^i + \hat{\imath} y^i$ and $\bar{z}^i = x^i - \hat{\imath} y^i$, for each $i = 1, 2, ..., m < n$. Note, that either of the following two pairs of vectors:

$$\begin{aligned} \mathbf{h}_i &= \partial_{z^i}(\mathbf{r}_Z + \mathbf{r}_{\bar{Z}}) = \partial_{z^i}(z^\nu \mathbf{e}_\nu + \bar{z}^\nu \bar{\mathbf{e}}_\nu) \text{ and} \\ \bar{\mathbf{h}}_i &= \partial_{\bar{z}^i}(\mathbf{r}_Z + \mathbf{r}_{\bar{Z}}) = \partial_{\bar{z}^i}(z^\nu \mathbf{e}_\nu + \bar{z}^\nu \bar{\mathbf{e}}_\nu), \end{aligned} \tag{2.2}$$

or

$$\begin{aligned} \mathbf{g}_i &= \partial_{x^i}(\mathbf{r}_X + \hat{\mathbf{r}}_Y) = \partial_{x^i}(x^\nu \bar{\mathbf{e}}^\star_\nu + y^\nu \hat{\mathbf{e}}^\star_\nu) \text{ and} \\ \hat{\mathbf{g}}_i &= \partial_{y^i}(\mathbf{r}_X + \hat{\mathbf{r}}_Y) = \partial_{y^i}(x^\nu \bar{\mathbf{e}}^\star_\nu + y^\nu \hat{\mathbf{e}}^\star_\nu) \ , \end{aligned} \tag{2.3}$$

can be taken to be the covariant basis for $M^{\hat{\mathbb{R}}^{2n}}$. Dual bases $\mathbf{h}^j$, $\bar{\mathbf{h}}^j$, $\bar{\mathbf{g}}^j$, and $\hat{\mathbf{g}}^j$ associated to these covariant bases are defined as follows

$$\mathbf{h}^j \cdot \mathbf{h}_i = \bar{\mathbf{h}}^j \cdot \bar{\mathbf{h}}_i = \mathbf{g}^j \cdot \mathbf{g}_i = \hat{\mathbf{g}}^j \cdot \hat{\mathbf{g}}_i = \delta^j_i, \tag{2.4}$$

where δ^j_i is *Kronecker's* delta symbol. Now, if each coordinate function $z^\nu\left(z^i, \bar{z}^i\right)$ on $\hat{\mathbb{R}}^{2n}$ is a holomorphic function on $M^{\hat{\mathbb{R}}^{2n}}$, meaning that it satisfies the *Cauchy-Riemann* equations: $\partial_{x^i}x^\nu = \partial_{y^i}y^\nu$ and $\partial_{x^i}y^\nu = -\partial_{y^i}x^\nu$, which can be expressed in a slightly nicer form $\partial_{\bar{z}^i}z^\nu \equiv 0$ or $\partial_{z^i}\bar{z}^\nu \equiv 0$, then

$$\mathbf{h}_i = \partial_{z^i}\mathbf{r}_Z = \partial_{z^i}z^\nu \mathbf{e}_\nu \text{ and } \bar{\mathbf{h}}_i = \partial_{\bar{z}^i}\mathbf{r}_{\bar{Z}} = \partial_{\bar{z}^i}\bar{z}^\nu \bar{\mathbf{e}}_\nu. \tag{2.5}$$

On the other hand, it follows from the aforementioned *Cauchy-Riemann* equations that

$$\partial_{z^i}x^\nu = \hat{\imath}\partial_{z^i}y^\nu \text{ and } \partial_{\bar{z}^i}x^\nu = -\hat{\imath}\partial_{\bar{z}^i}y^\nu. \tag{2.6}$$

Hence,

$$\begin{aligned} \mathbf{h}_i &= \partial_{z^i}(x^\nu + \hat{\imath}y^\nu)\mathbf{e}_\nu = 2\partial_{z^i}x^\nu \mathbf{e}_\nu = 2\hat{\imath}\partial_{z^i}y^\nu \mathbf{e}_\nu \text{ and} \\ \bar{\mathbf{h}}_i &= \partial_{\bar{z}^i}(x^\nu - \hat{\imath}y^\nu)\bar{\mathbf{e}}_\nu = 2\partial_{\bar{z}^i}x^\nu \bar{\mathbf{e}}_\nu = -2\hat{\imath}\partial_{\bar{z}^i}y^\nu \bar{\mathbf{e}}_\nu. \end{aligned} \tag{2.7}$$

This leads to

$$\begin{aligned} \mathbf{h}_i &= (\partial_{x^i} - \hat{\imath}\partial_{y^i})x^\nu \mathbf{e}_\nu \ = \partial_{x^i}\mathbf{r}_Z = -\hat{\imath}\partial_{y^i}\mathbf{r}_Z \text{ and} \\ \bar{\mathbf{h}}_i &= (\partial_{x^i} + \hat{\imath}\partial_{y^i})x^\nu \bar{\mathbf{e}}_\nu = \partial_{x^i}\mathbf{r}_{\bar{Z}} = \hat{\imath}\partial_{y^i}\mathbf{r}_{\bar{Z}}. \end{aligned} \tag{2.8}$$

Since

$$\partial_{z^i}\mathbf{r}_X = \partial_{z^i}x^\nu \bar{\mathbf{e}}^\star_\nu = \frac{1}{2}(\partial_{x^i} - \hat{\imath}\partial_{y^i})x^\nu \bar{\mathbf{e}}^\star_\nu = \frac{1}{2}\partial_{x^i}z^\nu \bar{\mathbf{e}}^\star_\nu = -\frac{\hat{\imath}}{2}\partial_{y^i}z^\nu \bar{\mathbf{e}}^\star_\nu \text{ and} \tag{2.9}$$

$$\partial_{z^i}\hat{\mathbf{r}}_Y = \partial_{z^i} y^\nu \hat{\mathbf{e}}^\star_\nu = \frac{1}{2}(\partial_{x^i} - \hat{\imath}\partial_{y^i})y^\nu \hat{\mathbf{e}}^\star_\nu = -\frac{\hat{\imath}}{2}\partial_{x^i} z^\nu \hat{\mathbf{e}}^\star_\nu = -\frac{1}{2}\partial_{y^i} z^\nu \hat{\mathbf{e}}^\star_\nu,$$

as well as,

$$\partial_{\bar{z}^j}\mathbf{r}_X = \partial_{\bar{z}^i} x^\nu \bar{\mathbf{e}}^\star_\nu = \frac{1}{2}(\partial_{x^i} + \hat{\imath}\partial_{y^i})x^\nu \bar{\mathbf{e}}^\star_\nu = \frac{1}{2}\partial_{x^i} \bar{z}^\nu \bar{\mathbf{e}}^\star_\nu = \frac{\hat{\imath}}{2}\partial_{y^i} \bar{z}^\nu \bar{\mathbf{e}}^\star_\nu \text{ and} \tag{2.10}$$

$$\partial_{\bar{z}^j}\hat{\mathbf{r}}_Y = \partial_{\bar{z}^i} y^\nu \hat{\mathbf{e}}^\star_\nu = \frac{1}{2}(\partial_{x^i} + \hat{\imath}\partial_{y^i})y^\nu \hat{\mathbf{e}}^\star_\nu = -\frac{1}{2}\partial_{y^i} \bar{z}^\nu \hat{\mathbf{e}}^\star_\nu = \frac{\hat{\imath}}{2}\partial_{x^i} \bar{z}^\nu \hat{\mathbf{e}}^\star_\nu,$$

we finally have that

$$\mathbf{h}_i = \partial_{z^i}(\mathbf{r}_X + \hat{\mathbf{r}}_Y) = \frac{\mathbf{g}_i - \hat{\imath}\hat{\mathbf{g}}_i}{2} \text{ and } \bar{\mathbf{h}}_i = \partial_{\bar{z}^i}(\mathbf{r}_X + \hat{\mathbf{r}}_Y) = \frac{\mathbf{g}_i + \hat{\imath}\hat{\mathbf{g}}_i}{2}, \tag{2.11}$$

that is,

$$\mathbf{g}_i = \mathbf{h}_i + \bar{\mathbf{h}}_i \text{ and } \hat{\mathbf{g}}_i = \hat{\imath}(\mathbf{h}_i - \bar{\mathbf{h}}_i). \tag{2.12}$$

Similarly,

$$\mathbf{h}^*_j = \partial_{\bar{z}^j}\mathbf{r}^*_Z = \partial_{\bar{z}^j}\bar{z}^\nu \mathbf{e}_\nu = \partial_{\bar{z}^j}(\mathbf{r}_X - \hat{\mathbf{r}}_Y) = \frac{\mathbf{g}^*_j + \hat{\imath}\hat{\mathbf{g}}^*_j}{2} \text{ and} \tag{2.13}$$

$$\bar{\mathbf{h}}^*_j = \partial_{z^j}\mathbf{r}^*_{\bar{Z}} = \partial_{z^j} z^\nu \bar{\mathbf{e}}_\nu = \partial_{z^j}(\mathbf{r}_X - \hat{\mathbf{r}}_Y) = \frac{\mathbf{g}^*_j - \hat{\imath}\hat{\mathbf{g}}^*_j}{2},$$

where $\mathbf{g}^*_j = \partial_{x^j}(\mathbf{r}_X - \hat{\mathbf{r}}_Y)$ and $\hat{\mathbf{g}}^*_j = \partial_{y^j}(\mathbf{r}_X - \hat{\mathbf{r}}_Y)$, that is,

$$\mathbf{g}^*_j = \mathbf{h}^*_j + \bar{\mathbf{h}}^*_j \text{ and } \hat{\mathbf{g}}^*_j = -\hat{\imath}(\mathbf{h}^*_j - \bar{\mathbf{h}}^*_j). \tag{2.14}$$

If we now introduce four vector operators as follows:

$$\mathbf{d}_{i,\nu} = \frac{1}{2}(\partial_{x^i}\bar{\mathbf{e}}^\star_\nu - \partial_{y^i}\hat{\mathbf{e}}^\star_\nu) = \partial_{z^i}\mathbf{e}_\nu + \partial_{\bar{z}^i}\bar{\mathbf{e}}_\nu, \tag{2.15}$$

$$\mathbf{d}^2_{ji,\nu} = \frac{1}{4}[(\partial^2_{x^j x^i} - \partial^2_{y^j y^i})\bar{\mathbf{e}}^\star_\nu - (\partial^2_{x^j y^i} + \partial^2_{y^j x^i})\hat{\mathbf{e}}^\star_\nu] = \partial^2_{z^j z^i}\mathbf{e}_\nu + \partial^2_{\bar{z}^j \bar{z}^i}\bar{\mathbf{e}}_\nu,$$

$$\bar{\mathbf{d}}_{j,\nu} = \frac{1}{2}(\partial_{x^j}\bar{\mathbf{e}}^\star_\nu + \partial_{y^j}\hat{\mathbf{e}}^\star_\nu) = \partial_{\bar{z}^j}\mathbf{e}_\nu + \partial_{z^j}\bar{\mathbf{e}}_\nu \text{ and}$$

$$\bar{\mathbf{d}}^2_{ji,\nu} = \frac{1}{4}[(\partial^2_{x^j x^i} - \partial^2_{y^j y^i})\bar{\mathbf{e}}^\star_\nu + (\partial^2_{x^j y^i} + \partial^2_{y^j x^i})\hat{\mathbf{e}}^\star_\nu] = \partial^2_{\bar{z}^j \bar{z}^i}\mathbf{e}_\nu + \partial^2_{z^j z^i}\bar{\mathbf{e}}_\nu,$$

then

$$\mathbf{h}_i = \mathbf{d}_{i,\nu} z^\nu,\ \bar{\mathbf{h}}_i = \mathbf{d}_{i,\nu}\bar{z}^\nu,\ \mathbf{h}^*_j = \bar{\mathbf{d}}_{j,\nu}\bar{z}^\nu,\ \bar{\mathbf{h}}^*_j = \bar{\mathbf{d}}_{j,\nu} z^\nu, \tag{2.16}$$

$$\mathbf{g}_i = 2\mathbf{d}_{i,\nu}x^\nu,\ \hat{\mathbf{g}}_i = -2\mathbf{d}_{i,\nu}y^\nu,\ \mathbf{g}^*_j = 2\bar{\mathbf{d}}_{j,\nu}x^\nu, \text{ and } \hat{\mathbf{g}}^*_j = -2\bar{\mathbf{d}}_{j,\nu}y^\nu.$$

The vector relation $\mathbf{d}_{i,\nu}\partial_{y^j}x^\nu = -\mathbf{d}_{i,\nu}\partial_{x^j}y^\nu$ obtained from the *Cauchy-Riemann* equations leads to the *Schwartz* integrability conditions $\partial_{y^j}\mathbf{g}_i - \partial_{x^j}\hat{\mathbf{g}}_i \equiv 0$ under which the differential $d(\mathbf{r}_X + \hat{\mathbf{r}}_Y) = \mathbf{g}_i dx^i + \hat{\mathbf{g}}_i dy^i$ becomes an exact one (also called a total differential). These conditions are often occur in their scalar component form

$$\partial^2_{y^j x^i}x^\nu = \partial^2_{x^j y^i}x^\nu \text{ and } \partial^2_{y^j x^i}y^\nu = \partial^2_{x^j y^i}y^\nu, \tag{2.17}$$

that is,

$$\partial^2_{y^j y^i}y^\nu = -\partial^2_{x^j x^i}y^\nu \text{ and } \partial^2_{y^j y^i}x^\nu = -\partial^2_{x^j x^i}x^\nu. \tag{2.18}$$

Thus,

$$\mathbf{d}_{j,\nu}\cdot\mathbf{g}_i = \mathbf{d}_{j,\nu}\cdot\partial_{x^i}(\mathbf{r}_X + \hat{\mathbf{r}}_Y) = \frac{1}{2}(\partial^2_{x^j x^i}\mathbf{r}_X\cdot\bar{\mathbf{e}}^\star_\nu - \partial^2_{y^j x^i}\hat{\mathbf{r}}_Y\cdot\hat{\mathbf{e}}^\star_\nu) = \tag{2.19}$$

$$= \partial^2_{x^j x^i}\mathbf{r}_X\cdot\bar{\mathbf{e}}^\star_\nu = -\partial^2_{y^j x^i}\hat{\mathbf{r}}_Y\cdot\hat{\mathbf{e}}^\star_\nu \text{ and}$$

$$\mathbf{d}^2_{ji,\nu}\cdot(\mathbf{r}_X+\hat{\mathbf{r}}_Y)=\frac{1}{4}(\partial^2_{x^jx^i}\mathbf{r}_X\cdot\bar{\mathbf{e}}^\star_\nu-\partial^2_{y^jy^i}\mathbf{r}_X\cdot\bar{\mathbf{e}}^\star_\nu-\partial^2_{x^jy^i}\hat{\mathbf{r}}_Y\cdot\hat{\mathbf{e}}^\star_\nu-\partial^2_{y^jx^i}\hat{\mathbf{r}}_Y\cdot\hat{\mathbf{e}}^\star_\nu)=$$
$$=\partial^2_{x^jx^i}\mathbf{r}_X\cdot\bar{\mathbf{e}}^\star_\nu=-\partial^2_{y^jy^i}\mathbf{r}_X\cdot\bar{\mathbf{e}}^\star_\nu=-\partial^2_{y^jx^i}\hat{\mathbf{r}}_Y\cdot\hat{\mathbf{e}}^\star_\nu=-\partial^2_{x^jy^i}\hat{\mathbf{r}}_Y\cdot\hat{\mathbf{e}}^\star_\nu=$$
$$=\partial_{x^j}[\mathbf{d}_{i,\nu}\cdot(\mathbf{r}_X+\hat{\mathbf{r}}_Y)]=\mathbf{d}_{j,\nu}\cdot\mathbf{g}_i.$$

since $\mathbf{d}_{i,\nu}\cdot(\mathbf{r}_X+\hat{\mathbf{r}}_Y)=(1/2)(\partial_{x^i}\mathbf{r}_X\cdot\bar{\mathbf{e}}^\star_\nu-\partial_{y^i}\hat{\mathbf{r}}_Y\cdot\hat{\mathbf{e}}^\star_\nu)=\partial_{x^i}\mathbf{r}_X\cdot\bar{\mathbf{e}}^\star_\nu=-\partial_{y^i}\hat{\mathbf{r}}_Y\cdot\hat{\mathbf{e}}^\star_\nu$. In addition,

(2.20)
$$\mathbf{d}^2_{ji,\nu}x^\nu=\frac{1}{4}[(\partial^2_{x^jx^i}x^\nu-\partial^2_{y^jy^i}x^\nu)\bar{\mathbf{e}}^\star_\nu-(\partial^2_{x^jy^i}x^\nu+\partial^2_{y^jx^i}x^\nu)\hat{\mathbf{e}}^\star_\nu]=$$
$$=\frac{1}{2}(\partial^2_{x^jx^i}x^\nu\bar{\mathbf{e}}^\star_\nu+\partial^2_{x^jx^i}y^\nu\hat{\mathbf{e}}^\star_\nu)=\frac{1}{2}\partial_{x^j}\mathbf{g}_i=-\frac{1}{2}\partial_{y^j}\hat{\mathbf{g}}_i \text{ and}$$
$$\mathbf{d}^2_{ji,\nu}y^\nu=\frac{1}{4}[(\partial^2_{x^jx^i}y^\nu-\partial^2_{y^jy^i}y^\nu)\bar{\mathbf{e}}^\star_\nu-(\partial^2_{x^jy^i}y^\nu+\partial^2_{y^jx^i}y^\nu)\hat{\mathbf{e}}^\star_\nu]=$$
$$=\frac{1}{2}(-\partial^2_{y^jx^i}x^\nu\bar{\mathbf{e}}^\star_\nu-\partial^2_{y^jx^i}y^\nu\hat{\mathbf{e}}^\star_\nu)=-\frac{1}{2}\partial_{x^j}\hat{\mathbf{g}}_i=-\frac{1}{2}\partial_{y^j}\mathbf{g}_i.$$

Adding the second equation multiplied by either $\hat{\imath}$ or $-\hat{\imath}$ to the first, we finally get that

(2.21)
$$\mathbf{d}^2_{ji,\nu}z^\nu=\partial_{z^j}\mathbf{g}_i=-\hat{\imath}\partial_{z^j}\hat{\mathbf{g}}_i=\partial_{x^j}\mathbf{h}_i=-\hat{\imath}\partial_{y^j}\mathbf{h}_i=\partial_{z^j}\mathbf{h}_i \text{ and}$$
$$\mathbf{d}^2_{ji,\nu}\bar{z}^\nu=\partial_{\bar{z}^j}\mathbf{g}_i=\hat{\imath}\partial_{\bar{z}^j}\hat{\mathbf{g}}_i=\partial_{x^j}\bar{\mathbf{h}}_i=\hat{\imath}\partial_{y^j}\bar{\mathbf{h}}_i=\partial_{\bar{z}^j}\bar{\mathbf{h}}_i.$$

Similarly,

(2.22)
$$\bar{\mathbf{d}}^2_{ji,\nu}x^\nu=\frac{1}{2}\partial_{x^j}\mathbf{g}^*_i=-\frac{1}{2}\partial_{y^j}\hat{\mathbf{g}}^*_i \text{ and } \bar{\mathbf{d}}^2_{ji,\nu}y^\nu=-\frac{1}{2}\partial_{x^j}\hat{\mathbf{g}}^*_i=-\frac{1}{2}\partial_{y^j}\mathbf{g}^*_i,$$

that is,

(2.23)
$$\bar{\mathbf{d}}^2_{ji,\nu}\bar{z}^\nu=\partial_{\bar{z}^j}\mathbf{g}^*_i=\hat{\imath}\partial_{\bar{z}^j}\hat{\mathbf{g}}^*_i=\partial_{x^j}\mathbf{h}^*_i=\hat{\imath}\partial_{y^j}\mathbf{h}^*_i=\partial_{\bar{z}^j}\mathbf{h}^*_i \text{ and}$$
$$\bar{\mathbf{d}}^2_{ji,\nu}z^\nu=\partial_{z^j}\mathbf{g}^*_i=-\hat{\imath}\partial_{z^j}\hat{\mathbf{g}}^*_i=\partial_{x^j}\bar{\mathbf{h}}^*_i=-\hat{\imath}\partial_{y^j}\bar{\mathbf{h}}^*_i=\partial_{z^j}\bar{\mathbf{h}}^*_i.$$

By (2.16), it is easy to see that

(2.24)
$$h_{ij}=\mathbf{h}_i\cdot\mathbf{h}^*_j=\mathbf{d}_{i,\nu}z^\nu\cdot\bar{\mathbf{d}}_{j,\nu}\bar{z}^\nu=\bar{\mathbf{h}}_i\cdot\bar{\mathbf{h}}^*_j=\mathbf{d}_{i,\nu}\bar{z}^\nu\cdot\bar{\mathbf{d}}_{j,\nu}z^\nu=$$
$$=\frac{1}{2}(\partial_{x^i}x^\nu\partial_{x^j}x^\nu+\partial_{y^i}x^\nu\partial_{y^j}x^\nu)=\frac{1}{2}g_{ij},$$

where

(2.25)
$$g_{ij}=\mathbf{g}_i\cdot\mathbf{g}^*_j=\hat{\mathbf{g}}_i\cdot\hat{\mathbf{g}}^*_j=4(\mathbf{d}_{i,\nu}x^\nu\cdot\bar{\mathbf{d}}_{j,\nu}x^\nu)=4(\mathbf{d}_{i,\nu}y^\nu\cdot\bar{\mathbf{d}}_{j,\nu}y^\nu).$$

Hence,

(2.26)
$$(d\sigma)^2=d(\mathbf{r}_Z+\mathbf{r}_{\bar{Z}})\cdot d(\mathbf{r}^*_Z+d\mathbf{r}^*_{\bar{Z}})=2h_{ij}dz^id\bar{z}^j=$$
$$=g_{ij}(dx^idx^j+dy^idy^j)=d(\mathbf{r}_X+\hat{\mathbf{r}}_Y)\cdot d(\mathbf{r}^*_X+\hat{\mathbf{r}}^*_Y).$$

Based on (2.18) we can see that x^ν and y^ν are harmonic functions on $\hat{\mathbb{R}}^{2n}$ since the *Riemannian Laplacian* of x^ν and y^ν vanishes identically on $\hat{\mathbb{R}}^{2n}$. In other words, $M^{\hat{\mathbb{R}}^{2n}}$ is a minimal hypersurface immersed in $\hat{\mathbb{R}}^{2n}$ [2]. In what follows we will show that $M^{\hat{\mathbb{R}}^{2n}}$is a trivial minimal hypersurface immersed in $\hat{\mathbb{R}}^{2n}$ [5].

For $\Lambda=1,2,...,n-m$ let $\mathbf{n}_\Lambda$ and $\bar{\mathbf{n}}_\Lambda$ be two mutually orthogonal corresponding sets of linearly independent unit vectors that span the orthogonal complement $M^{\perp\hat{\mathbb{R}}^{2n}}$ of $M^{\hat{\mathbb{R}}^{2n}}$ in $\hat{\mathbb{R}}^{2n}$ and let $\mathbf{n}^\Lambda$ and $\bar{\mathbf{n}}^\Lambda$ be dual bases associated to $\mathbf{n}_\Lambda$ and $\bar{\mathbf{n}}_\Lambda$, respectively. Then,

(2.27)
$$\mathbf{d}^2_{ji,\nu}z^\nu=\partial_{z^j}\mathbf{h}_i=(\partial_{z^j}\mathbf{h}_i\cdot\mathbf{h}^k)\mathbf{h}_k+(\partial_{z^j}\mathbf{h}_i\cdot\mathbf{n}_\Lambda)\mathbf{n}^\Lambda=(\nabla_j+\Delta_j)\mathbf{h}_i \text{ and}$$

$$\mathbf{d}^2_{ji,\nu}\bar{z}^\nu = \partial_{\bar{z}^j}\bar{\mathbf{h}}_i = (\partial_{\bar{z}^j}\bar{\mathbf{h}}_i \cdot \bar{\mathbf{h}}^k)\bar{\mathbf{h}}_k + (\partial_{\bar{z}^j}\bar{\mathbf{h}}_i \cdot \bar{\mathbf{n}}_\Lambda)\bar{\mathbf{n}}^\Lambda = (\nabla_j + \Delta_j)\bar{\mathbf{h}}_i,$$

where ∇_j and Δ_j are the connections on $M^{\hat{\mathbb{R}}^{2n}}$ and $M^{\perp\hat{\mathbb{R}}^{2n}}$, respectively, such that

$$\nabla_j\mathbf{h}_i = (\mathbf{d}^2_{ji,\nu}z^\nu \cdot \mathbf{h}^k)\mathbf{h}_k = \Gamma^k_{ji}\mathbf{h}_k,\ \nabla_j\bar{\mathbf{h}}_i = (\mathbf{d}^2_{ji,\nu}\bar{z}^\nu \cdot \bar{\mathbf{h}}^k)\bar{\mathbf{h}}_k = \bar{\Gamma}^k_{ji}\bar{\mathbf{h}}_k, \tag{2.28}$$

$$\Delta_j\mathbf{h}_i = (\partial_{z^j}\mathbf{h}_i \cdot \mathbf{n}_\Lambda)\mathbf{n}^\Lambda = \tau_{ji,\Lambda}\mathbf{n}^\Lambda \text{ and } \Delta_j\bar{\mathbf{h}}_i = (\partial_{\bar{z}^j}\bar{\mathbf{h}}_i \cdot \bar{\mathbf{n}}_\Lambda)\bar{\mathbf{n}}^\Lambda = \bar{\tau}_{ji,\Lambda}\bar{\mathbf{n}}^\Lambda.$$

Considering the fact that the contravariant vectors $\mathbf{h}^i$ and $\mathbf{h}^{*j}$ are a linear combination of the fundamental vectors $\mathbf{h}^*_l$ and $\mathbf{h}_l$, respectively, that means that $\mathbf{h}^k = \mathbf{h}^*_l h^{lk}$ and $\mathbf{h}^{*j} = \mathbf{h}_l h^{lj}$, where $h^{lk} = \mathbf{h}^l \cdot \mathbf{h}^{*k}$ are the contravariant components of the fundamental tensor h_{lk}, the *Christoffel* symbols Γ^k_{ji} and $\bar{\Gamma}^k_{ji}$ are defined as

$$\Gamma^k_{ji} = \partial_{z^j}\mathbf{g}_i \cdot \mathbf{g}^k = \mathbf{d}^2_{ji,\nu}z^\nu \cdot \mathbf{g}^k = \mathbf{d}^2_{ji,\nu}z^\nu \cdot (\mathbf{h}^k + \bar{\mathbf{h}}^k) = \mathbf{d}^2_{ji,\nu}z^\nu \cdot \mathbf{h}^k = \tag{2.29}$$

$$= \partial_{z^j}\mathbf{h}_i \cdot \mathbf{h}^k = \partial_{z^j}\mathbf{h}_i \cdot \mathbf{h}^*_l h^{lk} = \partial_{z^j}h_{il}h^{lk} = -h_{il}\partial_{z^j}h^{lk} \text{ and}$$

$$\bar{\Gamma}^k_{ji} = \partial_{\bar{z}^j}\mathbf{g}_i \cdot \mathbf{g}^k = \mathbf{d}^2_{ji,\nu}\bar{z}^\nu \cdot \mathbf{g}^k = \mathbf{d}^2_{ji,\nu}\bar{z}^\nu \cdot (\mathbf{h}^k + \bar{\mathbf{h}}^k) = \mathbf{d}^2_{ji,\nu}\bar{z}^\nu \cdot \bar{\mathbf{h}}^k =$$

$$= \partial_{\bar{z}^j}\bar{\mathbf{h}}_i \cdot \bar{\mathbf{h}}^k = \partial_{\bar{z}^j}\bar{\mathbf{h}}_i \cdot \bar{\mathbf{h}}^*_l h^{lk} = \partial_{\bar{z}^j}h_{il}h^{lk} = -h_{il}\partial_{\bar{z}^j}h^{lk},$$

so that

$$\partial_{x^j}\mathbf{g}_i \cdot \mathbf{g}^k = \Gamma^k_{ji} + \bar{\Gamma}^k_{ji} = \partial_{x^j}g_{il}g^{lk} \text{ and } \partial_{y^j}\mathbf{g}_i \cdot \mathbf{g}^k = \hat{\imath}(\Gamma^k_{ji} - \bar{\Gamma}^k_{ji}) = \partial_{y^j}g_{il}g^{lk}, \tag{2.30}$$

since $h^{lj} = 2g^{ij}$. In addition,

$$d^2(\mathbf{r}_Z + \mathbf{r}_{\bar{Z}}) = \mathbf{d}^2_{ji,\nu}z^\nu dz^j dz^i + \mathbf{d}^2_{ji,\nu}\bar{z}^\nu d\bar{z}^j d\bar{z}^i = \partial_{z^j}\mathbf{g}_i dz^j dz^i + \partial_{\bar{z}^j}\mathbf{g}_i d\bar{z}^j d\bar{z}^i = \tag{2.31}$$

$$= \partial_{x^j}\mathbf{g}_i(dx^j dx^i - dy^j dy^i) + \partial_{y^j}\mathbf{g}_i(dx^j dy^i + dy^j dx^i),$$

that leads to the second fundamental forms of $M^{\hat{\mathbb{R}}^{2n}}$ as follows

$$(d^2\sigma)_\Lambda = d^2(\mathbf{r}_Z + \mathbf{r}_{\bar{Z}}) \cdot (\mathbf{n}_\Lambda + \bar{\mathbf{n}}_\Lambda) = \tau_{ji,\Lambda}dz^j dz^i + \bar{\tau}_{ji,\Lambda}d\bar{z}^j d\bar{z}^i = \tag{2.32}$$

$$= \iota_{ji,\Lambda}(dx^j dx^i - dy^j dy^i) + t_{ji,\Lambda}(dx^j dy^i + dy^j dx^i) = d^2(\mathbf{r}_X + \hat{\mathbf{r}}_Y) \cdot (\mathbf{n}_\Lambda + \bar{\mathbf{n}}_\Lambda),$$

where $\iota_{ji,\Lambda} = \partial_{x^j}\mathbf{g}_i \cdot (\mathbf{n}_\Lambda + \bar{\mathbf{n}}_\Lambda) = -\partial_{y^j}\hat{\mathbf{g}}_i \cdot (\mathbf{n}_\Lambda + \bar{\mathbf{n}}_\Lambda)$ and $t_{ji,\Lambda} = \partial_{y^j}\mathbf{g}_i \cdot (\mathbf{n}_\Lambda + \bar{\mathbf{n}}_\Lambda) = \partial_{x^j}\hat{\mathbf{g}}_i \cdot (\mathbf{n}_\Lambda + \bar{\mathbf{n}}_\Lambda)$. Clearly,

$$\tau_{ji,\Lambda} = \frac{1}{2}(\iota_{ji,\Lambda} - \hat{\imath}t_{ji,\Lambda}) \text{ and } \bar{\tau}_{ji,\Lambda} = \frac{1}{2}(\iota_{ji,\Lambda} + \hat{\imath}t_{ji,\Lambda}). \tag{2.33}$$

As is well-known to us, the *Gaussian* curvature κ_G of a surface in $\mathbb{R}^3$ can be calculated as the ratio of the determinants of the second and first fundamental forms [1]. Hence, if we now introduce the *Gaussian* curvature tensor of $M^{\hat{\mathbb{R}}^{2n}}$ as follows

$$G_{jilk} = \tau_{ji,\Lambda}\bar{\tau}^\Lambda_{lk} = \frac{\iota_{ji,\Lambda}\iota^\Lambda_{lk} + t_{ji,\Lambda}t^\Lambda_{lk}}{4}, \tag{2.34}$$

then the *Gaussian* curvature κ_G of $M^{\hat{\mathbb{R}}^{2n}}$ can be calculated as

$$\kappa_G = \left|\kappa^j_i\right| = \left|\tau^l_{j,\Lambda}\bar{\tau}^{i,\Lambda}_l\right| = \frac{\left|G_{jilk}h^{ki}\right|}{|h_{jl}|} = \tag{2.35}$$

$$= \frac{\left|\tau_{ji,\Lambda}\bar{\tau}^\Lambda_{lk}h^{ki}\right|}{|h_{jl}|} = \frac{\left|(\iota_{ji,\Lambda}\iota^\Lambda_{lk} + t_{ji,\Lambda}t^\Lambda_{lk})g^{ki}\right|}{|g_{jl}|}.$$

However, since $\mathbf{h}^*_l = h_{il}\mathbf{h}^i$ and $\mathbf{h}_l = h_{il}\mathbf{h}^{*i}$, it follows from (2.29) that

$$\partial_{z^j}\mathbf{h}^k = \partial_{z^j}(\mathbf{h}^*_l h^{lk}) = -\Gamma^k_{ij}\mathbf{h}^i \text{ and } \partial_{\bar{z}^j}\mathbf{h}^{*k} = \partial_{\bar{z}^j}(\mathbf{h}_l h^{lk}) = -\bar{\Gamma}^k_{ij}\mathbf{h}^{*i}, \tag{2.36}$$

as well as

$$\partial_{\bar{z}^j}\mathbf{h}_l^* = \partial_{\bar{z}^j}(h_{il}\mathbf{h}^i) = \bar{\Gamma}_{lj}^k\mathbf{h}_k^* \text{ and } \partial_{z^j}\mathbf{h}_l = \partial_{z^j}(h_{il}\mathbf{h}^{*i}) = \Gamma_{lj}^k\mathbf{h}_k, \tag{2.37}$$

meaning that $\mathbf{h}_j^*$ and $\partial_{z^j}\mathbf{h}_l$ lie in the tangent vector space spanned by $\mathbf{h}_i$, more precisely, $M^{\hat{\mathbb{R}}^{2n}}$ is a flat space. In other words, since $\Delta_j\mathbf{h}_i \equiv \mathbf{0}$ and $\Delta_j\bar{\mathbf{h}}_i \equiv \mathbf{0}$, that leads to

$$G_{jilk} \equiv 0 \text{ and } \kappa_G \equiv 0, \tag{2.38}$$

it follows that if each coordinate function $z^\nu\left(z^i, \bar{z}^i\right)$ on $\hat{\mathbb{R}}^{2n}$ is a holomorphic function on $M^{\hat{\mathbb{R}}^{2n}}$, then $M^{\hat{\mathbb{R}}^{2n}}$ must be a flat space.

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