Commun.Fac.Sci.Univ.Ank.Series A1
Volume 56, Number 1, Pages 21–32 (2007)
ISSN 1303–5991

DECOMPOSITIONS OF SOME FORMS OF CONTINUITY

A. ACIKGOZ AND S. YUKSEL

Abstract. In this paper, α_I N_3 – sets [**3**] and α_I N_5 – sets [**3**] are introduced and characterizations of α – I – open [**6**], semi – I – open [**6**], α_I N_3 – and α_I N_5 – sets are investigated. Also, new decompositions of α – I – continuity and semi – I – continuity are obtained using these sets.

1. **Introduction**

Quite recently, Acikgoz and Yuksel [3] have introduced I - R closed sets and obtained a decomposition of continuity. Acikgoz et al. [1], [2] investigated some properties of $\alpha-$ I - open sets and obtained decompositions of $\alpha-$ I - continuity and semi - I - continuity.

The purpose of this paper is to introduce α_I N_3 – sets and α_I N_5 – sets via idealization and investigate characterizations of α – I – open, semi – I – open, α_I N_3 – and α_I N_5 – sets and also, to obtain new decompositions of α – I – continuity and semi – I – continuity using these sets.

2. **Preliminaries**

Throughout this paper Cl(A) and Int(A) denote the closure and the interior of A, respectively. Let (X,τ) be a topological space and let I be an ideal of subsets of X. An ideal is defined as a nonempty collection I of subsets of X satisfying the following two conditions: (1) If A$\in$ I and B $\subset$ A, then B$\in$ I; (2) If A$\in$ I and B$\in$ I, then A$\cup$B$\in$ I. An ideal topological space is a topological space (X,τ) with an ideal I on X and is denoted by (X, τ, I). For a subset A$\subset$X, $A^*(I)$ = {x$\in$X : U$\cap$A$\notin$ I for each neighborhood U of x} is called the local function of A with respect to I and τ [10]. We simply write A^* instead of $A^*(I)$ since there is no chance for confusion. X^* is often a proper subset of X. The hypothesis

$X = X^*$ [9] is equivalent to the hypothesis $\tau \cap I = \emptyset$ [13]. The ideal topological space which satisfies this hypothesis is called a *Hayashi – Samuels space* [10]. For

Received by the editors Nov. 3, 2006 ; Accepted: March 7, 2007.
2000 *Mathematics Subject Classification. Primary 54C08, 54A20; Secondary 54A05, 54C10.*
Key words and phrases. $\alpha_I N_3$ *– set,* $\alpha_I N_3$ *– set, decompositions of* α *– I – continuity and semi – I – continuity, topological ideal.*

every ideal topological space (X, τ, I), there exists a topology $\tau^*(I)$, finer than τ, generated by β (I,τ) = {U \ I : U$\in \tau$ and I$\in I$}, but in general β (I,τ) is not always a topology [10]. Additionally, Cl*(A)=A$\cup$A* defines a Kuratowski closure operator for $\tau^*(I)$.

First we shall recall some definitions used in the sequel.

Definition 2.1. A subset A of an ideal topological space (X, τ, I) is said to be

(1) *α – I – open* [6] if A $\subset$ Int(Cl*(Int(A))),
(2) *pre – I – open* [4] if A $\subset$ Int(Cl*(A)),
(3) *semi – I – open* [6] if A $\subset$ Cl*(Int(A)),
(4) *δ – I – open* [2] if Int(Cl*(A)) $\subset$ Cl*(Int(A)),
(5) *strong β – I – open* [8] if A $\subset$ Cl*(Int(Cl*(A))),
(6) *t – I – set* [6] if Int(A) = Int(Cl*(A)),
(7) *τ*– dense set* [9] if X = Cl*(A),
(8) *τ*– closed set* [10] if A = Cl*(A).

The family of all α – I – open (resp. pre – I – open, semi – I – open, strong β – I – open) sets in an ideal topological space (X, τ, I) is denoted by αIO (X,τ) (resp. PIO (X,τ), SIO (X,τ), SβIO (X,τ)).

Definition 2.2. A subset A of an ideal topological space (X, τ, I) is said to be

(1) *a semi – I – closed* [7] if Int(Cl*(A)) $\subset$ A,
(2) *a weakly I – locally – closed set* [11] if A = U $\cap$ V, where U is open and V is τ^*–closed,
(3) *aB_I – set* [6] if A = U $\cap$ V, where U is open and V is a t – I – set.

The family of all semi – I – closed (resp. weakly I – locally – closed, B_I –) sets in an ideal topological space (X, τ, I) is denoted by SIC (X,τ) (resp. W_ILC (X,τ), B_I (X,τ)).

Definition 2.3. A subset A of an ideal topological space (X, τ, I) is said to be a *nowhere τ*– dense set* if Int*(Cl(A))=$\emptyset$, where Int*(A) denotes the interior of A with respect to τ^*.

Theorem 2.4. *A subset A of a space (X, τ, I) is semi - I - closed if and only if Int(Cl*(A)) = Int(A).*

Proof. Necessity. Let A be semi - I - closed. Then we have Int(Cl*(A)) $\subset$ A. Then Int(Cl*(A)) $\subset$ Int(A) and hence Int(Cl*(A)) $\subset$ Int(A).

The sufficiency is clear. □

Corollary 1. *Let A be a subset of an ideal topological space (X, τ, I). A is a semi – I – closed set if and only if A = A $\cup$ Int(Cl*(A)).*

Proof. The necessity is clear as seen in Theorem 1.

Sufficiency.

$$\begin{aligned} Int(Cl^*(B) &= Int(Cl^*(A \cup Int(Cl^*(A)))) \\ &\subset Int(Cl^*(A) \cup Int(Cl^*(Int(Cl^*(A)))) \\ &= Int(Cl^*(A) \subset A \cup Int(Cl^*(A)) = B. \end{aligned}$$

Thus we obtain that Int(Cl*(B)) ⊂ B and hence B = A ∪ Int(Cl*(A)) is semi - I - closed. □

Definition 2.5. A subset A of a space (X, τ, I) is said to be $\beta-$ *I - closed* if its complement is $\beta-$ I - open.

The family of all β – I – closed sets in an ideal topological space (X, τ, I) is denoted by βIC (X,τ).

Theorem 2.6. *Let A be subset of an ideal topological space (X, τ, I). Then, If A is $\beta-$ I - closed, then Int(Cl*(Int(A))) ⊂ A.*

Proof. Since A is $\beta-$ I - closed, X - A∈ βIO (X,τ). Since τ*(I) is finer than τ, we have

$$\begin{aligned} X - A &\subset Cl(Int(Cl^*(X - A))) \\ &\subset Cl(Int(Cl(X - A))) \\ &= X - Int(Cl(Int(A))) \\ &\subset X - Int(Cl^*(Int(A))). \end{aligned}$$

Therefore, we obtain Int(Cl*(Int(A))) ⊂ A. □

3. α_I $\mathbf{N_3}$ – sets

Proposition 1. *Every semi - I - closed set of an ideal topological space is $\beta-$ I - closed.*

Proof. Let A be semi - I - closed. Then we have Int(Cl*(A))⊂A.
Then Int(Cl*(Int(A)))⊂Int(Cl*(A))⊂A and hence Int(Cl*(Int(A)))⊂A. □

Remark 3.1. The converses of Proposition 1 need not be true as shown in the following example.

Example 3.2. Let X = {1, 2, 3, 4}, τ = {Ø, X, {1, 2}, {4}, {1, 2, 4}} and I = {Ø, {3}}. Then A = {2, 3, 4} is a $\beta-$ I - closed set, but not semi - I - closed. For, Int(Cl*(Int(A))) = Int(Cl*(Int({2, 3, 4}))) = Int(Cl*({4})) = Int(({4})* ∪ {4}) = Int({3, 4} ∪ {4}) = Int({3, 4}) = {4} and hence Int(Cl*(Int({2, 3, 4}))) = {4} ⊂ {2, 3, 4} = A. This shows that A is a $\beta-$ I - closed set. But A is not a semi - I - closed. For Int(Cl*(A)) = Int(Cl*({2, 3, 4})) = Int(({2, 3, 4})* ∪ {2, 3, 4}) = Int(X ∪ {2, 3, 4}) = Int(X) = X ⊄ {2, 3, 4} = A and Int(Cl*(A)) ⊄ A.

Lemma 3.3. *(Acikgoz et. al* [1]*). Let (X, τ, I) be an ideal topological space and A a subset of X. Then the following properties hold:*

(1) If O is open in (X, τ, I), then O $\cap$ Cl*(A) $\subset$ Cl*(O $\cap$ A).
(2) If A $\subset$ X_0 $\subset$ X, then Cl*X_0(A) $\subset$ Cl*(A) $\cap$ X_0.

Proposition 2. *Let (X,τ,I) be an ideal topological space. A$\in$ αIO(X,τ) if and only if A $\cap$ S$\in$SIO (X,τ) for each S$\in$SIO (X,τ).*

Proof. Necessity. Let A$\in$ αIO (X,τ) and S$\in$SIO (X,τ). Using Lemma 1, we obtain

$$\begin{aligned} S\cap A &\subset Cl^*(Int(S))\cap Int(Cl^*(Int(A))) \\ &\subset Cl^*(Int(S)\cap Int(Cl^*(Int(A)))) \\ &\subset Cl^*(Int(S)\cap Cl^*(Int(A))) \\ &\subset Cl^*(Cl^*(Int(S)\cap Int(A))) \\ &\subset Cl^*(Int(S\cap A)). \end{aligned}$$

This shows that A $\cap$ S$\in$SIO (X,τ).

Sufficiency. Let S$\in$SIO (X,τ) and A $\cap$ S$\in$SIO (X,τ). Then in particular A$\in$SIO (X,τ). Assume x$\in$A $\cap$ **C**(Int(Cl*(Int(A)))) (**C** denoting complement). Then x$\in$Cl*(S) = Cl*(Int(S)) by [7], where S = **C**(Cl*(Int(A))). Hence we obtain

$$\begin{aligned} S\cup Int(\{x\}) &\subset Cl^*(Int(S)) \\ &= Cl^*(Int(Int(\{x\})\cup Int(S))) \\ &\subset Cl^*(Int(Int(S\cup\{x\}))) \\ &= Cl^*(Int(S\cup\{x\})). \end{aligned}$$

Thus S $\cup$ {x}$\in$SIO (X,τ) and consequently A $\cap$ (S $\cup$ {x})$\in$SIO (X,τ). But

$$\begin{aligned} A\cap(S\cup\{x\}) &= (A\cap S)\cup(A\cap\{x\}) \\ &= (A\cap S)\cup(A\cap\{x\}) \\ &= (A\cap S)\cup\{x\} \\ &= (A\cap\{x\})\cup(S\cap\{x\}) \\ &= \{x\} \end{aligned}$$

Hence {x} is open. As x$\in$Cl*(Int(A)), this implies x$\in$Int(Cl*(Int(A))), contrary to assumption. Thus x$\in$A implies x$\in$Int(Cl*(Int(A))), and A$\in$ αIO (X,τ). This completes the proof. □

Proposition 3. *Let (X, τ, I) be an ideal topological space. A$\in$ αIO (X,τ) if and only if A = U $\cap$ D where U$\in$ τ and Int(D) is τ^* – dense.*

Proof. Necessity. If A$\in$ αIO (X,τ), then we have

$$A = Int(Cl^*(Int(A))) - (Int(Cl^*(Int(A))) - A)$$

where Int(Cl*(Int(A))) = U$\in$ τ and Int(Cl*(Int(A))) – A is nowhere τ^*– dense dense.

Sufficiency. Let A = U $\cap$ D, where U$\in$ τ and Int(D) is τ^* – dense. Since A $\subset$ U

U = U∩X = U ∩ Cl*(Int(D)) ⊂ Int(U) ∩ Cl*(Int(D)) ⊂ Cl*(Int(U) ∩ Int(D)) = Cl*(Int(A)) and we obtain U ⊂ Int(Cl*(Int(A))). Hence A ⊂ Int(Cl*(Int(A))) so that A∈ αIO (X,τ). □

Definition 3.4. [3]. A subset H of an ideal topological space (X, τ, I) is called an α_I N_3 – set if H = A ∩ B where A∈ αIO (X,τ) and B is a t – I – set.

The family of all α_I N_3 – sets of (X, τ, I) is denoted by α_I N_3 (X,τ) in this paper, when there is no chance for confusion with the ideal.

Theorem 3.5. *For a subset A of an ideal topological space (X, τ, I), the following properties are equivalent:*

(1) A is semi – I – closed,
(2) A is β – I – closed and is an α_I N_3 – set,
(3) A is β – I – closed and δ– I – open.

Proof. a) ⇒ b). Let A∈SIC(X,τ). Since SIC(X,τ) ⊂ βIC(X,τ) by Proposition 1 and A = A ∩ X, where A is a t – I – set and X∈ αIO (X,τ). Therefore we have SIC(X,τ) ⊂ βIC(X,τ) ∩ α_I N_3(X,τ).

b) ⇒ c). The proof is seen in the Diagram.

c) ⇒ a). Let A be β – I – closed and δ– I – open. Since Int(Cl*(A)) ⊂ Int(Cl*(Int(A))) and Int(Cl*(Int(A))) ⊂ A, we obtain that A∈SIC(X,τ). □

Proposition 4. *Let (X, τ, I) be an ideal topological space. H∈ α_I N_3 (X,τ) if and only if H = B ∩ D where B∈B_I (X,τ) and Int(D) is τ* – dense.*

Proof. Necessity. Let H∈ α_I N_3 (X,τ) and write H = A ∩ B where A∈ αIO (X,τ) and B is a t – I – set. By Proposition 3, we write A = U ∩ D where U∈ τ and Int(D) is τ^* – dense. Thus H = A ∩ B = (U ∩ D) ∩ B = (U ∩ B) ∩ D where U ∩ B ∈ B_I (X,τ) and Int(D) is τ^* – dense as required.

Sufficiency. Assume that H = B ∩ D with B ∈ B_I (X,τ) and Int(D) is τ^* – dense. Then we have B = U ∩ B_2 where U∈ τ and B_2 is a t – I – set. Thus H = B ∩ D = (U ∩ B_2) ∩ D = (U ∩ D) ∩ B_2 where U ∩ D∈ αIO (X,τ) by Proposition 3 and B_2 is a t – I – set. Therefore we obtain H∈ α_I N_3 (X,τ). □

Proposition 5. *Let (X, τ, I) be an ideal topological space. H∈ α_I N_3 (X,τ) if and only if H = A ∩ (H ∪ Int(Cl*(H))) where A∈ αIO (X,τ).*

Proof. Necessity. Let H$\in$ α_I N_3 (X,τ) and assume H=A$\cap$B where A$\in$ αIO (X,τ) and B is a t – I – set. Since B is a t – I – set, we have

$$\begin{aligned} H &= A \cap H \\ &\subset A \cap (H \cup Int(Cl^*(H))) \\ &\subset A \cap (B \cup Int(Cl^*(B))) \\ &= A \cap (B \cup Int(B)) \\ &= A \cap B \\ &= H. \end{aligned}$$

So H = A $\cap$ (H $\cup$ Int(Cl*(H))), with A$\in$ αIO (X,τ) by Lemma 1 as required.Sufficiency. Assume that H $\subset$ X such that H = A $\cap$ (H $\cup$ Int(Cl*(H))) where A$\in$ αIO (X,τ). Since H $\cup$ Int(Cl*(H)) is semi – I – closed by Corollary 1 and hence it is a t – I – set. Therefore H$\in$ α_I N_3 (X,τ). □

Theorem 3.6. *Let (X, τ, I) be an ideal topological space.*

αIO (X,τ) = PIO (X,τ) $\cap$ α_I N_3 (X,τ).

Proof. Necessity. It is obvious that αIO (X,τ) $\subset$ PIO (X,τ) $\cap$ α_I N_3 (X,τ).

Sufficiency. Let H$\in$PIO (X,τ) $\cap$ α_I N_3 (X,τ). Then we have H $\subset$ Int(Cl*(H)) and by Proposition 5, H = A $\cap$ (Int(Cl*(H)) $\cup$ H) where A$\in$ αIO (X,τ) and T = (Int(Cl*(H)) $\cup$ H)$\in$SIC (X,τ) by Lemma 1, respectively. But

$$T = H \cup Int(Cl^*(H)) = Int(Cl^*(H))$$

Thus H = A $\cap$ Int(Cl*(H)) where A$\in$ αIO (X,τ) and Int(Cl*(H))$\in$ τ $\subset$ αIO (X,τ) and therefore H = A $\cap$ Int(Cl*(H))$\in$ αIO (X,τ), because αIO (X,τ) is a topology (see Corollary 3.2 of Acikgoz et.al [1]). □

It is seen in the following example that the decomposition provided by Theorem 4 is different from the decomposition of $\alpha-$ I - continuity given in Theorem 4.2 by Acikgoz et.al [2].

Example 3.7. Let X = {1, 2, 3, 4}, τ = {Ø, X, {1, 2}, {4}, {1, 2, 4}} and I = {Ø, {3}}. Then A = {3} is an α_I N_3 – set, which is not semi – I – open. For, Int(Cl*(A)) = Int(Cl*({3})) = Int({3}) = Ø = Int({3}), A = {3} = {3} $\cap$ X where A is a t – I – set and X$\in$ αIO (X). This shows that A is an α_I N_3 – set. On the other hand, since Cl*(Int(A)) = Ø and A $\not\subset$ Cl*(Int(A)), A is not semi – I – open.

Proposition 6. *For a subset A of an ideal topological space (X, τ, I), the following properties are equivalent:*

(1) A is α – I – open,
(2) A is pre – I – open and semi - I – open,
(3) A is pre – I – open and $\delta-$ I – open,
(4) A is pre – I – open and α_I N_3 – set.

Proof. The proof is obvious (Acikgoz et al. [2] and [1]). □

4. α_I N$_5$ - sets

Definition 4.1. A subset A of an ideal topological space (X, τ, I) is said to be an *I - R closed* [3] (resp. *strong $\beta-$ I- closed* [8]) set if A = Cl*(Int(A)) (resp. A $\subset$ Cl*(Int(Cl*(A)))).

Definition 4.2. A subset A of an ideal topological space (X, τ, I) is said to be an $A_{I-R}-$ *set* if A $\cap$ V, where U is open and V is an I − R closed set.

The family of all I - R closed (resp. strong $\beta-$ I - closed) sets in an ideal topological space (X, τ, I) is denoted by IRC (X,τ) (resp. SβIC (X, τ)).

Proposition 7. *Let (X, τ, I) be an ideal topological space. A$\in$SIO (X,τ) if and only if A = R $\cap$ D where R$\in$IRC (X,τ) and Int(D) is τ^* – dense.*

Proof. Necessity. Let A$\in$SIO (X,τ). Then we have U $\subset$ A $\subset$ Cl*(U) such that U$\in$ τ by Theorem 3.2 of [7]. Note that Cl*(A) = Cl*(U). We write A = Cl*(U) – (Cl*(U) – A) = Cl*(U) $\cap$ [X – (Cl*(U) – A)], where Cl*(U) – A $\subset$ Cl*(U) – U and Cl*(U) – U is nowhere τ^* – dense in (X, τ, I). We assume D = X – (Cl*(U) – A). Then X – Cl(Cl*(U) – A) is an open τ^* – dense subset of (X, τ, I) which is contained in D. Consequently, we use R = Cl*(U) to write A = R $\cap$ D where R is an I - R closed set and Int(D) is τ^* – dense, as required.

Sufficiency. Assume that A = R $\cap$ D where R is I – R closed and Int(D) is τ^*– dense. We write U$\in$ τ such that R = Cl*(U). We assume V = U $\cap$ Int(D). Then V$\in$ τ with V $\subset$ A. Finally, Cl*(V) = Cl*(U $\cap$ Int(D)) = Cl*(U) = R. Thus V $\subset$ A $\subset$ Cl*(V) and therefore A$\in$SIO (X,τ) using [7]. □

Definition 4.3. [3]. A subset H of an ideal topological space (X, τ, I) is called an α_I N$_5$ – set if H = A $\cap$ B where A$\in$ αIO (X,τ) and B is a τ^*– closed set.

The family of all α_I N$_5$ – sets of (X, τ, I) is denoted by α_I N$_5$ (X,τ) in this paper, when there is no chance for confusion with the ideal.

Proposition 8. *Let (X, τ, I) be an ideal topological space and A = U $\cap$ V a subset of X. Then the following hold:*

(1) If A is a weakly I – locally – closed set, then A is an α_I N$_5$ – set.
(2) If A is an α_I N$_5$ – set, then A is an α_I N$_3$ – set.
(3) If A is an α_I N$_3$ – set, then A is δ – I – open.

Proof. a) and b) The proof is a direct consequence of the Definition 2 and Definition 8.

c) Let A be an α_I N$_3$ – set. Then we have A = U $\cap$ V where U$\in$ αIO (X,τ) and V is a t – I – set. Since every α – I – open set is δ – I – open by [1] and every t – I – set is δ – I – open, therefore we obtain that A$\in$ δIO (X,τ). Because δIO (X,τ) is a topology (see Corollary 3.2 of Acikgoz et.al [14]). □

Remark 4.4. The converses of Proposition 3.1 need not be true as shown in the following example.

Example 4.5. Let X = {1, 2, 3, 4}, τ = {Ø, X, {1, 2}, {4}, {1, 2, 4}} and I = {Ø, {3}}. Then A = {3, 4} is an α_I N_5 – set, but not weakly I – locally – closed. αIO(X) = {Ø, X, {1, 2},{4},{1, 2, 4}, {3, 4}}. For a subset A = {3, 4} = X ∩ {3, 4}, where A is α – I – open and X is τ^* –closed. This shows that A is an α_I N_5 – set. But A is not weakly I – locally – closed. Because A$\notin \tau$.

Example 4.6. Let X = {a, b, c}, τ = {Ø, X, {a}, {c}, {a, c}} and I = {Ø, {a}}. Then A = {c} is an α_I N_3 – set but it is not an α_I N_5 – set. For Int(Cl*(A)) = Int(Cl*({c})) = Int({c} ∪ ({c})*) = Int({c} ∪ {b, c}} = Int({b, c}) = {c}, and so A = A ∩ X where A is a t – I – set and X∈ αIO (X,τ). This shows that A is an α_I N_3 – set. But A is not an α_I N_5 – set. For, Cl*(A)= Cl*({c}) = ({c})* ∪ {c} = {b,c} ∪ {c} = {b,c} ≠ {c} and Cl*(A) ≠ A.

Example 4.7. Let X = {a, b, c, d}, τ = {∅, X, {c}, {a, c}, {b, c}, {a, b, c}, {a, c, d}}, I = {∅, {c}, {d}, {c, d}}. Set A = {b, d}. Then A is a δ – I – open set which is not an α_I N_3 – set. For A = {b, d}, since Cl*(A) = {b, d} and Int(Cl*(A)) = ∅ so Int(Cl*(A)) ⊂ Cl*(Int(A)). This shows that A is a δ – I – open set. On the other hand, since A ⊄ Int(Cl*(Int(A))) = ∅ and A = A ∩ X where A$\notin \alpha$IO (X,τ), A is not an α_I N_3 – set.

The two classes α_I N_3 (X,τ) and α_I N_5 (X,τ) are related as seen in the next proposition, whose proof is omitted since it is similar to that of Proposition 3.

Proposition 9. *Let (X, τ, I) be an ideal topological space. H∈ α_I N_5 (X,τ) if and only if H = B ∩ D where B is a weakly I – locally – closed set and Int(D) is τ^* – dense.*

Theorem 4.8. *Let (X, τ, I) be an ideal topological space.*

$$\text{SIO }(X,\tau) = \text{S}\beta\text{IO }(X,\tau) \cap \alpha_I\ N_5\ (X,\tau).$$

Proof. Necessity. Let A∈SIO (X,τ). Then we have A∈SβIO (X,τ) by Remark 1.1 of [2]. Now, by Proposition 7 we write A = R ∩ D where R is I – R closed and Int(D) is τ^* – dense. Since R is a weakly I – locally – closed set (because R is τ^* – closed) then A is an α_I N_5 – set by Proposition 9.

Sufficiency. Let H∈SβIO (X,τ)∩α_I N_5 (X,τ). Then we have H⊂ Cl*(Int(Cl*(H))) and H = A ∩ F where A∈ αIO (X,τ) and F is τ^* – closed, respectively. Since H ⊂ F then Cl*(Int(Cl*(H))) ⊂ Cl*(Int(Cl*(F))) = Cl*(Int(F)) ⊂ Cl*(F) = F. Thus H ⊂ A ∩ Cl*(Int(Cl*(H))) ⊂ A ∩ Cl*(Int(Cl*(F))) ⊂ A ∩ F = H. So H = A ∩ Cl*(Int(Cl*(H))) where A∈ αIO (X,τ) and Cl*(Int(Cl*(H)))∈SIO (X,τ). Thus, we have H∈SIO (X,τ) by Proposition 2. □

Proposition 10. *For a subset A of an ideal topological space (X, τ, I), the following properties are equivalent:*

(1) A is semi – I – open,
(2) A is strong β – I – open and δ– I – open,
(3) A is strong β – I – open and is an α_I N_5 – set.

Proof. The proof is obvious. (Acikgoz et al. [2]). □

Remark 4.9. The relationships between the sets defined above, are shown in the following diagram.

DIAGRAM

Remark 4.10. By the examples stated below, we obtain the following results:

(1) β– I - closedness and α_I N_3 – set are independent of each other,
(2) δ– I - openness and β– I - closedness are independent of each other,
(3) t - I - set and α_I N_5 – set are independent of each other,
(4) Strong β– I - openness and α_I N_5 – set are independent of each other,
(5) Pre - I - openness and α_I N_3 – set are independent of each other.

Example 4.11. Let (X, τ, I) and A be the same ideal topological space and the set, respectively, as in Example 1. We obtain that A is β– I - closed but not is not an α_I N_3 – set. Because A = A ∩ X where A is not a t – I – set and X∈ αIO (X,τ).

Example 4.12. Let X = {a, b, c, d}, τ = {∅, X, {b}}, I = {∅, {c}}. Then A = {b} is an α_I N_3 – set which is not a β– I - closed set. For, A = {b} = {b} ∩ X where {b}∈ αIO (X,τ) and Int(Cl*(X)) = Int(X). This shows that A is an α_I N_3 – set. On the other hand, since Int(Cl*(Int(A))) = X and Int(Cl*(Int(A))) $\not\subset$ A, A is not a β– I - closed set.

Example 4.13. Let X = {a, b, c, d}, τ = {∅, X, {a}, {c}, {a, c}}, I = {∅, {a}}. Set A = {a, c}. Then A is a δ – I – open set but it is not a β– I - closed set. For A = {a, c}, since Int(Cl*(A)) = X, Cl*(Int(A)) = X and so Int(Cl*(A)) ⊂ Cl*(Int(A)). This shows that A is a δ – I – open set. On the other hand, since Int(Cl*(Int(A))) = X and Int(Cl*(Int(A))) $\not\subset$ A, A is not β– I - closed.

Example 4.14. Let (X, τ, I) and A be the same ideal topological space and the set, respectively, as in Example 1. We obtain that A is β– I - closed but not δ – I – open.

Example 4.15. Let (X, τ, I) and A be the same ideal topological space and the set, respectively, as in Example 7. We obtain that A is an α_I N_5 – set but not t – I – set.

Example 4.16. Let (X, τ, I) and A be the same ideal topological space and the set, respectively, as in Example 5. We obtain that A is a t – I – set but not an α_I N_5 – set.

Example 4.17. Let X = {a, b, c, d}, τ = {$\emptyset$, X, {d}, {a, b, c}}, I = {$\emptyset$, {c}}. Set A = {c, d}. Then A is an α_I N_5 – set but it is not a strong $\beta-$ I - open set. For A={c, d}, since Cl*(A) = {c,d} and Cl*(A) = A, so A = A $\cap$ X where X$\in$ αIO (X,τ) and A is τ^* - closed. This shows that A is an α_I N_5 – set. On the other hand, since A $\not\subset$ Cl*(Int(Cl*(A))), A is not strong $\beta-$ I - open.

Example 4.18. Let X = {a, b, c, d}, τ = {$\emptyset$, X, {a, b}}, I = {$\emptyset$, {c}}. Set A = {a, c}. Then A is a strong $\beta-$ I - open set but it is not an α_I N_5 – set. For A = {a, c}, since Cl*(Int(Cl*(A))) = X and A $\subset$ Cl*(Int(Cl*(A))), A is strong $\beta-$ I - open. On the other hand, since Cl*(A) = X $\neq$ A, A is not an α_I N_5 – set.

Example 4.19. Let X = {a, b, c, d}, τ = {$\emptyset$, X, {a, b}}, I = {$\emptyset$, {c}}. Set A = {b, c}. Then A is a pre – I – open set but it is not an α_I N_3 – set. For A = {b, c}, since Cl*(A) = X and Int(Cl*(A)) = X, so A $\subset$ X = Int(Cl*(A)). This shows that A is a pre – I – open set. On the other hand, since Int(Cl*(Int(A))) = $\emptyset$ and A $\not\subset$ Int(Cl*(Int(A))), A is not an α_I N_3 – set.

Example 4.20. Let (X, τ, I) and A be the same ideal topological space and the set, respectively, as in Example 5. We obtain that A is an α_I N_3 – set but it is not a pre – I – open set.

5. Decompositions of $\alpha-$ I - continuity and semi - I - continuity

Definition 5.1. A function f : (X, τ, I) $\to$ (Y, φ) is said to be $\alpha-$ I - continuous [6] (resp. semi - I - continuous [6], pre – I – continuous [4], semi $\delta-$ I - continuous [2], strong β – I – continuous [8]), if for every V $\in$ φ , f^{-1} (V) is an $\alpha-$ I – open set (resp. semi - I – open set, pre – I – open, δ – I – open, strong β – I – open set) of (X,τ,I).

Definition 5.2. A function f : (X, τ, I) $\to$ (Y, φ) is said to be α_I N_3 – continuous (resp. α_I N_5 – continuous) if for every V$\in$ φ, f^{-1} (V) is an α_I N_3 – set (resp. α_I N_5 – set) of (X, τ, I).

Theorem 5.3. *A function f : (X, τ, I) $\to$ (Y, φ) is $\alpha-$ I – continuous if and only if it is pre – I – continuous and α_I N_3 – continuous.*

Proof. This is a direct consequence of Theorem 4. □

Theorem 5.4. *For a function f : (X, τ, I) $\to$ (Y, φ) the following properties are equivalent:*

(1) f is $\alpha-$ I – continuous;
(2) f is pre – I – continuous and semi – I – continuous;
(3) f is pre – I – continuous and δ – I – continuous;
(4) f is pre – I – continuous and α_I N_3 – continuous.

Proof. This follows immediately from Proposition 6. □

Theorem 5.5. *A function f : (X, τ, I) $\rightarrow$ (Y, φ) is semi – I – continuous if and only if it is strong β – I – continuous and α_I N_5 – continuous.*

Proof. This is a direct consequence of Theorem 5. □

Theorem 5.6. *For a function f : (X, τ, I) $\rightarrow$ (Y, φ) the following properties are equivalent:*

(1) f is semi – I – continuous;
(2) f is strong β – I – continuous and δ – I – continuous;
(3) f is strong β – I – continuous and α_I N_5 – continuous.

Proof. This is an immediate consequence of Proposition 10. □

Ozet: Bu çalısmada, α_I N_3 – [3] and α_I N_5 – [3] kümeleri verilecek, α – I – açık, semi – I – açık, α_I N_3 – ve α_I N_5 – kümelerinin karakterizasyonları incelenecektir. Bu kümelerden yararlanarak α – I – sürekli ve semi – I – sürekliliğin yeni ayrışımları da elde edilecektir.

References

[1] A. Açıkgöz, T. Noiri and Ş. Yüksel, On α – I – continuous and α – I – open functions, Acta Math. Hungar., **105** (2004), 27 – 37.
[2] A. Acikgoz, T. Noiri and S. Yuksel, On $\delta-$ I - open sets and decomposition of α – I - continuity, Acta Math. Hungar., **102 (4)** (2004), 349 – 357.
[3] A. Acikgoz and S. Yüksel, Some new sets and decompositions of A_{I-R} continuity, α – I – continuity, continuity via idealization, Acta Math. Hungar., **114 (1-2)** (2007), 79-89.
[4] J. Dontchev, On pre – I – open sets and a decomposition of I – continuity, Banyan Math. J., Vol. **2** (1996).
[5] J. Dontchev, M. Ganster and D. Rose, Ideal resolvability, Topology and its applications, vol. **93** (1999), 1 – 16.
[6] E. Hatir and T. Noiri, On decompositions of continuity via idealization, Acta Math. Hungar., **96** (2002), 341 – 349.
[7] E. Hatir and T. Noiri, On semi – I – open sets and semi – I – continuous functions, Acta Math. Hungar., **10** (**4**) (2005), 345 – 353.
[8] E. Hatir, A. Keskin and T. Noiri, On a new decomposition of continuity via idealization, J. Geometry and Topology, **1** (2003), 55 – 64.
[9] E. Hayashi, Topologies defined by local properties, Math. Ann. **156** (1964), 205 – 215.
[10] D. Janković and T. R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly, **97** (1990), 295 – 310.
[11] A. Keskin, Ş. Yüksel and T. Noiri, "Decompositions of I – continuity and continuity", Commun. Fac. Sci. Üniv. Ank. Series A1, **53** (2004), 67 – 75.
[12] N. Levine, Semi – open sets and semi – continuity in topological spaces, Amer. Math. Monthly, Vol. **70** (1963), 36 – 41.
[13] P. Samuels, A topology formed from a given topology and ideal, J. London Math. Soc. **(2)**, **10** (1975), 409 – 416.
[14] S. Yuksel, A. Acikgoz and T. Noiri, On $\delta-$ I - continuous functions, Turk J Math., **29** (2005), 39-51.

Current address: A. ACIKGOZ,Aksaray University, Faculty of Arts and Sciences, Department of Mathematics, 68100 AKSARAY, S. YUKSEL Selçuk University,Faculty of Arts and Sciences, Department of Mathematics, 42031 CAMPUS – KONYA

E-mail address: syuksel@selcuk.edu.tr