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STRONG FORM OF PRE-I-CONTINUOUS FUNCTIONS

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Abstract. In this paper, semiopen and pre-I-open sets used to define and investigate a new class of functions called strongly pre-I-continuous. Relationships between the new class and other classes of functions are established

1. Introduction

In 1990, Jankovic and Hamlett [14] have defined the concept of I-open set via local function which was given by Vaidyanathaswamy [25]. The latter concept was also established utilizing the concept of an ideal whose topic in general topological spaces was treated in the classical text by Kuratowski [16]. In 1992, Abd El-Monsef et al [1] studied a number of properties of I-open sets as well as I-closed sets and I-continuous functions and investigated several of their properties. In 1999, Dontchev [10] has introduced the notion of pre-I-open sets which are weaker than that of I-open sets. In this paper, a new class of functions called strongly pre-I-continuous functions in ideal topological spaces is introduced and some characterizations and several basic properties are obtained.

2. Preliminaries

Throughtuout this paper, for a subset A of a topological space (X,τ), the closure of A and interior of A are denoted by (A) and (A), respectively. An ideal topological space is a topological space (X,τ) with an ideal I on X, and is denoted by (X,τ,I), where the ideal is defined as a nonempty collection of subsets of X satisfying the following two conditions. (i) If $A \in I$ and $B \subset A$, then $B \in I$; (ii) If $A \in I$ and $B \in I$, then $A \cup B \in I$. For a subset $A \subset X$, $A^*(I) = \{x \in X |\ U \cap A \notin I$ for each neighbourhood U of $x\}$ is called the local function of A with respect to I and τ [14]. When there is no chance of confusion, $A^*(I)$ is denoted by A^*. Note that often X^* is a proper subset of X. For every ideal topological space (X,τ,I), there exists topology $\tau^*(I)$, finer than τ, generated by the base $\beta(I,\tau) = \{U \backslash I \mid U \in \tau$ and

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$I \in I\}$, but in general $\beta(I,\tau)$ is not always a topology [14]. Observe additionally that $^*(A) = A^* \cup A$ defines a Kuratowski closure operator for $\tau^*(I)$. A subset S of an ideal topological space (X,τ,I) is said to be pre-I-open [10] (resp. semi I-open [12], *-dense-in-itself [13]) if $S \subset (^*(S))$ (resp. $S \subset^* ((S))$, $S \subset S^*$). The complement of a pre-I-open set is called pre-I-closed [10]. The intersection of all pre-I-closed sets containing S is called the pre-I-closure [26] of S and is denoted by $_{PI}(S)$. A set S is pre-I-closed if and only if $_{PI}(S) = S$. The pre-I-interior [26] of S is defined by the union of all pre-I-open sets of (X,τ,I) contained in S and is denoted by $_{PI}(S)$. The family of all pre-I-open (resp. pre-I-closed, semi-I-open) sets of (X,τ,I) is denoted by $PIO(X)$ [26] (resp. $PIC(X), SIO(X)$). The family of all pre-I-open (resp. pre-I-closed) sets of (X,τ,I) containing a point $x \in X$ is denoted by $PIO(X,x)$ (resp. $PIC(X,x)$).

Definition 2.1. A subset A of a topological space (X,τ) is said to be:

(i) semiopen if $A \subseteq ((A))$ [17].
(i) preopen if $A \subseteq ((A))$ [21].

The complement of semiopen set is called semiclosed. The intersection of all semiclosed sets of (X,τ) containing $A \subset X$ is called semiclosure [7] of A and is denoted by $s(A)$. The family of all semiopen subsets of (X,τ) is denoted by $SO(X)$.

Definition 2.2. A function $f : (X,\tau,I) \to (Y,\sigma)$ is called pre-I-continuous [10] (resp. I-irresolute [27], irresolute [7], semi continuous [17]) if for every open (resp. semiopen, semiopen, open), $f^{-1}(V) \in PIO(X)$ (resp. $f^{-1}(V) \in SIO(X)$, $f^{-1}(V) \in SO(X)$, $f^{-1}(V) \in SO(X)$).

Definition 2.3. An ideal space (X,τ,I) is said to be $**$-space [15] if A in a $*$-dense-in-itself for every $A \subseteq X$.

3. Strongly Pre-I-continuous functions

Definition 3.1. A function $f : (X,\tau,I) \to (Y,\sigma)$ is said to be strongly pre-I-continuous if $f^{-1}(V)$ is pre-I-open in X for every semiopen set V of Y.

It is clear that every strongly pre-I-continuous function is pre-I-continuous. But the converse is not always true as shown in the following example.

Example 3.2. Let $X = \{a,b,c\}$, $\tau = \{\varnothing, \{a\}, \{a,b\}, X\}$, $\sigma = \{\varnothing, \{a\}, X\}$ and $I = \{\varnothing, \{a\}\}$. Then the identity function $f : (X,\tau,I) \to (Y,\sigma)$ is pre-I-continuous but not strongly pre-I-continuous.

Definition 3.3. [5] A function $f : (X,\tau) \to (Y,\sigma)$ is said to be strongly precontinuous if $f^{-1}(V) \in PO(X)$ for every $V \in SO(Y)$.

Theorem 3.4. *Let (X,τ,I) be $**$-space. Then the function $f : (X,\tau,I) \to (Y,\sigma)$ is strongly pre-I-continuous if and only if it is strongly precontinuous.*

Proof. It follows from Theorem 6(a) of [15]. □

Recall that a topological space (X,τ) is said to be submaximal if every dense subset of X is open.

Definition 3.5. [3] A function $f:(X,\tau)\to(Y,\sigma)$ is said to be strongly semicontinuous if $f^{-1}(V)$ is open in (X,τ) for every semiopen set V of Y.

Theorem 3.6. *Let $f:(X,\tau,I)\to(Y,\sigma)$ be a function. Then*

(i) *If $I=\{\varnothing\}$, then f is strongly pre-I-continuous if and only if it is strongly precontinuous;*
(ii) *If $I=P(X)$, then f is strongly pre-I-continuous if and only if it is strongly semicontinuous;*
(iii) *If $I=\mathcal{N}$ (= nowhere dense subsets of (X,τ)), then f is strongly pre-I-continuous if and only if it is strongly precontinuous;*
(iv) *If (X,τ) is submaximal and I is any ideal on X, then f is strongly pre-I-continuous if and only if it is strongly semicontinuous.*

Proof. Follows from Proposition 2.7 and Corollary 2.13 of [9]. □

Theorem 3.7. *For a function $f:X\to Y$, the following are equivalent:*

(i) *f is strongly pre-I-continuous;*
(ii) *For each point $x\in X$ and each semiopen set V of Y containing $f(x)$, there exists a pre-I-open set V of X containing x and $f(V)\subseteq V$;*
(iii) *$f^{-1}(V)\subseteq({}^{*}(f^{-1}(V)))$ for every semiopen set V of Y;*
(iv) *$f^{-1}(F)$ is pre-I-closed in X, for every semiclosed set F of Y;*
(v) *$({}^{*}(f^{-1}(A)))\subseteq f^{-1}(s(A))$ for every subset A of Y;*
(vi) *$f(({}^{*}(B)))\subseteq s(f(B))$ for every subset B of X.*

Proof. (i)⇒(ii): Let $x\in X$ and V be any semiopen set of Y containing $f(x)$. Then $x\in f^{-1}(f(x))\subseteq f^{-1}(V)$. Set $V=f^{-1}(V)$, then by (i), V is a pre-I-open subset of X containing x and $f(V)=f(f^{-1}(V))\subseteq V$.
(ii)⇒(iii): Let U be any semiopen set of Y. Let x be any point in X such that $f(x)\in V$. Then $x\in f^{-1}(V)$. By (ii), there exists a pre-I-open set V of X such that $x\in V$ and $f(V)\subseteq V$. We obtain $x\in V\subseteq f^{-1}(f(V))\subseteq f^{-1}(V)$. This implies that $x\in V\subseteq f^{-1}(V)$. Thus, we have $x\in V\subseteq({}^{*}(V))\subseteq({}^{*}(f^{-1}(V)))$ and hence $f^{-1}(V)\subseteq({}^{*}(f^{-1}(V)))$.
(iii)⇒(iv): Let F be any semiclosed subset of Y. Then Y-F is semiopen in Y. By (iii), we obtain $f^{-1}(X\text{-}F)\subseteq({}^{*}(f^{-1}(X\text{-}F)))$. Then $Y\text{-}f^{-1}(F)\subseteq({}^{*}(Y\text{-}f^{-1}(F)))=Y\text{-}({}^{*}(f^{-1}(F)))$ and hence $f^{-1}(F)$ is pre-I-closed in X.
(iv)⇒(v): Let A be any subset of Y. Since $s(A)$ is a semiclosed subset of Y, then $f^{-1}(s(A))$ is pre-I-closed in X and hence $({}^{*}(f^{-1}(s(A))))\subseteq f^{-1}(s(A))$. Therefore, we obtain $({}^{*}(f^{-1}(A)))\subseteq f^{-1}(s(A))$.
(v)⇒(vi): Let B be any subset of X. By (v), we have $({}^{*}(B))\subseteq({}^{*}(f^{-1}(f(B))))\subseteq f^{-1}(s(f(B)))$ and hence $f(({}^{*}(B)))\subseteq s(f(B))$.
(vi)⇒(i): Let U be any semiopen subset of Y. Since $f^{-1}(Y\text{-}U)=$

Y-$f^{-1}(U)$ is a subset of X and by (vi), we obtain $f((^*(f^{-1}(X\text{-}V)))) \subseteq s(f(f^{-1}(X\text{-}U))) \subseteq s(X\text{-}U) = Y\text{-}s(U) = Y\text{-}U$ and hence $X\text{-}(^*(f^{-1}(U))) = (^*(X\text{-}f^{-1}(U))) = (^*(f^{-1}(Y\text{-}U))) \subseteq f^{-1}(f((^*(f^{-1}(U))))) \subseteq f^{-1}(X\text{-}U) = Y\text{-}f^{-1}(U)$. Therefore, we have $f^{-1}(U) \subseteq (^*(f^{-1}(U)))$ and hence $f^{-1}(U)$ is pre-I-open in X. Thus, f is strongly pre-I-continuous. □

Lemma 3.8. [23] *Let $(X_i, \tau_i)_{i\in\wedge}$ be any family of topological spaces. Let $X = \Pi_{i\in\wedge}X_i$, let A_{i_n} be any subset of X_{α_n}, $\alpha_n \in \wedge$, for each $n = 1$ to m. Let $A = \Pi_{n=1}^{m} A_{i_n} \times \Pi_{\beta\neq i_n} X_\beta$ be any subset of X. Then λ is semiopen set in X if and only if A_{i_n} is semiopen set in X_{i_n}, for each $n = 1$ to m.*

Theorem 3.9. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is strongly pre-I-continuous, if the graph function $g : (X, \tau, I) \to X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, strongly pre-I-continuous.*

Proof. Let $x \in X$ and $V \in SO(Y)$ containing $f(x)$. Then $X \times V$ is a semi-open set of $X \times Y$ by Lemma 3.8 and contains $g(x)$. Since g is strongly pre-I-continuous, there exists a pre-I-open set U of X containing x such that $g(U) \subset X \times V$. This shows that $f(U) \subset V$. By Theorem 3.7, f is strongly pre-I-continuous. □

Theorem 3.10. *If a function $f : X \to \Pi Y_i$ is strongly pre-I-continuous, then $P_i \circ f : X \to Y_i$ is strongly pre-I-continuous, where P_i is the projection of ΠY_i onto Y_i.*

Proof. Let A_i be an arbitrary semiopen set of Y_i. Since P_i is continuous and open, it is irresolute [[8], Theorem 1.2] and hence $P_i^{-1}(V_i)$ is a semiopen set in ΠY_i. Since f is strongly pre-I-continuous, then $f^{-1}(P_i^{-1}(V_i)) = (P_i \circ f)^{-1}(V_i)$ is pre-I-open in X. Hence, $P_i \circ f$ is strongly pre-I-continuous for each $i \in \wedge$. □

Recall that a subset A of X is said to be $*$-perfect if $A = A^*$[13]. A subset of X is said to be I-locally closed if it is the intersection of an open subset and a $*$-perfect subset of X [9]. An ideal space (X, τ, I) is I-submaximal if every subset of X is I-locally closed [4].

Proposition 1. *If $f : (X, \tau, I) \to (Y, \sigma)$ is a strongly pre-I-continuous function and (X, τ, I) is an I-submaximal space, then f is strongly semi-continuous.*

Proof. Follows from Lemma 4.4 of [4]. □

Definition 3.11. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be strongly irresolute if $f^{-1}(V)$ is semi-I-open in (X, τ, I) for every semiopen set V of Y.

Definition 3.12. An ideal space (X, τ, I) is said to be P-I-disconnected [4] if the $\varnothing \neq A^* \in \tau$ for each $A \in \tau$.

Proposition 2. *If $f : (X, \tau, I) \to (Y, \sigma)$ is a strongly irresolute function and (X, τ, I) is a P-I-disconnected space, then f is strongly pre-I-continuous.*

Proof. Follows from Proposition 4.2 of [4]. □

Theorem 3.13. *If $f : (X, \tau, I) \to (Y, \sigma)$ is strongly pre-I-continuous and A is a semiopen subset of (X, τ), then the restriction $f_{|A} : (A, \tau_{|A}, I_{|A}) \to (Y, \sigma)$ is strongly pre-continuous.*

Proof. Let V be any semiopen set of (Y, σ). Since f is strongly pre-I-continuous, we have $f^{-1}(V)$ is pre-I-open in (X, τ, I). Since A is semiopen in (X, τ), by Proposition 2.10(V) of [9], $(f_{|A})^{-1}(V) = A \cap f^{-1}(V)$ is preopen in A and hence $f_{|A}$ is strongly precontinuous. □

Recall that a function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be pre-I-irresolute if $f^{-1}(V) \in PIO(X)$ for every preopen set V of Y [10].

Definition 3.14. An ideal space (X, τ, I) is said to be pre-I-connected if X is not the union of two disjoint non-empty pre-I-open sets of X.

Definition 3.15. [24] A topological space (X, τ) is said to be semiconnected if X cannot be expressed as the union of two nonempty disjoint semiopen sets of X.

Theorem 3.16. *For the functions $f : (X, \tau, I) \to (Y, \sigma, J)$ and $g : (Y, \sigma, J) \to (Z, \eta, K)$, the following properties hold:*

(i) *If f is pre-I-continuous and g is strongly semicontinuous, then $g \circ f$ is strongly pre-I-continuous;*
(ii) *If f is strongly pre-I-continuous and g is semicontinuous, then $g \circ f$ is pre-I-continuous;*
(iii) *If f is strongly pre-I-continuous and g is irresolute, then $g \circ f$ is strongly pre-I-continuous;*
(iv) *If f is pre-I-irresolute and g is strongly pre-I-continuous, then $g \circ f$ is strongly pre-I-continuous.*

Proof. Follows from their respective definitions. □

Theorem 3.17. *If $f : (X, \tau, I) \to (Y, \sigma)$ is strongly pre-I-continuous surjective function and (X, τ, I) is pre-I-connected, then Y is semi-connected.*

Proof. Suppose Y is not semi-connected. Then there exist non-empty disjoint semiopen subsets U and V of Y such that $Y = U \cup V$. Since f is strongly pre-I-continuous, we have $f^{-1}(U)$ and $f^{-1}(V)$ are non-empty disjoint pre-I-open sets in X. Moreover, $f^{-1}(U) \cup f^{-1}(V) = X$. This shows that X is not pre-I-connected. This is a contradiction and hence Y is semi-connected. □

Lemma 3.18. [22] *For any function $f : (X, \tau, I) \to (Y, \sigma)$, $f(I)$ is an ideal on Y.*

Now, we recall the following definitions.

Definition 3.19. An ideal space (X, τ, I) is said to be pre-I-compact (resp. pre-I-Lindelöf, SI-compact [2], SI-Lindelof [2]) if for every pre-I-open (resp. pre-I-open, semiopen, semiopen) cover $\{W_\alpha : \alpha \in \triangle\}$ on X, there exists a finite (resp. countable) subset $\triangle_0$ of $\triangle$ such that $X - \bigcup\{W_\alpha \colon \alpha \in \triangle_0\} \in I$.

Theorem 3.20. *If $f : (X, \tau, I) \to (Y, \sigma, J)$ is strongly pre I-continuous surjection and (X, τ, I) is pre I-compact, then Y is S-$f(I)$-compact.*

Proof. Let $\{V_\alpha : \alpha \in \nabla\}$ be a semiopen cover of Y, then $\{f^{-1}(V_\alpha) : \alpha \in \nabla\}$ is a pre-I-open cover of X from strongly pre-I-continuity. By hypothesis, there exists a finite subcollection, $\{f^{-1}(V_{\alpha_i})$: i = 1, 2, ... $n\}$ such that X - $\bigcup\{f^{-1}(V_{\alpha_i})$: i = 1, 2,.... $n\} \in I$, implies , Y- $\bigcup\{V_{\alpha_i}$: i = 1, 2, ... $N\} \in f(I)$. Therefore, (Y, σ) is S-$f(I)$-compact. □

Theorem 3.21. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be a strongly pre-I-continuous surjection. If (X, τ, I) is pre-I-Lindelöf, then (Y, σ) is semi-$f(I)$-Lindelöf.*

Proof. Similar to the proof of Theorem 3.20. □

Definition 3.22. An ideal space (X, τ, I) is said to be:

(i) pre-I-T_1 if for each pair of distinct points x and y of X, there exist pre-I-open sets U and V of (X, τ, I) such that $x \in U$ and $y \notin U$, and $y \in V$ and $x \notin V$.
(ii) pre-I-T_2 if for each pair of distinct points x and y in X, there exists disjoint pre-I-open sets U and V in X such that $x \in U$ and $y \in V$.
(iii) semi-T_1 if for each pair of distinct points x and y of X, there exist semiopen sets U and V of (X, τ, I) such that $x \in U$ and $y \notin U$, and $y \in V$ and $x \notin V$ [20].
(iv) semi-T_2 if for each pair of distinct points x and y in X, there exist disjoint semiopen sets U and V in X such that $x \in U$ and $y \in V$ [20].

Theorem 3.23. *If $f : (X, \tau, I) \to (Y, \sigma)$ is a strongly pre-I-continuous injection and (Y, σ) is semi-T_1, then (X, τ, I) is pre-I-T_1.*

Proof. Suppose that (Y, σ) is semi-T_1. For any distinct points x and y in X, there exist $V, W \in SO(Y)$ such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is strongly pre-I-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are pre-I-open subsets of (X, τ, I) such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that (X, τ, I) is pre-I-T_1. □

Theorem 3.24. *If $f : (X, \tau, I) \to (Y, \sigma)$ is a strongly pre-I-continuous injection and Y is semi-T_2, then (X, τ, I) is pre-I-T_2.*

Proof. For any pair of distinct points x and y in X, there exist disjoint semiopen sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is strongly pre-I-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are pre-I-open sets in (X, τ, I) containing x and y, respectively. Therefore, $f^{-1}(U) \cap f^{-1}(V) = \varnothing$ because $U \cap V = \varnothing$. This shows that the space (X, τ, I) is pre-I-T_2. □

Theorem 3.25. *If $f : (X, \tau, I) \to (Y, \sigma)$ is strongly semi continuous function and $g : (X, \tau, I) \to (Y, \sigma)$ is strongly pre-I-continuous function and (Y, σ) is semi T_2, then the set $E = \{x \in X : f(x) = g(x)\}$ is pre-I-closed in (X, τ, I).*

Proof. If $x \in E^c$, then it follows that $f(x) \neq g(x)$. Since (Y, σ) is semi-T_2, there exist V, $W \in SO(Y)$ such that $f(x) \in V$ and $g(x) \in W$ and $V \cap W = \varnothing$. Since f is strongly semi continuous and g is strongly pre I-continuous, $f^{-1}(V)$ is open and $g^{-1}(W)$ is pre-I-open in X with $x \in f^{-1}(V)$ and $x \in g^{-1}(W)$. Put $A_x = f^{-1}(V) \cap g^{-1}(W)$. By Theorem 2.1 of [9](ii), A_x is pre-I-open. If a point $z \in A_x$, then $f(z) \in V$ and $g(z) \in W$. Hence $f(z) \neq g(z)$. This shows that $A_x \subset E^c$ and hence E is pre-I-closed in (X, τ, I). □

Definition 3.26. A space (X, τ) is said to be:

(i) s-regular if each pair of a point and a closed set not containing the point can be separated by disjoint semiopen sets [19].

(iii) semi-normal if every pair of disjoint closed sets of X can be separated by semiopen sets [18].

Definition 3.27. An ideal space (X, τ, I) is said to be:

(i) pre-I-regular if each pair of a point and a closed set not containing the point can be separated by disjoint pre-I-open sets.

(ii) pre-I-normal if every pair of disjoint closed sets of X can be separated by pre-I-open sets.

Theorem 3.28. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be a strongly pre-I-continuous injection. Then the following properties hold:*

(a) *If (Y, σ) is semi-T_2, then (X, τ, I) is pre-I-T_2,*

(b) *If (Y, σ) is semi regular and f is open or closed, then (X, τ, I) is pre-I-regular,*

(c) *If (Y, σ) is semi normal and f is closed, then (X, τ, I) is pre-I-normal.*

Proof. Follows from their respective definitions. □

ÖZET: Bu çalışmada; yarı açık ve ön-I-açık kümeler, kuvvetli ön-I-sürekli isimli fonksiyonların yeni bir sınıfını tanımlamak ve incelemek için kulllanıldılar. Fonksiyonların bu yeni sınıfı ile diğer sınıfları arasındaki ilişkiler elde edildi.

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