International Electronic Journal of Geometry
Volume 5 No. 2 pp. 36–58 (2012) ©IEJG

SCREEN ALMOST CONFORMAL LIGHTLIKE GEOMETRY IN INDEFINITE KENMOTSU SPACE FORMS

FORTUNÉ MASSAMBA

(Communicated by Ramesh SHARMA)

Abstract. We introduce a new class of lightlike hypersurfaces, namely, Screen Almost Conformal (SAC) lightlike hypersurfaces of indefinite Kenmotsu space forms, tangent to the structure vector field. We show that, under a certain condition, these hypersufaces and the leaves of its integrable screen distributions belong to the same class of η-Einstein, non extrinsic sphere, non-Ricci semi-symmetric and non-semi-parallel manifolds. We also prove that there is an integrable distribution whose leaves are space forms, Einstein and satisfy some symmetry properties. Theorems on integral manifolds and integrable, auto-parallel distributions are obtained. We finally characterize the relative nullity space in a proper totally contact umbilical SAC-lightlike hypersurface of an indefinite Kenmotsu space form.

1. introduction

Kenmotsu in [14] studied a class of almost contact Riemannian manifolds satisfying some special conditions. Such manifolds are called *Kenmotsu manifolds*. Several authors have studied properties of Kenmotsu manifolds since then. In [13], for instance, the authors partially classified Kenmotsu manifolds and considered manifolds admitting a transformation which keeps the Riemannian curvature tensor and Ricci tensor invariant.

As is well known, the geometry of lightlike submanifolds [3] is different because of the fact that their normal vector bundle intersects with the tangent bundle. Thus, the study becomes more difficult and strikingly different from the study of non-degenerate submanifolds. This means that one cannot use, in the usual way, the classical submanifold theory to define any induced object on a lightlike submanifold. To deal with this anomaly, the lightlike submanifolds were introduced and presented in a book by Duggal and Bejancu [3]. They introduced a non-degenerate screen distribution to construct a non-intersecting lightlike transversal vector bundle of the

Date: January 30, 2012 and, in revised form, June 26, 2012.
2010 *Mathematics Subject Classification.* Primary 53C15; Secondary 53C25, 53C50.
Key words and phrases. Indefinite Kenmostu space form; Lightlike hypersurface; Screen almost conformal distribution; Relative nullity foliation.
This work was carried out when the author was visiting the Max-Planck Institute for Mathematics in the Sciences (Leipzig, Germany), in the summer of 2011.

tangent bundle. Since then, a suitable choice of an integrable screen distribution has produced several new results on lightlike geometry (see, e.g, [9] and many more references therein). Also, see [15] for a different approach to deal with lightlike (degenerate) submanifolds. Jin, in a series of papers, studied and characterized the geometry of screen conformal lightlike hypersufaces of semi-Riemannian space forms, for instance Kähler, Lorentzian space form (see [12] and references therein). For lightlike cases of almost contact manifolds in general, some specific discussions on this matter can be found in [17], [18], [19], [20], [21], [22], [23], [24] and references therein.

We know that the shape operator plays a key role in studying the geometry of submanifolds [3]. Motivated by above line of direction, the aim of this paper is to introduce the concept of screen almost conformal distributions of lightlike hypersufaces of Kenmotsu space forms. That is, we study lightlike hypersufaces of Kenmotsu space forms, tangent to the structure vector field, whose shape operators are almost conformal to shape operators of their corresponding screen distributions. We also investigate the effect of almost conformal condition on the geometry of leaves of some integrable distributions and relative nullity foliations.

The paper is organized as follows. In Section 2, we recall some basic definitions for indefinite Kenmotsu manifolds and lightlike hypersurfaces of semi-Riemannian manifolds. In Section 3, we introduce a new class of screen almost conformal (SAC)-lightlike hypersurface M of an indefinite Kenmotsu space form $\overline{M}(c)$, tangent to the structure vector field, supported by an example. We prove that the proper totally contact umbilical SAC-lightlike hypersurface M belongs to the class of totally contact geodesic, η-Einstein, non-Ricci semi-symmetric and non-semi-parallel manifolds. We also discuss the effect of the change of the screen distribution on different results found. In Section 4, we investigate the geometry of leaves in a screen almost conformal lightlike hypersuface M of an indefinite Kenmotsu space form $\overline{M}(c)$, tangent to the structure vector field. These leaves are η-Einstein, non-extrinsic sphere, non-Ricci semi-symmetric and non-semi-parallel manifolds, under a certain condition. We prove that any integral manifold M' of $S(TM)$ is proper totally umbilical and locally a product manifold $\widehat{M}' \times L_\xi$, where $\widehat{M}'$ is proper totally umbilical leaf of $\widehat{D}$ and L_ξ is a non-degenerate curve. We show that there is an integrable distribution $\widehat{D}$, subbundle of TM, whose leaves are space forms of constant curvature $2\varphi\lambda^2$, proper totally umbilical, Einstein, locally symmetric and Ricci semi-symmetric (Theorem 4.5). Under a certain condition, we prove that the distribution $D \perp < \xi >$ in (3.4) is integrable, auto-parallel and M is locally a product $M^* \times C$, where M^* is a proper totally contact leaf of $D \perp \langle \xi \rangle$ and C is a lightlike curve tangent to the distribution $\overline{\phi}(N(TM))$ (Theorem 4.6). By Theorem 5.1, Section 5, we characterize the relative nullity space in a proper totally contact umbilical SAC-lightlike hypersurface of an indefinite Kenmotsu space form.

2. Preliminaries

Let $\overline{M}$ be a $(2n+1)$-dimensional manifold endowed with an almost contact structure $(\overline{\phi}, \xi, \eta)$, i.e., $\overline{\phi}$ is a tensor field of type $(1,1)$, ξ is a vector field, and η is a 1-form satisfying

$$\overline{\phi}^2 = -\mathbb{I} + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \eta \circ \overline{\phi} = 0 \quad \text{and} \quad \overline{\phi}\xi = 0. \tag{2.1}$$

Then $(\overline{\phi}, \xi, \eta, \overline{g})$ is called an indefinite almost contact metric structure on $\overline{M}$ if $(\overline{\phi}, \xi, \eta)$ is an almost contact structure on $\overline{M}$ and $\overline{g}$ is a semi-Riemannian metric on $\overline{M}$ such that [4], for any vector field $\overline{X}$, $\overline{Y}$ on $\overline{M}$

$$\overline{g}(\overline{\phi}\,\overline{X}, \overline{\phi}\,\overline{Y}) = \overline{g}(\overline{X}, \overline{Y}) - \eta(\overline{X})\,\eta(\overline{Y}). \tag{2.2}$$

It follows that, for any vector field $\overline{X}$ on $\overline{M}$, $\eta(\overline{X}) = \overline{g}(\xi, \overline{X})$.

Now, we give the following definition by adapting the one for Riemannian case given in [14].

An indefinite almost contact metric structure $(\overline{\phi}, \xi, \eta, \overline{g})$ is called an indefinite Kenmostu structure if

$$\overline{\nabla}_{\overline{X}}\xi = \overline{X} - \eta(\overline{X})\xi, \quad (\overline{\nabla}_{\overline{X}}\overline{\phi})\overline{Y} = \overline{g}(\overline{\phi}\,\overline{X}, \overline{Y})\xi - \eta(\overline{Y})\overline{\phi}\,\overline{X}, \tag{2.3}$$

where $\overline{\nabla}$ is the Levi-Civita connection for the semi-Riemannian metric $\overline{g}$. We call $\overline{M}$ an indefinite Kenmotsu manifold (see [11] for details). Here, without loss of generality, the vector field ξ is assumed to be spacelike, that is, $\overline{g}(\xi, \xi) = 1$. The Kenmotsu structure defined in [14] differs from the indefinite Kenmotsu one only by the positiveness of the metric involved and so, the main results in [14] remain unchanged for the indefinite case. We denote by $\Gamma(\Xi)$ the set of smooth sections of the vector bundle Ξ.

A plane section σ in $T_p\overline{M}$ is called a $\overline{\phi}$-section if it is spanned by $\overline{X}_0$ and $\overline{\phi}\,\overline{X}_0$, where $\overline{X}_0$ is a unit tangent vector field orthogonal to ξ. The sectional curvature of a $\overline{\phi}$-section σ is called a $\overline{\phi}$-sectional curvature. If an indefinite Kenmotsu manifold $\overline{M}$ has constant $\overline{\phi}$-sectional curvature c, then, by virtue of the Proposition 12 in [14], the curvature tensor $\overline{R}$ of $\overline{M}$ is given by

$$\begin{aligned}\overline{R}(\overline{X}, \overline{Y})\overline{Z} = {} & \frac{c-3}{4}\{\overline{g}(\overline{Y}, \overline{Z})\overline{X} - \overline{g}(\overline{X}, \overline{Z})\overline{Y}\} + \frac{c+1}{4}\{\eta(\overline{X})\eta(\overline{Z})\overline{Y} \\ & -\eta(\overline{Y})\eta(\overline{Z})\overline{X} + \overline{g}(\overline{X}, \overline{Z})\eta(\overline{Y})\xi - \overline{g}(\overline{Y}, \overline{Z})\eta(\overline{X})\xi + \overline{g}(\overline{\phi}\,\overline{Y}, \overline{Z})\overline{\phi}\,\overline{X} \\ & -\overline{g}(\overline{\phi}\,\overline{X}, \overline{Z})\overline{\phi}\,\overline{Y} - 2\overline{g}(\overline{\phi}\,\overline{X}, \overline{Y})\overline{\phi}\,\overline{Z}\}, \quad \overline{X}, \overline{Y}, \overline{Z} \in \Gamma(T\overline{M}).\end{aligned} \tag{2.4}$$

An indefinite Kenmotsu manifold $\overline{M}$ of constant $\overline{\phi}$-sectional curvature c will be called *indefinite Kenmotsu space form* and denoted $\overline{M}(c)$.

Example 2.1. We consider the 7-dimensional manifold $\overline{M}^7 = \{x \in \mathbb{R}^7 : x_7 > 0\}$, where $x = (x_1, x_2, ..., x_7)$ are the standard coordinates in $\mathbb{R}^7$. Let us consider the vector fields $e_1, e_2, ..., e_7$, linearly independent at each point of $\overline{M}^7$, as a combination of frames $\{\frac{\partial}{\partial x_i}\}$. Let $\overline{g}$ be the semi-Riemannian metric defined by $\overline{g}(e_i, e_j) = 0$, $\forall\, i \neq j$, $i, j = 1, 2, ..., 7$ and $\overline{g}(e_k, e_k) = 1$, $\forall\, k = 1, 2, 3, 4, 7$, $\overline{g}(e_m, e_m) = -1$, $\forall\, m = 5, 6$. Let η be the 1-form defined by $\eta(\cdot) = \overline{g}(\cdot, e_7)$. Let $\overline{\phi}$ be the $(1, 1)$ tensor field defined by $\overline{\phi}e_1 = -e_2$, $\overline{\phi}e_2 = e_1$, $\overline{\phi}e_3 = -e_4$, $\overline{\phi}e_4 = e_3$, $\overline{\phi}e_5 = -e_6$, $\overline{\phi}e_6 = e_5$, $\overline{\phi}e_7 = 0$. Using the linearity of $\overline{\ phi}$ and $\overline{g}$, we have $\overline{\phi}^2\overline{X} = -\overline{X} + \eta(\overline{X})e_7$, $\overline{g}(\overline{\phi}\,\overline{X}, \overline{\phi}\,\overline{Y}) = \overline{g}(\overline{X}, \overline{Y}) - \eta(\overline{X})\eta(\overline{Y})$. Thus, for $e_7 = \xi$, $(\overline{\phi},\ \xi,\ \eta,\ \overline{g})$ defines an almost contact metric structure on $\overline{M}^7$. Let $\overline{\nabla}$ be the Levi-Civita connection with respect to the metric $\overline{g}$ and let us choose the vector fields $e_1, e_2, ..., e_7$ to be $e_i = e^{x_7}\sum_{j=1}^{7} f_{ij}(x_1, ..., x_6)\frac{\partial}{\partial x_j}$, $\det(f_{ij}) \neq 0$ and $e_7 = \xi = -\frac{\partial}{\partial x_7}$, where functions f_{ij} are defined such that the action of $\overline{\nabla}$ on the basis $\{e_1, ..., e_7\}$ is given by $\overline{\nabla}_{e_i}e_i = -\xi$, $\forall i = 1, 2, 3, 4$, $\overline{\nabla}_{e_m}e_m = \xi$, $\forall m = 5, 6$, $\overline{\nabla}_{e_1}e_5 = e^{x_7}e_6$, $\overline{\nabla}_{e_1}e_6 = -e^{x_7}e_5$, $\overline{\nabla}_{e_2}e_5 = e^{x_7}e_6$, $\overline{\nabla}_{e_2}e_6 = -e^{x_7}e_5$, $\overline{\nabla}_{e_5}e_1 = e^{x_7}e_6$, $\overline{\nabla}_{e_5}e_2 = e^{x_7}e_6$, $\overline{\nabla}_{e_6}e_1 = -e^{x_7}e_5$, $\overline{\nabla}_{e_6}e_2 = -e^{x_7}e_5$, and other covariant derivatives $\overline{\nabla}_{e_i}e_j = 0$, $\forall i \neq j, i, j = 1, 2, ..., 6$. The non-vanishing brackets are given by, for $i = 1, 2, 3, ..., 6$, $[e_i, e_7] = e_i$. By Koszul's formula, we have $\overline{\nabla}_{e_i} e_7 = e_i$, $\forall i = 1, 2, ..., 6$, and $\overline{\nabla}_{e_7} e_7 = 0$. Using these relations, $(\overline{\phi}, \xi, \eta, \overline{g})$ is an indefinite Kenmostu structure in $\overline{M}^7$. Therefore, $(\overline{M}^7, \overline{\phi}, \xi, \eta, \overline{g})$ is an indefinite Kenmostu manifold with constant sectional curvature $c = -1$.

Let $(\overline{M}, \overline{g})$ be a $(2n+1)$-dimensional semi-Riemannian manifold with index s, $0 < s < 2n+1$ and let (M, g) be a hypersurface of $\overline{M}$, with $g = \overline{g}_{|M}$. M is said to be a lightlike hypersurface of $\overline{M}$ if g is of constant rank $2n-1$ and the orthogonal complement $TM^\perp$ of tangent space TM, defined as $TM^\perp = \bigcup_{p\in M}\{Y_p \in T_p\overline{M} : \overline{g}_p(X_p, Y_p) = 0, \forall X_p \in T_pM\}$, is a distribution of rank 1 on M [3]: $TM^\perp \subset TM$ and then coincides with the radical distribution $\mathrm{Rad}\, TM = TM \cap TM^\perp$. A complementary bundle of $TM^\perp$ in TM is a rank $2n-1$ non-degenerate distribution over M. It is called a *screen distribution* and is denoted by $S(TM)$. Existence of $S(TM)$ is secured provided M is paracompact. However, in general, $S(TM)$ is not canonical (thus it is not unique) and the lightlike geometry depends on its choice but it is canonically isomorphic to the vector bundle $TM/\mathrm{Rad}\, TM$ [15].

A lightlike hypersurface endowed with a specific screen distribution is denoted by the triple $(M, g, S(TM))$. As $TM^\perp$ lies in the tangent bundle, the following result has an important role in studying the geometry of a lightlike hypersurface.

Theorem 2.1. [3] *Let $(M, g, S(TM))$ be a lightlike hypersurface of $(\overline{M}, \overline{g})$. Then, there exists a unique vector bundle $N(TM)$ of rank 1 over M such that for any non-zero section E of $TM^\perp$ on a coordinate neighborhood $\mathcal{U} \subset M$, there exists a unique section N of $N(TM)$ on $\mathcal{U}$ satisfying $\overline{g}(N, E) = 1$ and $\overline{g}(N, N) = \overline{g}(N, W) = 0$, for any $W \in \Gamma(S(TM)|_{\mathcal{U}})$.*

Throughout the paper, all manifolds are supposed to be paracompact and smooth. We denote by $\perp$ and $\oplus$ the orthogonal and nonorthogonal direct sum of two vector bundles. By Theorem 2.1, we may write down the following decompositions

$$TM = S(TM) \perp TM^\perp, \tag{2.5}$$

$$T\overline{M} = TM \oplus N(TM) = S(TM) \perp (TM^\perp \oplus N(TM)). \tag{2.6}$$

Let $\overline{\nabla}$ be the Levi-Civita connection on $(\overline{M}, \overline{g})$, then, using the decomposition (2.6), we have the Gauss and Weingarten formulae in the form,

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y) \quad \text{and} \quad \overline{\nabla}_X V = -A_V X + \nabla^\perp_X V, \tag{2.7}$$

for any X, $Y \in \Gamma(TM|_{\mathcal{U}})$, $V \in \Gamma(N(TM))$, where $\nabla_X Y$, $A_V X \in \Gamma(TM)$ and $h(X, Y)$, $\nabla^\perp_X V \in \Gamma(N(TM))$. ∇ is an induced symmetric linear connection on M, $\nabla^\perp$ is a linear connection on the vector bundle $N(TM)$, h is a $\Gamma(N(TM))$-valued symmetric bilinear form and A_V is the shape operator of M in $\overline{M}$.

Equivalently, consider a normalizing pair $\{E, N\}$ as in Theorem 2.1. Then (2.7) takes the following form,

$$\overline{\nabla}_X Y = \nabla_X Y + B(X, Y)\, N \quad \text{and} \quad \overline{\nabla}_X N = -A_N X + \tau(X) N, \tag{2.8}$$

where B, A_N, τ and ∇ are called the local second fundamental form, the local shape operator, the transversal differential 1-form and the induced linear torsion-free connection, respectively, on $TM|_{\mathcal{U}}$.

From (2.8), we have the identities $B(\cdot, E) = 0$, $B(X, Y) = \overline{g}(\overline{\nabla}_X Y, E)$ and

$$\tau(X) = \overline{g}(\nabla^\perp_X N, E). \tag{2.9}$$

Let P be the projection morphism of TM on $S(TM)$ with respect to the orthogonal decomposition of TM. We have,

$$\nabla_X PY = \nabla^*_X PY + C(X, PY)E, \text{ and } \nabla_X E = -A^*_E X - \tau(X)E, \tag{2.10}$$

for any X, $Y \in \Gamma(TM)$, $E \in \Gamma(TM^\perp)$, where $\nabla^*_X PY$ and $A^*_E X$ belong to $\Gamma(S(TM))$. C, A^*_E and ∇^* are called the local second fundamental form, the local shape operator and the induced linear metric connection, respectively, on $S(TM)$. The induced linear connection ∇ is not a metric connection and we have

$$(\nabla_X g)(Y, Z) = B(X, Y)\theta(Z) + B(X, Z)\theta(Y), \tag{2.11}$$

where θ is a differential 1-form locally defined on M by $\theta(\cdot) := \overline{g}(N, \cdot)$. The local second fundamental forms B and C, respectively, of M and on $S(TM)$ are related to their shape operators by $g(A^*_E X, PY) = B(X, PY)$, $g(A^*_E X, N) = 0$, $g(A_N X, PY) = C(X, PY)$ and $g(A_N X, N) = 0$. We denote by $\overline{R}$, R and R^* the curvature tensors of $\overline{\nabla}$, ∇ and ∇^*, respectively. Using the Gauss-Weingarten equations for M and $S(TM)$, we obtain the Gauss-Codazzi equation for M and $S(TM)$ such that, for any X, Y, Z, $W \in \Gamma(TM)$,

$$\overline{g}(\overline{R}(X, Y)Z, PW) = g(R(X, Y)Z, PW) + B(X, Z)C(Y, PW) - B(Y, Z)C(X, PW), \tag{2.12}$$

$$\overline{g}(\overline{R}(X, Y)Z, E) = (\nabla_X B)(Y, Z) - (\nabla_Y B)(X, Z) + \tau(X)B(Y, Z) - \tau(Y)B(X, Z), \tag{2.13}$$

$$g(R(X, Y)PZ, PW) = g(R^*(X, Y)PZ, PW) + C(X, PZ)C(Y, PW) - C(Y, PZ)C(X, PW), \tag{2.14}$$

$$g(R(X, Y)PZ, N) = (\nabla_X C)(Y, PZ) - (\nabla_Y C)(X, PZ) + \tau(Y)C(X, PZ) - \tau(X)C(Y, PZ). \tag{2.15}$$

3. Screen almost conformal Lightlike hypersurfaces of indefinite Kenmotsu manifolds

Let $(\overline{M}, \overline{\phi}, \xi, \eta, \overline{g})$ be an indefinite Kenmotsu manifold and (M, g) be a lightlike hypersurface of $(\overline{M}, \overline{g})$, *tangent to the structure vector field* ξ *(*$\xi \in TM$*)*.

If E is a local section of $TM^\perp$, it is easy to check that $\overline{\phi}E \neq 0$ and $\overline{g}(\overline{\phi}E, E) = 0$, then $\overline{\phi}E$ is tangent to M. Thus $\overline{\phi}(TM^\perp)$ is a distribution on M of rank 1 such that $\overline{\phi}(TM^\perp) \cap TM^\perp = \{0\}$. This enables us to choose a screen distribution $S(TM)$ such that it contains $\overline{\phi}(TM^\perp)$ as a vector subbundle. If we consider a local section N of $N(TM)$, we have $\overline{\phi}N \neq 0$. Since $\overline{g}(\overline{\phi}N, E) = -\overline{g}(N, \overline{\phi}E) = 0$, we deduce that $\overline{\phi}E$ belongs to $S(TM)$ and $\overline{\phi}N$ is also tangent to M. At the same time, since $\overline{g}(\overline{\phi}N, N) = 0$, we see that the component of $\overline{\phi}N$, with respect to E, vanishes. Thus $\overline{\phi}N \in \Gamma(S(TM))$, i.e., $\overline{\phi}(N(TM))$ is also a vector subbundle of $S(TM)$ of rank 1. From (refequa1), we have $\overline{g}(\overline{\phi}N, \overline{\phi}E) = 1$. Therefore, $\overline{\phi}(TM^\perp) \oplus \overline{\phi}(N(TM))$ is a non-degenerate vector subbundle of $S(TM)$ of rank 2.

If M *is tangent to the structure vector field* ξ*, we may choose* $S(TM)$ *so that* ξ *belongs to* $S(TM)$. Using this, and since $\overline{g}(\overline{\phi}E, \xi) = \overline{g}(\overline{\phi}N, \xi) = 0$, there exists a non-degenerate distribution D_0 of rank $2n - 4$ on M such that

$$S(TM) = \{\overline{\phi}(TM^\perp) \oplus \overline{\phi}(N(TM))\} \perp D_0 \perp < \xi >, \tag{3.1}$$

where $\langle\xi\rangle$ is the distribution spanned by ξ. The distribution D_0 is invariant under $\overline{\phi}$, i.e. $\overline{\phi}(D_0) = D_0$. Moreover, from (2.5) and (3.1), we obtain the decompositions

$$TM = \{\overline{\phi}(TM^\perp) \oplus \overline{\phi}(N(TM))\} \perp D_0 \perp <\xi> \perp TM^\perp, \tag{3.2}$$

$$T\overline{M} = \{\overline{\phi}(TM^\perp) \oplus \overline{\phi}(N(TM))\} \perp D_0 \perp <\xi> \perp (TM^\perp \oplus N(TM)). \tag{3.3}$$

Now, we consider the distributions on M, $D := TM^\perp \perp \overline{\phi}(TM^\perp) \perp D_0$, $D' := \overline{\phi}(N(TM))$. Then, D is invariant under $\overline{\phi}$ and

$$TM = (D \oplus D') \perp \langle\xi\rangle. \tag{3.4}$$

Let us consider the local lightlike vector fields $U := -\overline{\phi}\,N$, $\quad V := -\overline{\phi}\,E$. Then, from (3.4), any $X \in \Gamma(TM)$ is written as $X = RX + QX + \eta(X)\xi$, $\;QX = u(X)\,U$, where R and Q are the projection morphisms of TM into D and D', respectively, and u is a differential 1-form locally defined on M by

$$u(X) := g(V, X), \quad \forall\, X \in \Gamma(TM). \tag{3.5}$$

Applying $\overline{\phi}$ to X and (2.1), one obtains $\overline{\phi}\,X = \phi\,X + u(X)\,N$, where ϕ is a tensor field of type $(1,1)$ defined on M by $\phi\,X := \overline{\phi}\,RX$. In addition, we obtain, $\phi^2\,X = -X + \eta(X)\xi + u(X)\,U$ and $\nabla_X\xi = X - \eta(X)\xi$. Using (2.1), we derive

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y) - u(Y)v(X) - u(X)v(Y), \tag{3.6}$$

where v is a differential 1-form locally defined on M by $v(\cdot) = g(U, \cdot)$. We have the following identities, for any $X \in \Gamma(TM)$, $\nabla_X\xi = X - \eta(X)\xi$ and

$$B(X,\xi) = 0, \tag{3.7}$$

$$C(X,\xi) = \theta(X). \tag{3.8}$$

Let M be a lightlike hypersurface of an indefinite Kenmotsu space form $\overline{M}(c)$ with $\xi \in TM$. Then, the relation (2.4) becomes, for any X, Y, $Z \in \Gamma(TM)$,

$$\overline{R}(X,Y)Z = g(X,Z)Y - g(Y,Z)X. \tag{3.9}$$

Using (2.4), (2.13) and (2.15), we obtain, for any X, Y, $Z \in \Gamma(TM)$,

$$\begin{aligned}(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) &= \tau(Y)B(X,Z)\\ &\quad - \tau(X)B(Y,Z),\end{aligned} \tag{3.10}$$

$$\begin{aligned}\text{and}\quad (\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) &= g(X,PZ)\theta(Y) - g(Y,PZ)\theta(X)\\ &\quad + \tau(X)C(Y,PZ) - \tau(Y)C(X,PZ).\end{aligned} \tag{3.11}$$

The relation between $\overline{R}$ and R is given by

$$\overline{R}(X,Y)Z = R(X,Y)Z + B(X,Z)A_NY - B(Y,Z)A_NX. \tag{3.12}$$

Using (3.9), the curvature tensor R of M is expressed as

$$R(X,Y)Z = g(X,Z)Y - g(Y,Z)X + B(Y,Z)A_NX - B(X,Z)A_NY. \tag{3.13}$$

Therefore, for any X, Y, Z, $W \in \Gamma(TM)$,

$$\begin{aligned}g(R(X,Y)PZ, PW) &= g(X,PZ)g(Y,PW) - g(Y,PZ)g(X,PW)\\ &\quad + B(Y,Z)C(X,PW) - B(X,Z)C(Y,PW),\end{aligned} \tag{3.14}$$

$$\text{and}\quad \overline{g}(R(X,Y)PZ, N) = g(X,PZ)\theta(Y) - g(Y,PZ)\theta(X). \tag{3.15}$$

Let us consider the following distribution

$$\widehat{D} = \left\{\overline{\phi}(TM^\perp) \oplus \overline{\phi}(N(TM))\right\} \perp D_0, \tag{3.16}$$

so that the tangent space of M is written

$$TM = \widehat{D} \perp \langle \xi \rangle \perp TM^{\perp}. \tag{3.17}$$

Let $\widehat{P}$ be the morphism of $S(TM)$ on $\widehat{D}$ with respect to the orthogonal decomposition of $S(TM)$ such that

$$\widehat{P}X = PX - \eta(X)\xi, \ \ \forall X \in \Gamma(TM). \tag{3.18}$$

It is easy to check that $\widehat{P}$ is also a projection. We have, for any X, $Y \in \Gamma(TM)$,

$$B(X, PY) = B(X, \widehat{P}Y), \ \ C(X, PY) = C(X, \widehat{P}Y) + \theta(X)\eta(Y). \tag{3.19}$$

Define the induced Ricci type tensor $R^{(0,2)}$ of M, respectively, as

$$R^{(0,2)}(X, Y) = \mathrm{trace}(Z \longrightarrow R(Z, X)Y), \forall X, Y \in \Gamma(TM). \tag{3.20}$$

Since the induced connection ∇ on M is not a Levi-Civita connection, in general, $R^{(0,2)}$ is not symmetric. Therefore, in general, it is just a tensor quantity and has no geometric or physical meaning similar to the symmetric Ricci tensor of $\overline{M}$.

Let consider a local quasi-orthogonal frame field $\{X_0, N, X_i\}_{i=1,\dots,2n-1}$ on $\overline{M}$, where $\{X_0, X_i\}$ is a local frame field on M with N, the unique section given in Theorem 2.1, and $E = X_0$. Locally, (3.20) is given by (see [3], for details)

$$R^{(0,2)}_{ls} - R^{(0,2)}_{sl} = 2d\tau(X_l, X_s) \ \text{ and } \ R^{(0,2)}_{0k} - R^{(0,2)}_{k0} = 2d\tau(X_0, X_k), \tag{3.21}$$

where $R^{(0,2)}_{ls} := R^{(0,2)}(X_s, X_l)$ and $R^{(0,2)}_{0k} := R^{(0,2)}(X_k, X_0)$. Using (2.12), a direct calculation gives

$$R^{(0,2)}(X, Y) = -(2n-1)g(X, Y) + B(X, Y)trA_N - B(A_N X, Y), \tag{3.22}$$

where trace tr is written with respect to g restricted to $S(TM)$. The Ricci tensor does not depend on the choice of E of $TM^{\perp}$. From (3.22), we have [10]

$$R^{(0,2)}(X, Y) - R^{(0,2)}(Y, X) = B(A_N X, Y) - B(A_N Y, X). \tag{3.23}$$

The tensor field $R^{(0,2)}$ of a lightlike hypersurface M of an indefinite Kenmotsu manifold $\overline{M}$ is called induced Ricci tensor [7] if it is symmetric.

For historical reasons, we still call $R^{(0,2)}$ an induced Ricci tensor, but, we denote it by Ric only if it is symmetric.

It is well known that the second fundamental form and the shape operator of a non-degenerate submanifold are related by means of the metric tensor field. Contrary to this we see from (2.8) and (2.10) that in case of lightlike hypersurface, there are interrelations between these geometric objects and those of its screen distribution. More precisely, the second fundamental forms of the lightlike hypersurface M and its screen distribution $S(TM)$ are related to their respective shape operator A_N and A^*_E. This consolidates the fact that the geometry of a lightlike hypersurface depends on a choice of screen distribution. As the shape operator is an information tool in studying geometry of submanifolds, we are led to consider lightlike hypersurfaces whose shape operators are almost the same as the one of their screen distribution.

Now, we introduce a new class, called screen almost conformal, briefly, SAC-lightlike hypersurface of an indefinite Kenmotsu manifold, as follows.

A lightlike hypersurface $(M, g, S(TM))$ of an indefinite Kenmotsu manifold $\overline{M}$ with $\xi \in TM$ is screen almost conformal (SAC) if the shape operator A_N and A^*_E of M and its screen distribution $S(TM)$, respectively, are related by

$$A_N = \varphi A^*_E + \alpha \otimes \xi, \tag{3.24}$$

where φ is non-vanishing smooth function and α is a differential 1-form on $\mathcal{U}$ in M. It is easy to see that $\alpha = \theta$ on $\mathcal{U}$ in M and (3.24) becomes

$$A_N = \varphi A^*_E + \theta \otimes \xi. \tag{3.25}$$

This is equivalent to

$$C = \varphi B + \theta \otimes \eta. \tag{3.26}$$

In particular, we say that M is screen almost homothetic if φ is a non-zero constant. In case $\mathcal{U} = M$ the screen almost conformality is said to be global. If θ vanishes, then M is a screen conformal lightlike hypersurface [9].

As an example of SAC-lightlike hypersurface, we have:

Example 3.1. Let M be a hypersurface of $(\overline{M}^7, \overline{\phi}, \xi, \eta, \overline{g})$, indefinite Kenmostu manifold defined in Example 2.1, given by $M = \{x \in \overline{M}^7 : x_5 = \sqrt{2}(x_4 + x_3),\ f_{3i} = f_{4i} = f_{5i} = 0, f_{33} = f_{44} = f_{55} = 1\}$. Thus, the tangent space TM is spanned by $\{U_i\}$, where $U_1 = e_1$, $U_2 = e_2$, $U_3 = e_4 - e_3$, $U_4 = \frac{1}{\sqrt{2}}(e_4 + e_3) - e_5$, $U_5 = e_6$, $U_6 = \xi$ and the 1-dimensional distribution $TM^{\perp}$ of rank 1 is spanned by E, where $E = \frac{1}{\sqrt{2}}(e_4 + e_3) - e_5$. It follows that $TM^{\perp} \subset TM$. Then M is a 6-dimensional lightlike hypersurface of $\overline{M}^7$. Also, the transversal bundle $N(TM)$ is spanned by $N = \frac{1}{2}\{\frac{1}{\sqrt{2}}(e_4 + e_3) + e_5\}$. Using the almost contact structure of $\overline{M}^7$ and the decomposition (3.1), D_0 is spanned by $\{F, \overline{\phi}F\}$, where $F = U_1$, $\overline{\phi}F = -U_2$ and the distributions $\langle\xi\rangle$, $\overline{\phi}(TM^{\perp})$ and $\overline{\phi}(N(TM))$ are spanned, respectively, by ξ, $\overline{\phi}E = -\frac{1}{\sqrt{2}}U_3 + U_5$ and $\overline{\phi}N = \frac{1}{2}\{-\frac{1}{\sqrt{2}}U_3 - U_5\}$. Hence, M is a lightlike hypersurface of $\overline{M}^7$. Denote by $\overline{\nabla}$ the Levi-Civita connection on $\overline{M}^7$. Then, by straightforward calculations, we obtain $\overline{\nabla}_{U_1}N = \frac{1}{2}e^{x_7}e_6$, $\overline{\nabla}_{U_2}N = \frac{1}{2}e^{x_7}e_6$, $\overline{\nabla}_{U_3}N = \overline{\nabla}_{U_5}N = \overline{\nabla}_{U_6}N = 0$, $\overline{\nabla}_E N = -\xi$, $\nabla_{U_1}E = -e^{x_7}e_6$, $\nabla_{U_2}E = -e^{x_7}e_6$, $\nabla_{U_3}E = \nabla_{U_5}E = \nabla_{U_6}E = \nabla_E E = 0$. Using these equations above, the differential 1-form τ vanishes i.e. $\tau(X) = 0$, for any $X \in \Gamma(TM)$. So, from the Gauss and Weingarten formulae we infer $A_N U_1 = -\frac{1}{2}e^{x_7}e_6$, $A_N U_2 = -\frac{1}{2}e^{x_7}e_6$, $A_N U_3 = A_N U_5 = A_N U_6 = 0$, $A_N E = \xi$, $A^*_E U_1 = e^{x_7}e_6$, $A^*_E U_2 = e^{x_7}e_6$, $A^*_E U_3 = A^*_E U_5 = A^*_E U_6 = A^*_E E = 0$. These imply that, for any $X \in \Gamma(TM)$, $A_N X = \varphi A^*_E X + \theta(X)\xi$, with $\varphi = -\frac{1}{2}$. Therefore, M is a screen almost homothetic lightlike hypersuface of an indefinite Kenmostu manifold $(\overline{M}^7, \overline{\phi}, \xi, \eta, \overline{g})$, tangent to the structure vector field ξ.

If $(M, g, S(TM))$ is a SAC-lightlike hypersurface of an indefinite manifold $\overline{M}$ with $\xi \in TM$, then, using (3.23) and $B(\xi, \cdot) = 0$, we have,

$$R^{(0,2)}(X, Y) - R^{(0,2)}(Y, X) = \varphi B(A^*_E X, Y) - \varphi B(A^*_E Y, X) = 0. \tag{3.27}$$

Therefore, a locally (or globally) SAC-lightlike hypersurface M of an indefinite Kenmotsu space form $\overline{M}(c)$ with $\xi \in TM$, admits an induced Ricci tensor.

In the sequel, we need the following relations, for any $X,\, Y \in \Gamma(TM)$,

$$(\nabla_X \theta)Y = -C(X,Y) + \tau(X)\theta(Y), \tag{3.28}$$

$$(\nabla_X \eta)Y = g(X,Y) - \eta(X)\eta(Y). \tag{3.29}$$

Using (3.28) and (3.29), the covariant derivative of C gives,

$$\begin{aligned}(\nabla_X C)(Y,PZ) &= X(\varphi)B(Y,PZ) + \varphi(\nabla_X B)(Y,PZ)\\ &\quad + \eta(PZ)\{-\varphi B(X,Y) + \tau(X)\theta(Y)\}\\ &\quad + \theta(Y)\{g(X,PZ) - \eta(X)\eta(PZ)\},\end{aligned} \tag{3.30}$$

and using (3.10), the left hand side of (3.11) is given by

$$\begin{aligned}(\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) &= \{X(\varphi) - \varphi\tau(X)\}B(Y,PZ)\\ &\quad - \{Y(\varphi) - \varphi\tau(Y)\}B(X,PZ) + g(X,PZ)\eta(Y)\\ &\quad - g(Y,PZ)\theta(X) + \{\tau(X)\theta(Y) - \tau(Y)\theta(X)\}\eta(PZ).\end{aligned} \tag{3.31}$$

On the other hand, using (3.18), the relation (3.11) becomes

$$\begin{aligned}(\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) &= g(X,PZ)\theta(Y) - g(Y,PZ)\theta(X)\\ &\quad + \varphi\tau(X)B(Y,PZ) - \varphi\tau(Y)B(X,PZ)\\ &\quad + \{\tau(X)\theta(Y) - \tau(Y)\theta(X)\}\eta(PZ).\end{aligned} \tag{3.32}$$

Putting the pieces (3.31) and (3.32) together, we have

$$\{X(\varphi) - 2\varphi\tau(X)\}B(Y,PZ) = \{Y(\varphi) - 2\varphi\tau(Y)\}B(X,PZ). \tag{3.33}$$

Theorem 3.1. *Let $(M, g, S(TM))$ be a SAC-lightlike hypersurface of an indefinite Kenmotsu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. Then, for any $X,\, Y \in \Gamma(TM)$,*

$$\{E(\varphi) - 2\varphi\tau(E)\}B(X,PY) = 0. \tag{3.34}$$

Proof. The proof follows from (3.33). □

Under the hypothesis of Theorem 3.1, one can prove that if $E(\varphi) - 2\varphi\tau(E) \neq 0$, then M is totally geodesic. Thus, M satisfies the symmetry proprieties studied in [16] for lightlike geometry in indefinite Sasakian manifolds. But if M is not totally geodesic, then, φ on M satisfies $E(\varphi) - 2\varphi\tau(E) = 0$.

A submanifold M is said to be a totally umbilical lightlike hypersurface of a semi-Riemannian manifold $\overline{M}$ if its local second fundamental form B satisfies

$$B(X,Y) = \lambda g(X,Y), \;\; \forall\, X,\, Y \in \Gamma(TM), \tag{3.35}$$

where λ is a smooth function on $\mathcal{U} \subset M$. If M is a totally umbilical SAC-lightlike hypersurface of an indefinite Kenmotsu manifold $\overline{M}$ with $\xi \in TM$, then, M is totally geodesic [18]. It follows that a Kenmotsu $\overline{M}$ does not admit any non-totally geodesic, totally umbilical SAC-lightlike hypersurface with almost conformal screen distribution. From this point of view, Bejancu considered the concept of totally contact umbilical submanifolds in [1].

A lightlike hypersurface (M, g) is said to be totally contact umbilical if its second fundamental form h satisfies ([18]),for any $X,\, Y \in \Gamma(TM)$,

$$h(X,Y) = H\,\{g(X,Y) - \eta(X)\eta(Y)\} + \eta(X)h(Y,\xi) + \eta(Y)h(X,\xi), \tag{3.36}$$

where $H = \rho N$ being the mean curvature vector of M (ρ a smooth function on $\mathcal{U} \subset M$). If ρ is nowhere vanishing on M, then the latter is said to be *proper totally contact umbilical.*

Let us assume that the screen distribution $S(TM)$ of M is integrable and let M' be a leaf of $S(TM)$. Then, using (2.8) and (2.10), we obtain

$$\overline{\nabla}_X Y = \nabla'_X Y + h'(X, Y), \tag{3.37}$$

for any X, $Y \in \Gamma(TM')$, where ∇' and h' are the Levi-Civita connection and the second fundamental form of M' in $\overline{M}$. Thus,

$$h'(X, Y) = C(X, Y)E + B(X, Y)N, \quad \forall\, X,\ Y \in \Gamma(TM'). \tag{3.38}$$

Lemma 3.1. *Let $(M, g, S(TM))$ be a SAC-lightlike hypersurface of an indefinite Kenmotsu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. Then, $S(TM)$ is integrable.*

The proof of this lemma is similar to the one for Theorem 2.3 in [3, page 89]. $S(TM)$ being integrable, M is locally a product manifold $L \times M'$, where L is an open subset of a lightlike geodesic ray in $\overline{M}$ and M' is a leaf of $S(TM)$.

If M is a totally contact umbilical SAC-lightlike hypersurface, the shape operators A^*_E and A_N are given by, for any $X \in \Gamma(TM)$,

$$A^*_E X = \rho \widehat{P} X \quad \text{and} \quad A_N X = \rho\varphi \widehat{P} X + \theta(X)\xi. \tag{3.39}$$

Using the relations (3.39), the trace of the shape operator A_N, with respect to g restricted to $S(TM)$, is given by,

$$tr A_N = 2(n-1)\varphi\rho. \tag{3.40}$$

Taking (3.39) in (3.13), the curvature tensor R of M is given by,

$$\begin{aligned} R(X, Y)Z &= g(X, Z)Y - g(Y, Z)X - \varphi\rho^2\{g(X, Z)\widehat{P}Y - g(Y, Z)\widehat{P}X\} \\ &\quad + \varphi\rho^2\{\eta(X)\widehat{P}Y - \eta(Y)\widehat{P}X\}\eta(Z) + \rho[\{g(Y, Z) - \eta(Y)\eta(Z)\}\theta(X) \\ &\quad - \{g(X, Z) - \eta(X)\eta(Z)\}\theta(Y)]\xi,\ \forall X, Y, Z \in \Gamma(TM). \end{aligned} \tag{3.41}$$

A lightlike hypersurface M is said to be η-Einstein if its induced Ricci tensor Ric satisfies

$$Ric = ag + b\eta \otimes \eta, \tag{3.42}$$

where the non-zero functions a and b are not necessarily constant on M.

For η-Einstein lightlike hypersurfaces, due to the symmetry of the induced degenerate metric g, the Ricci tensor is symmetric, and the notion of η-Einstein manifold does not depend on the choice of the screen distribution $S(TM)$.

Theorem 3.2. *Let $(M, g, S(TM))$ be a proper totally contact umbilical SAC-lightlike hypersurface of an indefinite Kenmotsu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. Then, M is η-Einstein.*

Proof. Let $(M, g, S(TM))$ be a proper totally contact umbilical SAC-lightlike hypersurface of an indefinite Kenmotsu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. From (3.39) and using (3.40), the induced Ricci type tensor (3.22) becomes

$$\begin{aligned} Ric(X, Y) &= -(2n-1)g(X, Y) + 2(n-1)\varphi\rho^2\{g(X, Y) - \eta(X)\eta(Y)\} \\ &\quad - \varphi\rho^2\{g(X, Y) - \eta(X)\eta(Y)\} = ag(X, Y) + b\eta(X)\eta(Y), \end{aligned} \tag{3.43}$$

where $a = -(2n-1) + (2n-3)\varphi\rho^2$ and $b = -(2n-3)\varphi\rho^2$. This induced Ricci type tensor is symmetric and then called an induced Ricci tensor which is satisfied the relation (3.42). Therefore, M is η-Einstein. $\square$

A lightlike submanifold M of a semi-Riemannian manifold $\overline{M}$ is said to be Ricci semi-symmetric if the following condition is satisfied ([6], [26])

$$(R(W_1, W_2) \cdot Ric)(X, Y) = 0, \ \forall W_1, W_2, X, Y \in \Gamma(TM), \tag{3.44}$$

where R and Ric are induced Riemannian curvature and Ricci tensor on M, respectively. The latter condition is equivalent to

$$-Ric(R(W_1, W_2)X, Y) - Ric(X, R(W_1, W_2)Y) = 0.$$

Let M is a SAC-lightlike hypersurface of a Kenmotsu space form $\overline{M}(c)$ with $\xi \in TM$. If M is proper totally contact umbilical, by Theorem 3.2, M is η-Einstein with $a = -(2n-1)+(2n-3)\varphi\rho^2$ and $b = -(2n-3)\varphi\rho^2$. Using (3.41) and (3.43), the left-hand side of (3.44) gives $-Ric(R(E, V)U, \xi) - Ric(U, R(E, V)\xi) = (2n-1)\rho \neq 0$. This implies that a proper totally contact umbilical SAC-lightlike hypersurface M is not Ricci semi-symmetric. Therefore,

Theorem 3.3. *There exist no proper totally umbilical SAC-lightlike hypersurfaces M of indefinite Kenmotsu space forms ($\overline{M}(c)$ with $\xi \in TM$ that are Ricci semi-symmetric.*

As an example to this theorem, let us consider the manifold to be $\overline{M}^7 = \{x \in \mathbb{R}^7 : x_7 > 0\}$, where $x = (x_1, x_2, ..., x_7)$ are the standard coordinates in $\mathbb{R}^7$. The vector fields, $e_p = x_7 \frac{\partial}{\partial x_p}$, $e_q = -x_7 \frac{\partial}{\partial x_q}$, for any $p = 1, 2, 3, 4$, $q = 5, 6, 7$ are linearly independent at each point of $\overline{M}^7$. Endowing with the same almost contact structure as in Example 2.1, $\overline{M}^7$ is an indefinite Kenmotsu space form with $c = -1$. The hypersurface M of $\overline{M}^7$ given by $x_5 = \sqrt{2}\,(x_2 + x_3)$ is lightlike and having a local quasi-orthogonal field of frames $U_1 = e_1$, $U_2 = e_2 - e_3$, $U_3 = \frac{1}{\sqrt{2}}(e_2 + e_3) - e_5$, $U_4 = e_4$, $U_5 = e_6$, $U_6 = \xi$, $N = \frac{1}{2}\{\frac{1}{\sqrt{2}}(e_2 + e_3)\} + e_5$ along M, we get $\overline{\nabla}_{U_3} N = -\xi$ and $\overline{\nabla}_X N = 0$, $\forall X \in \Gamma(TM)$, $X \neq U_3$ and $\tau(X) = 0$, $\forall X \in \Gamma(TM)$. By (2.8) and (2.10), we have, $A_N U_3 = \xi$, $A_N X = 0$, $\forall X \in \Gamma(TM)$, $X \neq U_3$, $A^*_E X = 0$ and $\nabla_X E = 0$, $\forall X \in \Gamma(TM)$. We have $A_N = \theta \otimes \xi$ and M is totally geodesic but not proper totally umbilical and its screen distribution is almost conformal. The components of $R \cdot Ric$ vanish, that is, $(R(\cdot, \cdot) \cdot Ric)(e_p, e_q) = 0$, $\forall\, p,\ q,\ r,\ s$, i.e. M is Ricci semi-symmetric.

A submanifold M is said to be semi-parallel if its second fundamental form h satisfies ([21]), for any W_1, W_2, X, $Y \in \Gamma(TM)$,

$$(R(W_1, W_2) \cdot h)(X, Y) = 0, \tag{3.45}$$

that is , $h(R(W_1, W_2)X, Y) + h(X, R(W_1, W_2)Y) = 0$.

Theorem 3.4. *Let M be a SAC-lightlike hypersurface of an indefinite Kenmotsu space form ($\overline{M}(c)$ with $\xi \in TM$. If the component of the second fundamental form $h(V, U) \neq 0$, then, M is not semi-parallel.*

Proof. From the left hand-side of (3.45), we have, $(R(V, \xi) \cdot h)(U, \xi) = h(V, U) \neq 0$, i.e. M is not semi-parallel. □

If a SAC-lightlike hypersurface M, of an indefinite Kenmotsu space form $\overline{M}(c)$ with $\xi \in TM$, is proper totally contact umbilical, then its second fundamental form h satisfies (3.36) and $h(V, U) = \rho \neq 0$. Therefore, there exist submanifolds on $\overline{M}$ which satisfy the hypotheses of the Theorem 3.4.

In [23], the author showed that if the second fundamental form h of M satisfies (3.36), then ρ satisfies the partial differential equations

$$E(\rho)+\rho\tau(E)-\rho^2=0,\ \ \xi(\rho)+\rho(\tau(\xi)+1)=0, \tag{3.46}$$

$$\text{and}\quad \widehat{P}X(\rho)+\rho\tau(\widehat{P}X)=0,\quad \forall X\in\Gamma(TM). \tag{3.47}$$

If a SAC-lightlike hypersurface M is proper totally contact umbilical, then, ρ satisfies the equations (3.46) and (3.47). Since $X=\widehat{P}X+\eta(X)\xi+\theta(X)E$, using (3.46) and (3.47), one obtains $X(\rho)+\rho\{\tau(X)+\eta(X)\}=\rho^2\theta(X)$ and the mean curvature vector $H=\rho N$ of M satisfies $\nabla^{\perp}_{E}H=\rho^2N$, $\nabla^{\perp}_{\xi}H=-\rho N$ and $\nabla^{\perp}_{PX}H=0$, $PX\neq\xi$, $\forall X\in\Gamma(TM)$. This means that H is not parallel, that is, the totally contact umbilical SAC-lightlike hypersurface M of a Kenmotsu space form $\overline{M}(c)$ with $\xi\in TM$ is not an extrinsic sphere (see [5] and [23] for details).

Let ϑ be the mean curvature 1-form, that is, the dual differential 1-form of the mean curvature vector H of M. Then ϑ is locally defined by

$$\vartheta(X)=\overline{g}(H,X)=\rho\theta(X),\quad \forall\, X\in\Gamma(TM), \tag{3.48}$$

which leads to $\tau(X)=\vartheta(X)-X(\ln|\rho|)-\eta(X)$. Now, putting this relation together with the relations (3.21), we have

$$\begin{aligned}R^{(0,2)}_{ls}-R^{(0,2)}_{sl}&=X_l(\vartheta(X_s))-X_l(X_s(\ln|\lambda|))-X_l(\eta(X_s))-\vartheta(\nabla_{X_l}X_s)\\&+\nabla_{X_l}X_s(\ln|\lambda|)+\eta(\nabla_{X_l}X_s)-X_s(\vartheta(X_l))+X_s(X_l(\ln|\lambda|))\\&+X_s(\eta(X_l))+\vartheta(\nabla_{X_s}X_l)-\nabla_{X_s}X_l(\ln|\lambda|)-\eta(\nabla_{X_s}X_l)\\&=2d\vartheta(X_l,X_s),\end{aligned} \tag{3.49}$$

and similarly, we have $R^{(0,2)}_{0k}-R^{(0,2)}_{k0}=2d\vartheta(X_0,X_k)$. This means that $R^{(0,2)}$ is symmetric on M if and only if $d\vartheta=0$ on $\mathcal{U}\subset M$, that is ϑ is closed. If $R^{(0,2)}$ is a symmetric Ricci tensor Ric, then, the 1-form ϑ is closed. Using (2.11), we have, for any X, Y, $Z\in\Gamma(TM)$,

$$(\nabla_Xg)(Y,Z)=\{g(X,Y)-\eta(X)\eta(Y)\}\vartheta(Z)+\{g(X,Z)-\eta(X)\eta(Z)\}\vartheta(Y).$$

If ϑ vanishes identically on M, then ∇ is a torsion-free metric connection on M. By Theorem 2.2 in [3, page 88], M is totally geodesic. This contradicts the above statement on the geodesibility of M. Therefore, $\vartheta\neq0$ and we obtain another class of lightlike hypersurface whose induced Ricci tensor is symmetric.

The geometry of lightlike hypersurfaces depends on the vector bundles $S(TM)$ and $N(TM)$. It is known that the local second fundamental form B of M on $\mathcal{U}$ is independent of the choice of the above vector bundles. This means that all results depending only on B are stable with respect to any change of those vector bundles. Suppose a screen $S(TM)$ changes to another screen $\widetilde{S(TM)}$. The following are the local transformation equations due to this change (see [3] for details):

$$\widetilde{W}_i = \sum_{j=1}^{2n-1} W_i^j \left(W_j - \epsilon_j \mathbf{f}_j E\right), \tag{3.50}$$

$$\widetilde{N} = N - \frac{1}{2} g(W,W)E + W, \tag{3.51}$$

$$\widetilde{\tau}(X) = \tau(X) + B(X, \widetilde{N} - N), \tag{3.52}$$

$$\widetilde{A^*_E} X = A^*_E X + B(X, N - \widetilde{N})E, \tag{3.53}$$

$$\widetilde{\nabla}_X Y = \nabla_X Y + B(X,Y)\{\frac{1}{2} g(W,W)E - W\}, \tag{3.54}$$

for any $X,\ Y \in \Gamma(TM|_{\mathcal{U}})$, where ϵ_i are signature of the orthonormal basis $\{W_i\}$, $W = \sum_{i=1}^{2n-1} \mathbf{f}_i W_i$ is the characteristic vector field of the screen change and W_i^j and $\mathbf{f}_i$ are smooth functions on $\mathcal{U}$ such that $\{W_i^j\}$ are $(2n-1)\times(2n-1)$ semi-orthogonal matrices. Denote by ω the dual 1-form of W with respect to the induced metric g of M [3], that is,

$$\omega(X) = g(X, W), \quad \forall\, X \in \Gamma(TM). \tag{3.55}$$

Let P and $\widetilde{P}$ be projections of TM on $S(TM)$ and $\widetilde{S(TM)}$, respectively, with respect to the orthogonal decomposition of TM. Using (3.51), it is easy to check that $\widetilde{P}X = PX - \omega(X)E$ and $\widetilde{C}(X, \widetilde{P}Y) = \widetilde{C}(X, PY)$, $\forall X,\ Y \in \Gamma(TM)$. The relationship between the second fundamental forms C and $\widetilde{C}$ of the screen distribution $S(TM)$ and $\widetilde{S(TM)}$, respectively, is given by (using (3.51) and (3.54)),

$$\widetilde{C}(X, PY) = C(X, PY) - \frac{1}{2}\omega(\nabla_X PY + B(X,Y)W). \tag{3.56}$$

If M is a proper totally contact umbilical SAC-lightlike hypersurface of a Kenmotsu space form $\overline{M}(c)$ with $\xi \in TM$, then M is not totally geodesic. Since $\widetilde{\theta} = \theta + \omega$, the 1-forms $\widetilde{\tau}$ and τ, and the shape operators $\widetilde{A^*_E}$ and A^*_E are related, respectively, as $\widetilde{\tau}(X) = \tau(X) + \rho\omega(\widehat{P}X)$ and $\widetilde{A^*_E}X = A^*_E X - \rho\omega(\widehat{P}X)E$. The dual differential 1-form ϑ of the mean curvature vector H of M depends on the subbundle $N(TM)$ and letting $\widetilde{\vartheta}$ be another induced object with respect to another transversal subbundle $\widetilde{N(TM)}$. Then, the mean curvature 1-forms $\widetilde{\vartheta}$ and ϑ are related as $\widetilde{\vartheta}(X) = \vartheta(X) + \rho\omega(X)$, $\forall\, X \in \Gamma(TM)$.

Theorem 3.5. *Let $(M, g, S(TM))$ be a proper totally contact umbilical SAC-lightlike hypersurface of an indefinite Kenmotsu manifold $(\overline{M}, \overline{g})$ with $\xi \in TM$. The 1-form τ in (2.9) and the shape operator A^*_E in (2.10) all two are independent of $S(TM)$ if and only if the 1-form ω in (3.55) is proportional to the 1-form η, i.e. $\omega = \omega(\xi)\eta$. Moreover, the mean curvature 1-form ϑ in (3.48) is independent of $S(TM)$ if and only if the 1-form ω vanishes identically on M.*

If the screen distribution $S(TM)'$ generated by $\{\widetilde{W'_i}\}$ as given in (3.50) is screen locally almost conformal, then by (3.26), (3.56) and the fact that $B = B'$, we obtain $g(\nabla_X PY, W) = \psi B(X,Y) - 2\eta(Y)\omega(X)$, for some smooth function ψ on M. Using the symmetry of B, we obtain $g(\nabla_X PY - \nabla_Y PX, W) = 2\{\eta(X)\omega(Y) - \eta(Y)\omega(X)\}$. Thus, we have $g(\nabla_X Y - \nabla_Y X, W) = 2\{\eta(X)\omega(Y) - \eta(Y)\omega(X)\}$, $\forall\, X,\ Y \in \Gamma(S(TM))$, that is, $\omega([X,Y]) = 2\{\eta(X)\omega(Y) - \eta(Y)\omega(X)\}$. Hence, the 1-form ω in (3.55) vanishes identically on the first derivative space distribution $Span\{[X,Y]_x : X_x,\ Y_x \in \Gamma(S(T_x))\}$ if and only if $\eta(X)\omega(Y) = \eta(Y)\omega(X)$.

4. Geometry of leaves of screen almost conformal lightlike hypersurface

Let $(M, g, S(TM))$ be a screen almost conformal (SAC) lightlike hypersurface of an indefinite Kenmotsu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. Now, we investigate the effects of almost conformality condition on the geometry of leaves of some integrable distributions with specific attention to those of screen distribution $S(TM)$, the distributions $\widehat{D}$ and $D \perp \langle\xi\rangle$. Since the screen distribution $S(TM)$ of a screen almost conformal lightlike hypersurface M is integrable, let M' be a leaf of $S(TM)$. Then, using (3.37), we obtain

$$\overline{\nabla}_X Y = \nabla'_X Y + h'(X,Y) = \nabla'_X Y + B(X,Y)(\varphi E + N), \tag{4.1}$$

for any $X,\ Y \in \Gamma(TM')$, where ∇' and $h' = B \otimes (\varphi E + N)$ are the Levi-Civita connection on M' and the second fundamental form h' of M', respectively.

Theorem 4.1. *Let $(M, g, S(TM))$ be a SAC-lightlike hypersurface of an indefinite Kenmotsu manifold $(\overline{M}, \overline{g})$ with $\xi \in TM$ and a leaf M' of $S(TM)$. Then,*

(i) *M is totally geodesic,*
(ii) *M is proper totally contact umbilical,*
(iii) *M is minimal,*

if and only if and only if M' is so immersed as a submanifold of $\overline{M}$.

Suppose that M is a proper totally contact umbilical SAC-lightlike hypersurface. By Theorem 4.1, the leaf M' is proper totally contact geodesic and we have,

$$\overline{\nabla}_X Y = \nabla'_X Y + h'(X,Y),\ \forall X, Y \in \Gamma(TM'), \tag{4.2}$$

where $h'(X,Y) = H'\{g(X,Y) - \eta(X)\eta(Y)\}$ is the second fundamental form on M' and $H' = \rho\varphi E + \rho N$ is the mean curvature vector of the leaf M'.

From (3.14), we have, for any $X,\ Y,\ Z,\ W \in \Gamma(TM)$,

$$\begin{aligned} g(R(X,Y)PZ, PW) &= g(X,PZ)g(Y,PW) - g(Y,PZ)g(X,PW) \\ &+ \varphi\rho^2\{g(Y,PZ) - \eta(Y)\eta(PZ)\}\{g(X,PW) - \eta(X)\eta(PW)\} \\ &- \varphi\rho^2\{g(X,PZ) - \eta(X)\eta(PZ)\}\{g(Y,PW) - \eta(Y)\eta(PW)\} \\ &+ \rho\theta(X)\eta(PW)\{g(Y,PZ) - \eta(Y)\eta(PZ)\} \\ &- \rho\theta(Y)\eta(PW)\{g(X,PZ) - \eta(x)\eta(PZ)\}. \end{aligned} \tag{4.3}$$

On the other hand, using the Gauss and Weingarten equations, the curvature tensors R and R' of ∇ and ∇^*, respectively, are related by

$$\begin{aligned} R(X,Y)PZ &= R'(X,Y)PZ + C(X,PZ)A^*_E Y - C(Y,PZ)A^*_E X \\ &+ \{(\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) + \tau(Y)C(X,PZ) \\ &- \tau(X)C(Y,PZ)\}E, \end{aligned} \tag{4.4}$$

where $(\nabla_X C)(Y, PZ) = X(C(Y, PZ)) - C(\nabla^*_X Y, PZ) - C(Y, \nabla^*_X PZ)$. Thus,

$$\begin{aligned} &g(R(X,Y)PZ, PW) = g(R'(X,Y)PZ, PW) - \varphi\rho^2\{g(Y,PZ) - \eta(Y)\eta(PZ)\} \\ &\times \{g(X,PW) - \eta(X)\eta(PW)\} + \varphi\rho^2\{g(X,PZ) - \eta(X)\eta(PZ)\}\{g(Y,PW) \\ &- \eta(Y)\eta(PW)\} - \rho\theta(Y)\eta(PZ)\{g(X,PW) - \eta(X)\eta(PW)\} \\ &+ \rho\theta(X)\eta(PZ)\{g(Y,PW) - \eta(Y)\eta(PW)\}. \end{aligned} \tag{4.5}$$

From (4.3) and (4.5) the curvature tensor R' of M' is given by

$$\begin{aligned} R'(X,Y)Z &= g(X,Z)Y - g(Y,Z)X + 2\varphi\rho^2\{g(Y,Z) - \eta(Y)\eta(Z)\}\widehat{P}X \\ &- 2\varphi\rho^2\{g(X,Z) - \eta(X)\eta(Z)\}\widehat{P}Y, \ \forall X, Y, Z \in \Gamma(TM'), \end{aligned} \tag{4.6}$$

and he non-zero functions ρ and φ satisfy $X(\rho) + \rho(\tau(X) + \eta(X)) = 0$ and $X(\varphi) - 2\varphi\tau(X) = 0$, for any $X \in \Gamma(TM')$. Using this, the Ricci type tensor Ric' of M' gives $Ric'(X,Y) = \{-(2n-1) + 4(n-1)\varphi\rho^2\}g(X,Y) - 4(n-1)\varphi\rho^2\eta(X)\eta(Y)$. Therefore, we have

Theorem 4.2. *Let $(M, g, S(TM))$ be a proper totally contact umbilical SAC-lightlike hypersurface of an indefinite Kenmotsu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. Let M' be a leaf of $S(TM)$. Then, M' is η-Einstein.*

It is easy to check that $(R'(\xi, V) \cdot Ric')(U, \xi) = -(2n-1) \neq 0$ and $(R'(V,\xi) \cdot h')(\xi, U) = H' \neq 0$. Then, we have

Lemma 4.1. *Let $(M, g, S(TM))$ be a proper totally contact umbilical SAC-lightlike hypersurface of an indefinite Kenmotsu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. There exist no leaves of $S(TM)$ that are Ricci semi-symmetric and semi-parallel.*

From partial differential equation on ρ and φ above, one obtains, for any $X \in \Gamma(TM')$, $\overline{g}(\nabla'^{\perp}_X H', E) = X(\rho) + \rho\tau(X) = -\rho\eta(X)$ and $\overline{g}(\nabla'^{\perp}_X H', N) = \varphi X(\rho) + \rho\{X(\varphi) - \varphi\tau(X)\} = -\rho\varphi\eta(X)$, where $\nabla'^{\perp}$ is a linear connection on $N(TM) \oplus TM^{\perp}$ along M' defined by $\nabla'^{\perp}_X E = \nabla^{*\perp}_X E = -\tau(X)E$ and $\nabla'^{\perp}_X N = \nabla^{\perp}_X N = \tau(X)N$. We have, $\overline{g}(\nabla'^{\perp}_X H', E) \neq 0$ and $\overline{g}(\nabla'^{\perp}_X H', N) \neq 0$, $\forall X \in \Gamma(TM)$. That is, the mean curvature vector H' of the leaf M' is not parallel. We have,

Theorem 4.3. *Let $(M, g, S(TM))$ be a proper totally contact umbilical SAC-lightlike hypersurface of an indefinite Kenmostu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. Let M' be a leaf of $S(TM)$. Then M' is not an extrinsic sphere.*

The result of this theorem on proper totally contact umbilical SAC-lightlike hypersurfaces is similar to the one found in [23, Theorem 5.10].

Now, referring to the decomposition (3.17), for any $X \in \Gamma(TM)$, $Y \in \Gamma(\widehat{D})$, we have

$$\nabla_X Y = \widehat{\nabla}_X Y + \widehat{h}(X, Y), \tag{4.7}$$

where $\widehat{\nabla}$ is a linear connection on the bundle $\widehat{D}$ and $\widehat{h} : \Gamma(TM) \times \Gamma(\widehat{D}) \longrightarrow \Gamma(\langle\xi\rangle \perp TM^{\perp})$ is $\mathcal{F}(M)$-bilinear. Let $\mathcal{U} \subset M$ be a coordinate neighborhood. Then, using (3.17), (4.7) can be rewritten (locally) in the following way:

$$\begin{aligned} \nabla_X Y &= \widehat{\nabla}_X Y + g(\nabla_X Y, \xi)\xi + g(\nabla_X Y, N)E \\ &= \widehat{\nabla}_X Y - g(X, Y)\xi + C(X, Y)E, \end{aligned} \tag{4.8}$$

and the local expression of $\widehat{h}$ is defined as $\widehat{h}(X,Y) = -g(X,Y)\xi + C(X,Y)E$. The tensor $\widehat{h}$ is not symmetric, in general. Using (4.8), then, the distribution $\widehat{D}$ is integrable if and only if it is symmetric, i.e.

$$C(X,Y) = C(Y,X), \quad \forall X, Y \in \Gamma(\widehat{D}). \tag{4.9}$$

Lemma 4.2. *Let $(M, g, S(TM))$ be a SAC-lightlike hypersurface of an indefinite Kenmotsu space form $\overline{M}(c)$ with $\xi \in TM$. Then, the distribution $\widehat{D}$ in (3.16) is integrable.*

The proof follows from Lemma 3.1 and $C(\cdot,\cdot) = C(\widehat{P}\cdot, \widehat{P}\cdot)$ on $S(TM)$. This result means that the distribution $\widehat{D}$ is always integrable on a SAC-lightlike hypersurface M of an indefinite Kenmotsu space form $\overline{M}(c)$ with $\xi \in TM$. Let $\widehat{M'}$ be a leaf of $\widehat{D}$. Using (3.26), we have, for any X, Y, $Z \in \Gamma(TM)$,

$$(\nabla_X C)(Y, \widehat{P}Z) = X(\varphi)B(Y, \widehat{P}Z) + \varphi(\nabla_X B)(Y, \widehat{P}Z). \tag{4.10}$$

Mimicing the technique used to derive (3.33), one obtains,

$$\{E(\varphi) - 2\varphi\tau(E)\}B(X, \widehat{P}Z) = -g(X, \widehat{P}Z). \tag{4.11}$$

By relation (4.11), since $X = \widehat{P}X + \eta(X)\xi + \theta(X)E$ and $B(\cdot, \xi) = 0$, we have

$$\{E(\varphi) - 2\varphi\tau(E)\}B(\widehat{P}X, \widehat{P}Z) = -g(\widehat{P}X, \widehat{P}Z). \tag{4.12}$$

This relation implies that $\{E(\varphi) - 2\varphi\tau(E)\} \neq 0$ and $B \neq 0$ along $\widehat{M'}$. From (3.26) and (4.12), we have

$$B(\widehat{P}X, \widehat{P}Y) = \rho g(\widehat{P}X, \widehat{P}Y) \ \text{ and } \ C(\widehat{P}X, \widehat{P}Y) = \varphi\rho g(\widehat{P}X, \widehat{P}Y), \tag{4.13}$$

$$\text{where} \quad \lambda = -(E(\varphi) - 2\varphi\tau(E))^{-1} \neq 0. \tag{4.14}$$

Theorem 4.4. *Let $(M, g, S(TM))$ be a SAC-lightlike hypersurface of an indefinite Kenmotsu space form $\overline{M}(c)$ with $\xi \in TM$. Then, any integral manifold M' of $S(TM)$ is proper totally umbilical and locally a product manifold $\widehat{M'} \times L_\xi$, where $\widehat{M'}$ is proper totally umbilical leaf of $\widehat{D}$ and L_ξ is a non-degenerate curve tangent to the distribution spanned by ξ.*

Proof. Let $(M, g, S(TM))$ be a SAC-lightlike hypersurface of an indefinite Kenmotsu space form $\overline{M}(c)$ with $\xi \in TM$. Then, as it is mentioned above, $S(TM)$ and $\widehat{D}$ are integrable. This means that they determine foliations. The distribution $< \xi >$ being a 1-dimensional non-degenerate distribution, it defines a foliation. Since $S(TM) = \widehat{D} \perp < \xi >$ in (3.1), then, any integral manifold M' of $S(TM)$, immersed as a submanifold in $\overline{M}$, is locally a product manifold $\widehat{M'} \times L_\xi$, where $\widehat{M'}$ is leaf of $\widehat{D}$ and L_ξ is a non-degenerate curve tangent to ξ. By combining the first equations of (2.8) and (2.10), we obtain

$$\begin{aligned} \overline{\nabla}_X Y &= \widehat{\nabla}_X Y - g(X,Y)\xi + B(X,Y)\{\varphi E + N\} \\ &= \widehat{\nabla}'_X Y + \widehat{h}'(X,Y), \end{aligned} \tag{4.15}$$

for any X, $Y \in \Gamma(\widehat{M'})$, where $\widehat{\nabla}'$ and $\widehat{h}' = -g \otimes \xi + B \otimes \{\varphi E + N\}$ are the Levi-Civita connection and second fundamental form of $\widehat{M'}$ in $\overline{M}$, respectively. Since

$\{E(\varphi) - 2\varphi\tau(E)\} \neq 0$ and $B \neq 0$ along $\widehat{M}'$, then, by (4.13) $\widehat{M}'$ is not totally geodesic and its second fundamental form $\widehat{h}'$ is given by

$$\widehat{h}'(X,Y) = \{-\xi + \lambda(\varphi E + N)\}g(X,Y), \tag{4.16}$$

for any X, $Y \in \Gamma(\widehat{M}')$, which implies that $\widehat{M}'$ is proper totally umbilical, since $-\xi + \lambda(\varphi E + N) \neq 0$. The second fundamental form h' of M' in (4.1) is related to $\widehat{h}'$ by $\widehat{h}' = -g \otimes \xi + h'$. Therefore, M' is proper totally umbilical. □

The second fundamental form $\widehat{h}'$ of $\widehat{M}'$ defined in (4.15) is deduced as

$$\widehat{h}'(X,Y) = \widehat{H}'g(X,Y), \ \ \forall\, X,\ Y \in \Gamma(T\widehat{M}'), \tag{4.17}$$

where $\widehat{H}' = -\xi + \lambda(\varphi E + N)$ is the mean curvature vector of the leaf $\widehat{M}'$. The relation (4.15) becomes $\overline{\nabla}_X Y = \widehat{\nabla}'_X Y + \widehat{H}'g(X,Y)$, which implies

$$\begin{aligned}\overline{\nabla}_X\overline{\nabla}_Y Z = {}& \widehat{\nabla}'_X\widehat{\nabla}'_Y Z + \widehat{H}'g(X,\widehat{\nabla}'_Y Z) + (\overline{\nabla}_X\widehat{H}')g(Y,Z)\\ &+ \widehat{H}'X(g(Y,Z)),\end{aligned} \tag{4.18}$$

$$\text{and} \quad \overline{\nabla}_{[X,Y]}Z = \widehat{\nabla}'_{[X,Y]}Z + \widehat{H}'g([X,Y],Z), \tag{4.19}$$

for any X, Y, $Z \in \Gamma(T\widehat{M}')$. Form (4.18) and (4.19), we have

$$\overline{R}(X,Y)Z = \widehat{R}'(X,Y)Z + (\overline{\nabla}_X\widehat{H}')g(Y,Z) - (\overline{\nabla}_Y\widehat{H}')g(X,Z). \tag{4.20}$$

Since $\widehat{P}X = X$ and $\eta(X) = 0$, $\forall\, X \in \Gamma(T\widehat{M}')$, the relation (4.20) reduces,

$$\overline{R}(X,Y)Z = \widehat{R}'(X,Y)Z - (1 + 2\varphi\lambda^2)\{g(Y,Z)X - g(X,Z)Y\}, \tag{4.21}$$

$$\text{and} \ \ X(\lambda) + \lambda\tau(X) = 0 \ \ \text{and} \ \ X(\varphi) - 2\varphi\tau(X) = 0. \tag{4.22}$$

On the other hand, we have

$$\overline{R}(X,Y)Z = R(X,Y)Z + \varphi\lambda^2\{g(X,Z)Y - g(Y,Z)X\}. \tag{4.23}$$

Putting (4.21) and (4.23) together, we obtain

$$R(X,Y)Z = \widehat{R}'(X,Y)Z - (1 + \varphi\lambda^2)\{g(Y,Z)X - g(X,Z)Y\}. \tag{4.24}$$

Also, using (3.41), the curvature R is expressed along the leaf $\widehat{M}'$ as

$$R(X,Y)Z = -(1 - \varphi\lambda^2)\{g(Y,Z)X - g(X,Z)Y\}. \tag{4.25}$$

Using this and (4.24), the curvature tensor $\widehat{R}'$ of $\widehat{M}'$ is given by

$$\widehat{R}'(X,Y)Z = 2\varphi\lambda^2\{g(Y,Z)X - g(X,Z)Y\}. \tag{4.26}$$

Therefore, $\widehat{M}'$ is a semi-Riemannian manifold of constant curvature $2\varphi\lambda^2$.

Theorem 4.5. *Let $(M, g, S(TM))$ be a SAC-lightlike hypersurface of an indefinite Kenmotsu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. Let $\widehat{M}'$ be a leaf of $\widehat{D}$, immersed in $\overline{M}$ as non-degenerate submanifold. Then the following assertions hold*

(i) *$\widehat{M}'$ is a space form of constant curvature $2\varphi\lambda^2$,*
(ii) *$\widehat{M}'$ is Einstein,*
(iii) *$\widehat{M}'$ is locally symmetric, and*
(iv) *$\widehat{M}'$ is Ricci semi-symmetric*

Proof. Using (4.26), the Ricci tensor $\widehat{Ric}'$ of the leaf $\widehat{M}'$ is given by

$$\widehat{Ric}'(X,Y) = 2(2n-3)\varphi\lambda^2 g(X,Y), \tag{4.27}$$

for any W, X, Y, $Z \in \Gamma(T\widehat{M}')$. The covariant derivative of $\widehat{R}'$ is

$$\begin{aligned}(\widehat{\nabla}'_W \widehat{R}')(X,Y)Z &= 2W(\varphi\lambda^2)\{g(Y,Z)X - g(X,Z)Y\} + 2\varphi\lambda^2\{W(g(Y,Z))X \\ &+ g(Y,Z)\widehat{\nabla}'_W X - W(g(X,Z))Y - g(X,Z)\widehat{\nabla}'_W Y\} - 2\varphi\lambda^2\{g(Y,Z)\widehat{\nabla}'_W X \\ &- g(\widehat{\nabla}'_W X, Z)Y\} - 2\varphi\lambda^2\{g(\widehat{\nabla}'_W Y, Z)X - g(X,Z)\widehat{\nabla}'_W Y\} \\ &- 2\varphi\lambda^2\{g(Y,\widehat{\nabla}'_W Z)X - g(X,\widehat{\nabla}'_W Z)Y\} = 2W(\varphi\lambda^2)\{g(Y,Z)X - g(X,Z)Y\}.\end{aligned}$$

Using (4.22), $W(\varphi\lambda^2) = 2\varphi\lambda^2\tau(W) - 2\varphi\lambda^2\tau(W) = 0$ and, for any W, X, Y, $Z \in \Gamma(T\widehat{M}')$, $(\widehat{\nabla}'_W \widehat{R}')(X,Y)Z = 0$, that is, the leaf $\widehat{M}'$ is locally symmetric. Now we want to show that

$$(\widehat{R}'(W_1,W_2)\cdot \widehat{Ric}')(X,Y) = 0, \quad \forall\, W_1,\ W_2,\ X,\ Y \in \Gamma(TM). \tag{4.28}$$

From (4.27), one obtains

$$\begin{aligned}\widehat{Ric}'(\widehat{R}'(W_1,W_2)X,Y) &= 4(2n-3)(\varphi\lambda^2)^2\{g(W_1,Y)g(W_2,X) \\ &- g(W_2,Y)g(W_1,X)\},\end{aligned} \tag{4.29}$$

and

$$\begin{aligned}\widehat{Ric}'(X,\widehat{R}'(W_1,W_2)Y) &= 4(2n-3)(\varphi\lambda^2)^2\{g(W_1,X)g(W_2,Y) \\ &- g(W_2,X)g(W_1,Y)\}.\end{aligned} \tag{4.30}$$

Putting the pieces (4.29) and (4.30) together into (4.28), one obtains that

$$(\widehat{R}'(W_1,W_2)\cdot \widehat{Ric}')(X,Y) = 0, \ \ \forall\, W_1, W_2, X, Y \in \Gamma(T\widehat{M}'), \tag{4.31}$$

that is, the leaf $\widehat{M}'$ is Ricci semi-symmetric, which completes the proof. □

Using (4.17) and (4.26). it is easy to see that the leaves of the integrable distribution $\widehat{D}$ are semi-parallel.

Let $\widehat{\nabla}'^{\perp}$ be a linear connection on $N(TM)\oplus TM^{\perp}$ along $\widehat{M}'$ defined by $\widehat{\nabla}'^{\perp}_X E = \nabla^{*\perp}_X E = -\tau(X)E$ and $\widehat{\nabla}'^{\perp}_X N = \nabla^{\perp}_X N = \tau(X)N$, for any $X \in \Gamma(TM')$. Using the relations (4.22), the covariant derivative of the mean curvature vector $\widehat{H}'$ of the leaf $\widehat{M}'$ satisfies $\overline{g}(\widehat{\nabla}'^{\perp}_X \widehat{H}', E) = 0$ and $\overline{g}(\widehat{\nabla}'^{\perp}_X \widehat{H}', N) = 0$. This means that the mean curvature vector $\widehat{H}'$ of the leaf $\widehat{M}'$ is parallel. Therefore,

Lemma 4.3. *Let $(M, g, S(TM))$ be a SAC-lightlike hypersurface of an indefinite Kenmotsu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. Then, all the integral manifolds of $\widehat{D}$ are semi-parallel and extrinsic spheres.*

Next we deal with the geometry of the distribution $D \perp \langle\xi\rangle$ in (3.4). As is known the screen distribution $S(TM)$ of a SAC-lightlike hypersurface M is integrable. Let Φ be the fundamental 2-form on M, locally defined by

$$\Phi(X,Y) = \overline{g}(X, \overline{\phi}Y).$$

Note that the differential 1-form u in (3.5) is related to the fundamental Φ as

$$u(X) = -\Phi(X,E),\ \forall\, X \in \Gamma(TM).$$

Suppose that the distribution $D \perp \langle\xi\rangle$ is integrable. Let M^* be a leaf of $D \perp \langle\xi\rangle$. Using the decomposition (3.3) and for any $X \in \Gamma(TM)$, $Y \in \Gamma(D \perp \langle\xi\rangle)$, we have

$$\overline{\nabla}_X Y = \nabla_X^{D\perp\langle\xi\rangle} Y + h^{D\perp\langle\xi\rangle}(X, Y), \tag{4.32}$$

where $\nabla^{D\perp\langle\xi\rangle}$ is a linear connection on $D \perp \langle\xi\rangle$ and $h^{D\perp\langle\xi\rangle} : \Gamma(TM) \times \Gamma(D \perp \langle\xi\rangle) \longrightarrow D' \oplus N(TM)$ is $\mathcal{F}(M)$-bilinear. Let $\mathcal{U} \subset M$ be a coordinate neighborhood as fixed in Theorem 2.1. By (3.3), for any $X \in \Gamma(TM)$, $Y \in \Gamma(D \perp \langle\xi\rangle)$, we have

$$\begin{aligned} \overline{\nabla}_X Y &= \nabla_X^{D\perp\langle\xi\rangle} Y + \overline{g}(\overline{\nabla}_X Y, E)N + \overline{g}(\overline{\nabla}_X Y, V)U \\ &= \nabla_X^{D\perp\langle\xi\rangle *} Y + h^{D\perp\langle\xi\rangle *}(X, Y), \end{aligned} \tag{4.33}$$

where $h^{D\perp\langle\xi\rangle *}(X, Y) = B(X, Y)N + B(X, \phi Y)U$ is the second fundamental form of the leaf M^*.

Theorem 4.6. *Let $(M, g, S(TM))$ be a proper totally contact umbilical SAC-lightlike hypersurface of an indefinite Kenmotsu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. If the fundamental 2-form Φ vanishes on $D \perp \langle\xi\rangle$, then the following statements hold:*

- (i) *the distribution $D \perp \langle\xi\rangle$ is integrable;*
- (ii) *the distribution $D \perp \langle\xi\rangle$ is auto-parallel with respect to the induced connection ∇;*
- (iii) *M is locally a product $M^* \times C$, where M^* is a proper totally contact leaf of $D \perp \langle\xi\rangle$ and C is a lightlike curve tangent to the distribution $\overline{\phi}(N(TM))$.*

Proof. Using (2.2) and the fact that $h(X, \overline{\phi}Y) = \rho g(X, \overline{\phi}Y)N$ with $\rho \neq 0$, we have, for any X, $Y \in \Gamma(D \perp \langle\xi\rangle)$,

$$2\Phi(X, Y) = \frac{1}{\rho}\{\overline{g}(h(X, \overline{\phi}Y), E) - \overline{g}(h(\overline{\phi}X, Y), E)\} = \frac{1}{\rho}u([X, Y]). \tag{4.34}$$

If Φ vanishes on $D \perp \langle\xi\rangle$, then $u([X, Y]) = 0$, for any X, $Y \in \Gamma(D \perp \langle\xi\rangle)$, that is, the distribution $D \perp \langle\xi\rangle$ is integrable (i). To prove (ii), we need to check $g(\nabla_X E, V) = 0$, $g(\nabla_X V, V) = 0$, $g(\nabla_X Y_0, V) = 0$ and $g(\nabla_X \xi, V) = 0$, for any $X \in \Gamma(D \perp \langle\xi\rangle)$ and $Y_0 \in \Gamma(D_0)$. Hence, using (3.36), we obtain $g(\nabla_X E, V) = \rho\Phi(X, E) = 0$, $g(\nabla_X V, V) = 0$, $g(\nabla_X Y_0, V) = \rho\Phi(X, Y_0) = 0$, $g(\nabla_X \xi, V) = \Phi(X, E) = 0$. Finally, from (i) we deduce that $D \perp \langle\xi\rangle$ determines a foliation. $D' := \overline{\phi}(N(TM))$ being a 1-dimensional distribution, it defines a foliation. Let M^* be a leaf of $D \perp \langle\xi\rangle$. Then, by (3.36) and for any X, $Y \in \Gamma(TM^*)$, the second fundamental form $h^{D\perp\langle\xi\rangle *}$ in (4.33) of M^* reduces

$$h^{D\perp\langle\xi\rangle *}(X, Y) = \rho\{g(X, Y) - \eta(X)\eta(Y)\}N + \rho\Phi(X, Y)U.$$

If Φ vanishes on $D \perp \langle\xi\rangle$, $h^{D\perp\langle\xi\rangle *}$ becomes $h^{D\perp\langle\xi\rangle *}(X, Y) = \rho\{g(X, Y) - \eta(X)\eta(Y)\}N$. The leaf M^* of $D \perp \langle\xi\rangle$ is totally contact umbilical. So being $TM = (D \perp \langle\xi\rangle) \oplus D'$, we obtain (iii). □

5. Relative nullity foliations of screen almost conformal lightlike hypersurfaces

This section is devoted to investigate relative nullity foliations in screen almost conformal lightlike hypersurfaces M of Kenmotsu space form $\overline{M}(c)$, tangent to the structure vector field ξ (see [22], for more details). We show that, under a certain condition, their leaves are totally geodesic.

The relative nullity space at a point x is defined by

$$T^{*0}(x) = \{X \in T_xM : A^*_E X = 0, \ \forall\, E \in T_xM^\perp\}. \tag{5.1}$$

The dimension $\nu(x)$ of $T^{*0}(x)$ is called the index of relative nullity at x. The value $\nu_0 = \min_{x\in M} \nu(x)$ is called the index of minimum relative nullity [5].

Writing A^*_E as, for any $X \in \Gamma(TM)$,

$$A^*_E X = \sum_{i=1}^{2n-4} \frac{B(X, F_i)}{g(F_i, F_i)} F_i + B(X, V)U + B(X, U)V, \tag{5.2}$$

with $g(F_i, F_i) \neq 0$ and using $B(., \xi) = 0$, it is easy to check that $A^*_E\xi = A^*_E E = 0$. Therefore, $\dim T^{*0}(x) \geq 2, \ \forall\, x \in M$. Moreover

$$T_xM^\perp \perp \langle\xi\rangle_x \subset T^{*0}(x). \tag{5.3}$$

Hence, $T^{*0}(x)$ is a degenerate distribution along M and $\nu_0 = 2$.

The orthogonal complement $(T^{*0}(x))^\perp$ of $T^{*0}(x)$ in T_xM is denoted by $T^{*1}(x)$.

Proposition 5.1. *Let M be a lightlike hypersurface of indefinite Kenmotsu space form $\overline{M}(c)$ with $\xi \in TM$. The orthogonal complement $T^{*1}(x)$ of $T^{*0}(x)$ in T_xM is given by*

$$T^{*1}(x) = \mathrm{span}\{A^*_E Y, \ \ Y \in T_xM, \ E \in T_xM^\perp\} \perp T_xM^\perp.$$

Proof. It is obvious to check that $T_xM^\perp \subset T^{*1}(x)$. Then, there exists a set $\Delta(x)$ such that

$$T^{*1}(x) = \Delta(x) \perp T_xM^\perp.$$

Now we want to show that $\Delta(x) = \mathrm{span}\{A^*_E Y\}$. Given any $E \in T_xM^\perp$, $Y \in T_xM$ and $X \in T^{*0}(x)$, $g(X, A^*_E Y) = g(A^*_E X, Y) = 0$, so, $A^*_E Y \in \Delta(x)$. On the other hand, let $Z \in \mathrm{span}\{A^*_E Y\}^{\perp_S}$ and $Y \in T_xM$, where $\perp_S$ denotes the orthogonality symbol in the screen distribution $S(TM)$. We have $0 = g(Z, A^*_E Y) = g(A^*_E Z, Y), \forall\, Y \in T_xM$. Then, $A^*_E Z \in S(TM) \cap T_xM^\perp = \{0\}$, that is, $A^*_E Z = 0$ and $Z \in T^{*0}(x)$.

Thus $\mathrm{span}\{A^*_E Y\}^{\perp_S} \subset T^{*0}(x)$ and $T^{*1}(x) \subset \mathrm{span}\{A^*_E Y\}$. Since $A^*_E Y \notin T_xM^\perp$, then $\Delta(x) \subset \mathrm{span}\{A^*_E Y\}$ which completes the proof. □

Let G be the set of points in M where $\nu(x) = \nu_0$. By Theorem 4.4 in [22], G is an open set in M. We now show that the relative nullity space $T^{*0}(x)$ is a smooth distribution. Let x_0 be an element of G. From (5.3), we have

$$T^{*0}(x_0) = P(T^{*0}(x_0)) \perp T_{x_0}M^\perp \perp \langle\xi\rangle_{x_0}. \tag{5.4}$$

Let $\perp_S$ denotes the orthogonality symbol in the screen distribution $S(TM)$. For $Y \in T_{x_0}M$, $E \in T_{x_0}M^\perp$ and $X \in P(T^{*0}(x_0))$, we have, $g(A^*_E Y, X) = g(Y, A^*_E X) = 0$, so we obtain,

$$\mathrm{span}\{A^*_E Y\} \subset P(T^{*0}(x_0))^{\perp_S}.$$

Let $Z \in \mathrm{span}\{A^*_E Y\}^{\perp_S}$ and $Y \in T_{x_0}M$. We have $0 = g(Z, A^*_E Y) = g(A^*_E Z, Y)$, $\forall\, Y \in T_xM$. Then $A^*_E Z \in S(TM) \cap T_{x_0}M^\perp = \{0\}$, that is, $A^*_E Z = 0$ and $Z \in P(T^{*0}(x_0))$. Thus, $\mathrm{span}\{A^*_E Y\}^{\perp_S} \subset P(T^{*0}(x_0))$ and $P(T^{*0}(x_0))^{\perp_S} \subset \mathrm{span}\{A^*_E Y\}$. Thus, $P(T^{*0}(x_0))^{\perp_S} = \mathrm{span}\{A^*_E Y\}$ and $T^{*1}(x_0) = \mathrm{span}\{A^*_E Y\} \perp T_{x_0}M^\perp$. There exist vector fields $Y_1, ..., Y_{2n-\nu+1} \in T_{x_0}M$ such that

$$\{E(x_0), A^*_{E(x_0)} Y_1, ..., A^*_{E(x_0)} Y_{2n-\nu+1}\},$$

represents a basis of $T^{*1}(x)$.

Take smooth local extensions of $E(x_0)$ and $Y_1, ..., Y_{2n-\nu+1} \in T_{x_0}M$ in $TM^\perp$ and TM respectively. By continuity, the vector fields $\{E(x_0), Y_1, ..., Y_{2n-\nu+1}\}$ remain linearly independent in a neighborhood $\mathcal{V} \subset G$ of x_0 and then T^{*1} is a smooth distribution. Consequently, T^{*0} is smooth distribution.

Suppose that M is a proper totally contact umbilical SAC-lightlike hypersurface of indefinite Kenmotsu space form $\overline{M}(c)$ with $\xi \in TM$. Let x be an element of G. If $X \in T^{*0}(x)$, then $A^*_E X = 0$. Using the fact that $X = PX + \theta(X)E$ and $A^*_E E = 0$, we get $A^*_E PX = 0$, which implies that

$$B(PX, PY) = 0, \forall\, Y \in T_xM. \tag{5.5}$$

Since $B \neq 0$ on M and ξ is the only vector field in $S(TM)$ such that $B(\xi, \cdot) = 0$, the relation (5.5) implies that PX is proportional to ξ, that is $PX = \eta(X)\xi$. Thus, the vector field is now

$$X = \eta(X)\xi + \theta(X)E. \tag{5.6}$$

That is, $X \in T_xM^\perp \perp \langle\xi\rangle_x$ and $P(T^{*0}(x)) = \{0\}$. Therefore

$$T_xM^\perp \perp \langle\xi\rangle_x \subset T^{*0}(x). \tag{5.7}$$

We have the following result.

Theorem 5.1. *Let $(M, g, S(TM))$ be a proper totally contact umbilical SAC-lightlike hypersurface of an indefinite Kenmotsu space form $(\overline{M}(c), \overline{g})$ with $\xi \in TM$. Then, on G*

$$T^{*0} = TM^\perp \perp \langle\xi\rangle. \tag{5.8}$$

Moreover, the relative nullity distribution T^{*0} is integrable and the leaves are totally geodesic in M and $\overline{M}$.

Proof. From (5.3) and (5.7), we obtain the relation (5.8). From Gauss and Codazzi equations, we have, for any $E \in \Gamma(TM^\perp)$ and X, Y, $Z \in \Gamma(TM)$,

$$\overline{g}(\overline{R}(X,Y)Z, E) = \overline{g}((\nabla_X h)(Y,Z) - (\nabla_Y h)(X,Z), E). \tag{5.9}$$

Take $X \in \Gamma(TM)$ and Y, $Z \in T^{*0}(x)$, $x \in G$. Since $(\nabla_X h)(Y,Z) = \nabla^\perp_X h(Y,Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z)$, then

$$\begin{aligned}\overline{g}((\nabla_X h)(Y,Z) - (\nabla_Y h)(X,Z), E) &= X(B(Y,Z)) - Y(B(X,Z)) \\ - \tau(X)B(Y,Z) + \tau(Y)B(X,Z) &- B(\nabla_X Y, Z) - B(Y, \nabla_X Z) \\ + B(\nabla_Y X, Z) + B(X, \nabla_Y Z).\end{aligned} \tag{5.10}$$

Using (3.9) the left hand side of (5.9) vanishes and the relation (5.10) becomes

$$\begin{aligned}0 &= X(B(Y,Z)) - Y(B(X,Z)) - \tau(X)B(Y,Z) + \tau(Y)B(X,Z) \\ &- B(\nabla_X Y, Z) - B(Y, \nabla_X Z) + B(\nabla_Y X, Z) + B(X, \nabla_Y Z).\end{aligned} \tag{5.11}$$

From (5.6), $Z \in T^{*0}(x)$ implies that Z takes the form $Z = \eta(Z)\xi + \theta(Z)E$ and $B(Y,Z) = \eta(Z)B(\xi, PY) + \theta(Z)B(E, PY) = 0$. Similarly, $B(X,Z) = 0$.

On the other hand, since $B(X, A^*_E Y) = B(A^*_E X, Y)$, we have

$$B(\nabla_X Y, Z) = B(\nabla^*_X PY, Z) + \theta(Y)B(X, A^*_E Z) = 0, \tag{5.12}$$

for $Z \in T^{*0}(x)$. Also $B(\nabla_Y X, Z) = 0$.

The relation (5.11) becomes $B(X, \nabla_Y Z) - B(Y, \nabla_X Z) = 0$. But

$$B(Y, \nabla_X Z) = B(Y, \nabla^*_X PZ) - \theta(X)B(A^*_E Y, X) = 0.$$

Consequently $h(\nabla_Y Z, PX) = 0$, $\forall\, X \in \Gamma(TM)$. Since M is not parallel, we deduce that $\nabla_Y Z \in T_x M^\perp \perp \langle\xi\rangle_x = T^{*0}(x)$, i.e., $\nabla_Y X \in T^{*0}(x)$. This implies that $T^{*0}(x)$ is involutive with totally geodesic leaves in both M and $\overline{M}$. □

Acknowledgments

The author expresses his deepest gratitude for support and hospitality of Max Planck Institute for Mathematics in the Sciences (Leipzig, Germany), in summer of 2011. He also thanks the referees for their valuable comments and suggestions.

References

[1] Bejancu, A., Umbilical Semi-invariant submanifolds of a Sasakian manifold, Tensor N. S. 37 (1982), 203-213.

[2] Blair, D. E., Riemannian geometry of contact and symplectic manifolds, Progress in Mathematics 203. Birkhauser Boston, Inc., Boston, MA 2002.

[3] Duggal, K. L. and Bejancu, A., Lightlike submanifolds of semi-Riemannian manifolds and applications, Mathematics and Its Applications, Kluwer Academic Publishers, Dordrecht, 1996.

[4] Bonome, A., Castro, R., García-Rio, E. and Hervella, L., Curvature of indefinite almost contact manifolds, J. geom., 58 (1997), 66-86.

[5] Dajczer, M. et al., Submanifolds and isometric immersions. Mathematics lecture series 13. Publish or Perish, Inc., Houston, Texas, 1990.

[6] Defever, F., Ricci semi-symmetric hypersurfaces, Balkan J. of Geometry and Its Appl., 5 (1) (2000), 81-91.

[7] Duggal, K. L., On scalar curvature in lightlike geometry, J. Geom. Phys. 57 (2)(2007), 473-481.

[8] Duggal, K. L. and Jin, D. H., Null curves and hypersurfaces of semi-Riemannian manifolds, World Scientific Publishing Co. Pte. Ltd, 2007.

[9] Duggal, K. L. and Sahin, B., Differential Geometry of Lightlike Submanifolds, Frontiers in Mathematics, 2010.

[10] Günes, R., Sahin, B., and Kiliç, E., On lightlike hypersurfaces of semi-Riemannian space form, Turkish J. Math. 27 (2003), 283-297.

[11] Janssens, D. and Vanhecke, L., Almost contact structures and curvature tensors, Kodai Math. J., 4 (1981), 1-27.

[12] Jin, D. H., Screen conformal lightlike hypersurfaces of a semi-Riemannian space form, J. Korea Soc. Math. Educ. Ser. B: Pure Appl. Math. 16 no. 3 (2009), 271-276.

[13] Jun, J.-B., De, U. C. and Pathak, G., On Kenmotsu manifolds, J. Korean Math. Soc., 42, 3 (2005), 435-445.

[14] Kenmotsu, K., A class of almost contact Riemannian manifolds, Tohoku Math. J., 24 (1972), 93-103.

[15] Kupeli, D. N., Singular semi-invariant geometry, vol. 366 of Mathematics and Applications, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1996.

[16] Lungiambudila, O., Massamba, F. and Tossa, J., Symmetry properties of lightlike hypersurface in indefinite Sasakian manifolds, SUT J. Math. 46 (2)(2010), 177-204.

[17] Massamba, F., Lightlike hypersurfaces of indefinite Sasakian manifolds with parallel symmetric bilinear forms, Differ. Geom. Dyn. Syst., 10 (2008), 226-234.

[18] Massamba, F., Totally contact umbilical lightlike hypersurfaces of indefinite Sasakian manifolds, Kodai Math. J., 31 (2008), 338-358.

[19] Massamba, F., Killing and geodesic lightlike hypersurfaces of indefinite Sasakian manifolds, Turk J. Math., 32 (2008), 325-347.

[20] Massamba, F., Screen integrable lightlike hypersurfaces of indefinite Sasakian manifolds, Mediterr. J. Math, 6 (2009), 27-46.

[21] Massamba, F., On semi-parallel lightlike hypersurfaces of indefinite Kenmotsu manifolds, J. Geom., 95 (2009), 73-89.

[22] Massamba, F., Relative nullity foliations and lightlike hypersurfaces in indefinite Kenmotsu manifolds, Turk. J. Math., 35 (2011), 129-149

[23] Massamba, F., On lightlike geometry in indefinite Kenmotsu manifolds, Math. Slovaca 62 (2)(2012), 315-344.

[24] Massamba, F., Lightlike geometry of leaves in indefinite Kenmotsu manifolds, Cubo 13 (3) (2011), 69-89.

[25] O'Neill, B., Semi-Riemannian Geometry with Applications to Relativity, Vol. 103 of Pure and Applied Mathematics, Academic Press, New York, NY, USA, 1983.

[26] Sahin, B., Lightlike hypersurfaces of semi-Euclidean spaces satisfying curvature conditions of semi-symmetry, Turkish J. Math. 31 (2007) 139-162.

Department of Mathematics, University of Botswana, Private Bag 0022, Gaborone, Botswana

E-mail address: massfort@yahoo.fr, massambaf@mopipi.ub.bw