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V_n- SLANT HELICES IN MINKOWSKI n-SPACE E_1^n

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ABSTRACT. In this paper we give a definition of harmonic curvature functions in terms of V_n and define a new kind of slant helix which we call V_n-slant helix in $n-$dimensional Minkowski space E_1^n by using the new harmonic curvature functions . Also we define a vector field D_L which we call Darboux vector field of V_n-slant helix in $n-$dimensional Minkowski space E_1^n and we give some characterizations about slant helices.

1. Introduction

Hayden gave more restrictive definition for generalized helices in [6]: If the fixed direction makes a constant angle with all the vectors of the Frenet frame then the curve is a generalized helix in E^n. This definition only works in the odd dimensional case. Moreover, in the same reference, it is proved that the definition is equivalent to the fact that the ratios $\frac{k_{n-1}}{k_{n-2}}, \frac{k_{n-3}}{k_{n-4}}, ..., \frac{k_2}{k_1}$ being the curvatures, are constant. This statement is related with the Lancret Theorem for generalized helices in E^3 (the ratio of torsion to curvature is constant).

Later, Izumiya and Takeuchi defined a new kind of helix i.e.,slant helix and gave a characterization of slant helices in Euclidean $3-$space E^3 [8]. And then Kula and Yaylı investigated spherical images; the tangent indicatrix and binormal indicatrix of a slant helix [10] . Morever, they gave a characterization for slant helices in E^3 : "For involute of a curve γ, γ is a slant helix if and only if its involute is a general helix". If a curve α in E^n, for which all the ratios $\frac{k_{n-1}}{k_{n-2}}, \frac{k_{n-3}}{k_{n-4}}, ..., \frac{k_2}{k_1}$ are constant was called *ccr* curves[11]. In the same reference, it is shown that in the even dimensional case, a curve has constant curvature ratios if and only if its tangent indicatrix is a geodesic in the flat torus. In 2008, Önder *et al.* [12] defined a new kind of slant helix in Euclidean $4-$space E^4 which they called B_2-slant helix and they gave some characterizations of this slant helix in Euclidean $4-$space E^4 . Özdamar and Hacısalihoğlu defined harmonic curvature functions [13]. They generalized inclined

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curves in E^3 to E^n. Gök *et al.* gave the definition a vector field D in Euclidean $n-$space E^n, it is a new characterization for V_n-slant helix [4].

In this study, we define a new kind of slant helix in Minkowski $n-$space E_1^n, where we use the constant angle in between a fixed direction X and the nth Frenet vector field V_n of the curve, this means that

$$g(V_n, X) = \lambda_n \varepsilon_{n-1} = constant \quad , \lambda_n \neq 0.$$

Since nth Frenet vector field V_n of the curve makes a constant angle with a fixed direction X, we call it V_n-slant helix in Minkowski $n-$space E_1^n. In this paper, at first we give a generalization of Hacısalihoğlu's harmonic curvature functions [13]. In this case we define a new characterization in E_1^n such as:

$$\alpha : I \subset \mathbb{R} \longrightarrow E_1^n \text{ is a } V_n-\text{slant helix, then } \sum_{i=1}^{n-2} \varepsilon_{n-(i+2)} H_i^{*^2} = constant$$

where H_i^* is i^{th} harmonic curvature function in terms of V_n.

2. Preliminaries

Let E_1^n be the n $-$dimensional pseudo-Euclidean space with index 1 endowed with the indefinite inner product given by

$$g(x, y) = -x_1 y_1 + \sum_{i=2}^{n} x_i y_i,$$

where $x = (x_1, x_2, \cdots, x_n)$, $y = (y_1, y_2, \cdots, y_n)$ is the usual coordinate system. Then v is said to be spacelike, timelike or null according to $g(v, v) > 0$, $g(v, v) < 0$, or $g(v, v) = 0$ and $v \neq 0$, respectively. Notice that the vector $v = 0$ is spacelike. The category into which a given tangent vector falls is called its causal character. These definitions can be generalized for curves as follows. A curve α in E_1^n is said to be spacelike if all of its velocity vectors $\alpha'$are spacelike, similarly for timelike and null [1].

Let us recall from [15, 7] the definition of the Frenet frame and curvatures.

Let $\alpha : I \subset \mathbb{R} \to E_1^n$ be non-null curve in E_1^n. A non-null curve $\alpha(s)$ is said to be a unit speed curve if $g(\alpha'(s), \alpha'(s)) = \varepsilon_0$, ($\varepsilon_0$ being $+1$ or -1 according to α is spacelike or timelike respectively). Let $\{V_1, V_2, ..., V_n\}$ be the moving Frenet frame along the unit speed curve α, where V_i $(i = 1, 2, ..., n)$ denote ith Frenet vector fields and k_i be i^{th} curvature functions of the curve $(i = 1, 2, ..., n-1)$. Then the Frenet formulas are given as

$$\begin{aligned} \nabla_{V_1} V_1 &= k_1 V_2, \\ \nabla_{V_1} V_i &= -\varepsilon_{i-2}\varepsilon_{i-1} k_{i-1} V_{i-1} + k_i V_{i+1}, \quad 1 < i < n \\ \nabla_{V_1} V_n &= -\varepsilon_{n-2}\varepsilon_{n-1} k_{n-1} V_{n-1} \end{aligned} \tag{2.1}$$

where $g(V_i, V_i) = \varepsilon_{i-1}$, and ∇ is the Levi-Civita connection of E_1^n [7].

3. V_n-Slant Helix in E_1^n

In this section we define V_n-slant helices in Minkowski n-space E_1^n and give some characterizations by using the new harmonic curvatures H_i^* for V_n-slant helix.

Definition 3.1. Let $\alpha : I \subset \mathbb{R} \to E_1^n$ be non-null curve with nonzero curvatures $k_i (i = 1, 2, ..., n)$ in E_1^n and $\{V_1, V_2, ..., V_n\}$ denotes the Frenet frame of the curve α. We call α as a V_n-slant helix in E_1^n if n^{th} unit vector field V_n makes a constant angle with a fixed direction X, that is,

$$g(V_n, X) = \lambda_n \varepsilon_{n-1} = constant \ , \ \lambda_n \neq 0 \ .$$

Therefore, X is in the subspace $Sp\{V_1, V_2, ..., V_{n-1}, V_n\}$ and can be written as

$$X = \sum_{i=1}^{n} x_i V_i \quad , \quad g(X, X) = 1 \ .$$

Definition 3.2. Let $\alpha : I \subset \mathbb{R} \longrightarrow E_1^n$ be a unit speed non-null curve with nonzero curvatures $k_i (i = 1, 2, ..., n)$ in E_1^n. Harmonic curvature functions in terms of V_n for α are defined by

$$H_i^* : I \subset \mathbb{R} \longrightarrow \mathbb{R}$$

$$\begin{aligned} H_0^* &= 0, \\ H_1^* &= \varepsilon_{n-3}\varepsilon_{n-2}\frac{k_{n-1}}{k_{n-2}}, \\ H_i^* &= \left(k_{n-i}H_{i-2}^* - \nabla_{V_1}H_{i-1}^*\right)\frac{\varepsilon_{n-(i+2)}\varepsilon_{n-(i+1)}}{k_{n-(i+1)}}, \quad 2 \leq i \leq n-2. \end{aligned} \tag{3.1}$$

Theorem 3.3. *Let $\alpha : I \subset \mathbb{R} \longrightarrow E_1^n$ be a non-null curve in E_1^n arc-lengthed parameter and X a unit constant vector field and $\{V_1, V_2, ..., V_n\}$ denote the Frenet frame of the curve α, $\{H_1^*, H_2^*, ..., H_{n-2}^*\}$ denote the harmonic curvature functions of the curve α. If $\alpha : I \subset \mathbb{R} \longrightarrow E_1^n$ is a V_n-slant helix then we have*

$$g(V_{n-(i+1)}, \ X) = H_i^* \ g(V_n, \ X), \quad 1 \leq i \leq n-2, \tag{3.2}$$

where X is axis of the V_n-slant helix.

Proof. We will use the induction method.

Let $i = 1$:

Since X is the axis of the V_n-slant helix α, we get

$$X = \lambda_1 V_1 + \lambda_2 V_2 + ... + \lambda_n V_n.$$

From the definition of V_n-slant helix we have

$$g(V_n, X) = \lambda_n \varepsilon_{n-1}. \tag{3.3}$$

A differentiation in Eq.(3.3) and the Frenet formulas give us that

$$g(V_{n-1}, X) = 0. \tag{3.4}$$

Again, differentiation in Eq.(3.4) and the Frenet formulas give

$$\begin{aligned} g(\nabla_{V_1} V_{n-1}, X) &= 0, \\ -\varepsilon_{n-3}\varepsilon_{n-2}k_{n-2}g(V_{n-2}, X) + k_{n-1}g(V_n, X) &= 0, \\ g(V_{n-2}, X) &= \varepsilon_{n-3}\varepsilon_{n-2}\frac{k_{n-1}}{k_{n-2}}g(V_n, X) \\ g(V_{n-2}, X) &= H_1^* \, g(V_n, X), \end{aligned}$$

respectively. Hence it is shown that the Eq.(3.2) is true for $i = 1$.

We now assume the Eq.(3.2) is true for the first $i-1$. Then we have

$$g(V_{n-i}, \ X) = H_{i-1}^* \, g(V_n, X). \tag{3.5}$$

A differentiation in Eq.(3.5) and the Frenet formulas give us that

$$-\varepsilon_{n-i-2}\varepsilon_{n-i-1}k_{n-i-1} \, g(V_{n-i-1}, \ X) + k_{n-i} \, g(V_{n-i+1}, \ X) = \nabla_{V_1} H_{i-1}^* \, g(V_n, X).$$

Since we have the induction hypothesis, $g(V_{n-i+1}, X) = H_{i-2}^* g(V_n, X)$, we get

$$\left(k_{n-i}H_{i-2}^* - \nabla_{V_1} H_{i-1}^*\right) \frac{\varepsilon_{n-(i+2)}\varepsilon_{n-(i+1)}}{k_{n-(i+1)}} \, g(V_n, X) = g(V_{n-(i+1)}, X),$$

which gives

$$g(V_{n-(i+1)}, X) = H_i^* \, g(V_n, X).$$

□

Theorem 3.4. *Let $\alpha : I \subset \mathbb{R} \longrightarrow E_1^n$ be a non-null curve in E_1^n arc-lengthed parameter and X a unit constant vector field and $\{V_1, V_2, ..., V_n\}$ and $\{H_1^*, H_2^*, ..., H_{n-2}^*\}$ denote the Frenet frame and the harmonic curvature functions of the curve α, respectively. If $\alpha : I \subset \mathbb{R} \longrightarrow E_1^n$ is a V_n-slant helix then we have*

$$X = g(V_n, X)\left(\sum_{i=1}^{n-2} H_i^* V_{n-(i+1)}\varepsilon_{n-(i+2)} + \varepsilon_{n-1}V_n\right).$$

Proof. If the axis of V_n-slant helix α in E_1^n is X, then we can write

$$X = \sum_{i=1}^{n} \lambda_i V_i.$$

By using the Theorem(3.3) we get

$$\begin{aligned} \lambda_1 &= \varepsilon_0 \, H_{n-2}^* g(V_n, \ X), \\ \lambda_2 &= \varepsilon_1 \, H_{n-3}^* g(V_n, \ X), \\ &\vdots \\ \lambda_{n-2} &= \varepsilon_{n-3} H_1^* \, g(V_n, \ X), \\ \lambda_{n-1} &= 0, \\ \lambda_n &= \varepsilon_{n-1} \, g(V_n, \ X). \end{aligned}$$

Thus we can easily obtain that

$$X = g(V_n, X)\left(\sum_{i=1}^{n-2} H_i^* V_{n-(i+1)}\varepsilon_{n-(i+2)} + \varepsilon_{n-1}V_n\right).$$

□

Theorem 3.5. *Let $\alpha : I \subset \mathbb{R} \longrightarrow E_1^n$ be a non-null curve in E_1^n arc-lengthed parameter, X be a unit constant vector field and $\{V_1, V_2, ..., V_n\}$, $\{H_1^*, H_2^*, ..., H_{n-2}^*\}$ denote the Frenet frame and the harmonic curvature functions of the curve α,respectively. If $\alpha : I \subset \mathbb{R} \longrightarrow E_1^n$ is a V_n-slant helix, then $\sum_{i=1}^{n-2} \varepsilon_{n-(i+2)} {H_i^*}^2 = constant$.*

Proof. Let α be a V_n-slant helix with the arc length parameter s . Since X is a unit vector field, by using Theorem(3.4) we obtain

$$(g(V_n, X))^2\left(\varepsilon_{n-1} + \sum_{i=1}^{n-2} \varepsilon_{n-(j+2)} {H_i^*}^2\right) = 1. \tag{3.6}$$

Thus we get

$$\sum_{i=1}^{n-2} \varepsilon_{n-(i+2)} {H_i^*}^2 = \frac{1-\varepsilon_{n-1}\lambda_n^2}{\lambda_n^2}.$$

for some non-zero constant λ_n, which completes the proof. □

Definition 3.6. If X is the axis of V_n-slant helix α in E_1^n, then from Theorem(3.4) we can write

$$X = g(V_n, X)\left(\sum_{i=1}^{n-2} H_i^* V_{n-(i+1)}\varepsilon_{n-(i+2)} + \varepsilon_{n-1}V_n\right)$$

where $g(V_n,\ X) = \lambda_n \varepsilon_{n-1} = constant$. And then we can define a new vector field as

$$D_L = \varepsilon_0 H_{n-2}^* V_1 + \varepsilon_1 H_{n-3}^* V_2 + ... + \varepsilon_{n-3} H_1^* V_{n-2} + \varepsilon_{n-1}V_n$$

which is an axis of the V_n-slant helix α.

Theorem 3.7. *Let $\alpha : I \subset \mathbb{R} \longrightarrow E_1^n$ be a non-null curve in E_1^n arc-lengthed parameter, X be a unit constant vector field and $\{V_1, V_2, ..., V_n\}$ and $\{H_1^*, H_2^*, ..., H_{n-2}^*\}$ denote the Frenet frame and the harmonic curvature functions for V_n-slant helix α, respectively. Then α is a V_n-slant helix if and only if D_L is a constant vector field.*

Proof. Suppose that α is a V_n-slant helix in E_1^n and X is the axis of α. From Theorem(3.4), we get

$$X = g(V_n, X)\left(\sum_{i=1}^{n-2} H_i^* V_{n-(i+1)}\varepsilon_{n-(i+2)} + \varepsilon_{n-1}V_n\right). \tag{3.7}$$

From the Eq.(3.3) $g(V_n, X)$ is a constant and so D_L is a constant vector field.

Conversely, since D_L is a constant vector field then we can write that

$$X = g(V_n, X)D_L$$

and then

$$g(X, X) = g(V_n, X)g(X, D_L)$$

or since X is a unit vector field, we have

$$g(V_n, X) = \frac{1}{g(X, D_L)}$$

where $g(X, D_L) =$ constant. So, $g(V_n, X)$ is constant and thus α is a V_n-slant helix. □

Corollary 1. *Let α be a unit speed curve in E_1^3, $\{V_1, V_2, V_3\}$ and $\{k_1, k_2\}$ denote the Frenet frame and curvature functions of the curve α, respectively. Then α is a V_3-slant helix if and only if $\frac{k_2}{k_1} = constant$.*

Proof. Let α be V_3-slant helix in E_1^3, from Theorem(3.7) for $n = 3$,

$$D_L = \varepsilon_1 \frac{k_2}{k_1} V_1 + \varepsilon_2 V_3 = constant \tag{3.8}$$

Differentation in(3.8) gives

$$\nabla_{V_1} D_L = \varepsilon_1 \left(\frac{k_2}{k_1}\right)' V_1 = 0,$$

or $\frac{k_2}{k_1} = constant$.

Conversely, if $\frac{k_2}{k_1}$ is constant, $\nabla_{V_1} D_L = 0$ and $D_L = constant$. From Theorem(3.7) α is a V_3-slant helix, which completes the proof. □

Corollary 2. *Let α be a non-degenerate W-curve i.e., all curvatures of the curve are constant in E_1^3, $\{V_1, V_2, V_3\}$, $\{k_1, k_2\}$ denote the Frenet frame and curvature functions of the curve α, respectively. In this case the curve α is a V_3-slant helix.*

Proof. It is obvious from Corollary 1. □

Corollary 3. *Let α be a non-degenerate W-curve i.e., all curvatures of the curve are constant in E_1^4, $\{V_1, V_2, V_3, V_4\}$, $\{k_1, k_2, k_3\}$ denote the Frenet frame and curvature functions of the curve α, respectively. In this case the curve α is not a V_4-slant helix i.e., B_2-slant helix.*

Proof. Let α be a non-degenerate W-curve i.e., all curvatures of the curve are constant in E_1^4. From the Definition(3.2) and Definition(3.6) we can write

$$D_L = -\varepsilon_1 \frac{1}{k_1} \left(\frac{k_3}{k_2}\right)' + \varepsilon_2 \frac{k_3}{k_2} V_2 + \varepsilon_3 V_4.$$

where k_1, k_2 and k_3 are curvatures of the curve. If all curvatures of the curve are constants, i.e., the curve is a $W-$curve, then we get

$$D_L = \varepsilon_2 \frac{k_3}{k_2} V_2 + \varepsilon_3 V_4.$$

If we take the derivative of W we get

$$\nabla_{V_1} D_L = -\varepsilon_0 \varepsilon_1 \varepsilon_2 \frac{k_1 k_3}{k_2} V_1.$$

Since α is a non-degenerate curve, we obtain that $\nabla_{V_1} D_L \neq 0$ or D_L is constant vector field. So, from Theorem (3.7) the curve is not V_4-slant helix i.e., B_2-slant helix. □

Corollary 4. *Let α be a non- degenerate curve in E_1^4. If the curve α is a V_4-slant helix i.e., B_2-slant helix then,*

$$\left[\frac{1}{k_1}\left(\frac{k_3}{k_2}\right)'\right]' + \varepsilon_0 \varepsilon_1 k_1 \frac{k_3}{k_2} = 0.$$

Proof. Let α be V_4-slant helix i.e., B_2-slant helix. From Theorem(3.5) for $n = 4$, we have $\varepsilon_1 {H_1^*}^2 + \varepsilon_0 {H_2^*}^2$ =constant. By using the Definition(3.2)

$$\varepsilon_1 \left(\frac{k_3}{k_2}\right)^2 + \varepsilon_0 \left[\frac{1}{k_1}\left(\frac{k_3}{k_2}\right)'\right]^2 = \text{constant}. \tag{3.9}$$

By taking the derivative of Eq.(3.9) we obtain

$$\left[\frac{1}{k_1}\left(\frac{k_3}{k_2}\right)'\right]' + \varepsilon_0 \varepsilon_1 k_1 \frac{k_3}{k_2} = 0. \tag{3.10}$$

□

Theorem 3.8. *Let α be a non- degenerate curve in E_1^{2m+1}, and $\left\{H_1^*, H_2^*, ..., H_{2m-1}^*\right\}$ be the harmonic curvature functions of the curve α. If the ratios $\frac{k_2}{k_1}, \frac{k_4}{k_3}, \frac{k_6}{k_5} \cdots \frac{k_{2m-2}}{k_{2m-3}}, \frac{k_{2m}}{k_{2m-1}}$ are constant, then we have for $2 \leq i \leq m$*

$$H_{2i-2}^* = 0$$

and

$$H_{2i-1}^* = \frac{k_{2m}}{k_{2m-1}} \cdot \frac{k_{2m-2}}{k_{2m-3}} \cdots \frac{k_{2m+1-(2i-1)}}{k_{2m+1-(2i)}} \varepsilon_{2m-1} \varepsilon_{2m-2} \ldots \varepsilon_{2m+1-(2i)}.$$

Proof. We apply the induction method for the proof .
Let $i = 1$:

From Definition(3.2) we may write

$$H_2^* = (k_{2m-1}H_0^* - \nabla_{V_1} H_1^*)\frac{\varepsilon_{2m-3}\varepsilon_{2m-2}}{k_{2m-2}}$$
$$H_2^* = \left(-\varepsilon_{2m-2}\varepsilon_{2m-1}\frac{k_{2m}}{k_{2m-1}}\right)'\frac{\varepsilon_{2m-3}\varepsilon_{2m-2}}{k_{2m-2}}$$

where $\frac{k_{2m}}{k_{2m-1}} = constant$, so

$$H_2^* = 0,$$

and again Definition(3.2) gives us

$$H_3^* = (k_{2m-2}H_1^* - \nabla_{V_1} H_2^*)\frac{\varepsilon_{2m-4}\varepsilon_{2m-3}}{k_{2m-3}} .$$

By using $H_2^* = 0$ and Definition (3.2) we can write

$$H_3^* = \frac{k_{2m}}{k_{2m-1}}.\frac{k_{2m-2}}{k_{2m-3}}\varepsilon_{2m-1}\varepsilon_{2m-2}\varepsilon_{2m-3}\varepsilon_{2m-4} .$$

Let us assume that Theorem 3.8 is true for the case $i = p$, then we may write that

$$H_{2p-2}^* = 0$$

and

$$H_{2p-1}^* = \frac{k_{2m}}{k_{2m-1}}.\frac{k_{2m-2}}{k_{2m-3}}...\frac{k_{2m+1-(2p-1)}}{k_{2m+1-(2p)}}\varepsilon_{2m-1}\varepsilon_{2m-2}...\varepsilon_{2m+1-(2p)} .$$

Definition (3.2) gives us $H_{2p}^* = 0$ and

$$H_{2p+1}^* = \left(k_{2m-2p}H_{2p-1}^* - \nabla_{V_1} H_{2p}^*\right)\frac{\varepsilon_{2m-2p-2}\varepsilon_{2m-2p-1}}{k_{2m-2p-1}} .$$

By using $H_{2p}^* = 0$ and Definition (3.2) we can write

$$H_{2p+1}^* = \frac{k_{2m}}{k_{2m-1}}.\frac{k_{2m-2}}{k_{2m-3}}...\frac{k_{2m+1-(2p+1)}}{k_{2m+1-(2p+2)}}\varepsilon_{2m-1}\varepsilon_{2m-2}...\varepsilon_{2m+1-(2p+2)} ,$$

which completes the proof. □

Definition 3.9. Let α be a non- degenerate curve in E_1^{2m+1}, and $\left\{H_1^*, H_2^*, ..., H_{2m-1}^*\right\}$ be the harmonic curvature functions of the curve α. If the ratios $\frac{k_2}{k_1}, \frac{k_4}{k_3}, \frac{k_6}{k_5}...\frac{k_{2m-2}}{k_{2m-3}}, \frac{k_{2m}}{k_{2m-1}}$ are constant, then the curve α is called V_n- slant helix in the sense of Hayden, where $2 \le i \le m$.

Corollary 5. *Let α be a non- degenerate curve in E_1^{2m+1}, and $\left\{H_1^*, H_2^*, ..., H_{2m-1}^*\right\}$ be the harmonic curvature functions of the curve α. If the ratios $\frac{k_2}{k_1}, \frac{k_4}{k_3}, \frac{k_6}{k_5}...\frac{k_{2m-2}}{k_{2m-3}}, \frac{k_{2m}}{k_{2m-1}}$ are constant, then from Theorem (3.7) and Theorem(3.8) we can easily see that the axis of a V_n- slant helix in the sense of Hayden α is*

$$D_L = \varepsilon_0 H_{2m-1}^* V_1 + \varepsilon_2 H_{2m-3}^* V_3 + ... + \varepsilon_{2m-2} H_1^* V_{2m-1} + \varepsilon_{2m} V_{2m+1}.$$

Proof. According to Definition (3.6) for $n = 2m+1$ we have

$$D_L = \varepsilon_0 H^*_{2m-1} V_1 + \varepsilon_1 H^*_{2m-2} V_2 + \cdots + \varepsilon_{2m-2} H^*_1 V_{2m-1} + \varepsilon_{2m} V_{2m+1}$$

where from Theorem(3.8) we get

$$D_L = \varepsilon_0 H^*_{2m-1} V_1 + \varepsilon_2 H^*_{2m-3} V_3 + ... + \varepsilon_{2m-2} H^*_1 V_{2m-1} + \varepsilon_{2m} V_{2m+1},$$

which completes the proof. □

ÖZET: Bu çalışmada E_1^n n-boyutlu Minkowski uzayında yeni tanımlanan Harmonik eğrilik fonksiyonları yardımıyla V_n- slant helis adını verdiğimiz yeni bir slant helis tanımlanmış ve bu helisin V_n cinsinden Harmonik eğrilik fonksiyonları verilmiştir. Ayrıca E_1^n n-boyutlu Minkowski uzayında V_n- slant helis eğrisi boyunca D_L ile gösterilen bir vektör alanı tanımlanmış ve buna V_n- slant helisin Darboux vektör alanı denilmiştir. Bu vektör alanı sayesinde slant helislerin yeni bazı karakterizasyonları verilmiştir.

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