Commun.Fac.Sci.Univ.Ank.Series A1
Volume 62, Number 1, Pages 45–59 (2013)
ISSN 1303–5991

SURVEY ON THE DOMAIN OF THE MATRIX LAMBDA IN THE NORMED AND PARANORMED SEQUENCE SPACES*

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Abstract. In the present paper, we summarize the literature on the normed and paranormed sequence spaces derived by the domain of the matrix lambda. Moreover, we establish some inclusion relations concerning with those spaces and determine their alpha-, beta- and gamma-duals. Finally, we record some open problems and further suggestions related with Λ summability.

1. Introduction

By ω, we denote the space of all real valued sequences. Any vector subspace of ω is called a *sequence space*. We shall write ℓ_∞, c and c_0 for the spaces of all bounded, convergent and null sequences, respectively. Also by bs, cs, ℓ_1 and ℓ_p; we denote the spaces of all bounded, convergent, absolutely convergent and $p-$absolutely convergent series, respectively; where $1 < p < \infty$. A sequence space X is called an $FK-$space if it is a complete linear metric space with continuous coordinates $p_n : X \longrightarrow \mathbb{C}$ with $p_n(x) = x_n$ for all $x = (x_k) \in X$ and every $n \in \mathbb{N}$, where $\mathbb{C}$ denotes the complex field and $\mathbb{N} = \{0, 1, 2, \ldots\}$. A normed $FK-$spaces is called a $BK-$space, that is, a $BK-$space is a Banach space with continuous coordinates. The sequence spaces ℓ_∞, c and c_0 are $BK-$spaces with the usual sup-norm defined by $\|x\|_\infty = \sup_{k\in\mathbb{N}} |x_k|$.

If a normed sequence space X contains a sequence (b_n) with the property that for every $x \in X$ there is a unique sequence of scalars (α_n) such that

$$\lim_{n\to\infty} \|x - (\alpha_0 b_0 + \alpha_1 b_1 + \cdots + \alpha_n b_n)\| = 0$$

Received by the editors Nov. 11, 2012; Accepted: May 20, 2013.
2010 *Mathematics Subject Classification.* Primary 46A45; Secondary 40C05.
Key words and phrases. Normed and paranormed sequence spaces, matrix domain, triangle matrices, lambda matrix, almost convergence, alpha-, beta- and gamma-duals, matrix transformations.
The main results of this paper were presented in part at the conference Algerian-Turkish International Days on Mathematics 2012 (ATIM' 2012) to be held October 9–11, 2012 in Annaba, Algeria at the Badji Mokhtar Annaba University.

then (b_n) is called a *Schauder basis* (or briefly *basis*) for X. The series $\sum_{k=0}^{\infty} \alpha_k b_k$ which has the sum x is then called the expansion of x with respect to (b_n), and is written as $x = \sum_{k=0}^{\infty} \alpha_k b_k$.

Let X, Y be any two sequence spaces and $A = (a_{nk})$ be an infinite matrix of complex numbers a_{nk}, where $k, n \in \mathbb{N}$. Then, we say that A defines a matrix mapping from X into Y, and we denote it by writing $A : X \longrightarrow Y$, if for every sequence $x = (x_k) \in X$ the sequence $Ax = \{(Ax)_n\}$, the $A-$transform of x, is in Y; where

$$(Ax)_n := \sum_{k=0}^{\infty} a_{nk} x_k \quad \text{for each} \quad n \in \mathbb{N}. \tag{1.1}$$

By $(X : Y)$, we denote the class of all matrices A such that $A : X \to Y$. Thus, $A \in (X : Y)$ if and only if the series on the right side of (1.1) converges for each $n \in \mathbb{N}$ and every $x \in X$, and we have $Ax = \{(Ax)_n\}_{n \in \mathbb{N}} \in Y$ for all $x \in X$. A sequence x is said to be $A-$summable to l if Ax converges to l which is called as the $A-$limit of x.

The shift operator P is defined on ω by $P(x_n) = x_{n+1}$ for all $n \in \mathbb{N}$. A Banach limit L is defined on ℓ_∞, as a non-negative linear functional, such that $L(Px) = L(x)$ and $L(e) = 1$, where $e = (1, 1, 1, \ldots)$. A sequence $x = (x_k) \in \ell_\infty$ is said to be almost convergent to the generalized limit l if all Banach limits of x are l, and is denoted by $f - \lim x_k = l$. Lorentz [25] proved that

$$f - \lim \ x_k = l \quad \text{if and only if} \quad \lim_{m \to \infty} \sum_{k=0}^{m} \frac{x_{n+k}}{m+1} = l \quad \text{uniformly in} \quad n.$$

It is well-known that a convergent sequence is almost convergent such that its ordinary and generalized limits are equal. By f_0 and f, we denote the space of all almost null and all almost convergent sequences, that is,

$$f_0 := \left\{ x = (x_k) \in \omega : \lim_{m \to \infty} \sum_{k=0}^{m} \frac{x_{n+k}}{m+1} = 0 \text{ uniformly in } n \right\},$$
$$f := \left\{ x = (x_k) \in \omega : \exists l \in \mathbb{C} \ni \ \lim_{m \to \infty} \sum_{k=0}^{m} \frac{x_{n+k}}{m+1} = l \text{ uniformly in } n \right\}.$$

Assume here and after that (p_k) is a bounded sequence of strictly positive real numbers with $\sup p_k = H$ and $M = \max\{1, H\}$. Then the linear spaces $\ell_\infty(p)$,

$c(p)$, $c_0(p)$ and $\ell(p)$ were defined by Maddox [27] (see also Simons [45]) as follows:

$$\ell_\infty(p) := \left\{ x = (x_k) \in \omega : \sup_{k \in \mathbb{N}} |x_k|^{p_k} < \infty \right\},$$

$$c(p) := \left\{ x = (x_k) \in \omega : \exists l \in \mathbb{C} \ni \lim_{k \to \infty} |x_k - l|^{p_k} = 0 \right\},$$

$$c_0(p) := \left\{ x = (x_k) \in \omega : \lim_{k \to \infty} |x_k|^{p_k} = 0 \right\},$$

$$\ell(p) := \left\{ x = (x_k) \in \omega : \sum_{k=0}^{\infty} |x_k|^{p_k} < \infty \right\}, \ (0 < p_k < \infty).$$

We shall assume throughout that $p_k^{-1} + (p_k')^{-1} = 1$ provided $0 < \inf p_k \le H < \infty$ and denote the collection of all finite subsets of $\mathbb{N}$ by $\mathcal{F}$.

Define the functions g_1 and g_2 on the spaces $\ell_\infty(p)$, $c(p)$ or $c_0(p)$ and $\ell(p)$ by

$$g_1(x) := \sup_{k \in \mathbb{N}} |x_k|^{p_k/M} \quad \text{and} \quad g_2(x) := \left(\sum_{k=0}^{\infty} |x_k|^{p_k} \right)^{1/M}.$$

Then, $c_0(p)$ and $c(p)$ are complete paranormed spaces paranormed by g_1 if $p \in \ell_\infty$; (cf. [28, Theorem 6]). It is known from [26] that the inclusion $c_0(p) \subset c_0(q)$ holds if and only if $\liminf q_k/p_k > 0$. $\ell_\infty(p)$ is also complete paranormed space by g_1 if and only if $\inf p_k > 0$. Also, $\ell(p)$ is complete paranormed space paranormed by g_2 and $\{e^{(k)}\}_{k \in \mathbb{N}}$ is a basis for the space $\ell(p)$, where $e^{(k)}$ denotes the sequences whose only non-zero entry is a 1 in k^{th} place for each $k \in \mathbb{N}$.

An infinite matrix $T = (t_{nk})$ is called a triangle if $t_{nn} \neq 0$ and $t_{nk} = 0$ for all $k > n$. The *domain* X_A of an infinite matrix A in a sequence space X is defined by

$$X_A := \left\{ x = (x_k) \in \omega : Ax \in X \right\} \tag{1.2}$$

which is also a sequence space. If A is triangle, then one can easily observe that the sequence spaces X_A and X are linearly isomorphic, i.e., $X_A \cong X$.

The idea constructing a new sequence space by means of the domain of a triangle matrix was employed by Wang [48], Ng and Lee [43], Malkowsky [29], Altay and Başar [1, 2, 3, 4, 5, 6, 7, 8], Malkowsky and Savaş [33], Başarır [17, 18], Başarır and Kayıkçı [19], Başarır and Öztürk [20], Kara and Başarır [21], Kara et al. [22], Aydın and Başar [9, 10, 11, 12, 13], Başar et al. [16], Şengönül and Başar [47], Altay [1], Polat and Başar [44] and, Malkowsky et al. [30]. Additionally, $c_0(u,p)$ and $c(u,p)$ are the spaces consisting of the sequences $x = (x_k)$ such that $ux = (u_k x_k)$ is in the spaces $c_0(p)$ and $c(p)$ for $u \in \mathcal{U}$, the set of sequences with non-zero entries, respectively, and studied by Başarır [17]. Finally, the new technique for deducing certain topological properties, for example $AB-$, $KB-$, $AD-$properties, solidity and monotonicity etc., and determining the $\beta-$ and $\gamma-$duals of the domain of a triangle matrix in a sequence space is given by Altay and Başar [7].

Although in most cases the new sequence space X_A generated by the triangle matrix A from a sequence space X is the expansion or the contraction of the original space X, it may be observed in some cases that those spaces overlap. Define the summation operator S and the backward difference operator Δ respectively defined by $(Sx)_n = \sum_{k=0}^{n} x_k$ and $(\Delta x)_n = x_n - x_{n-1}$, $(x_{-1} \equiv 0)$, for all $n \in \mathbb{N}$, where $x = (x_k) \in \omega$. Then, one can easily see that the inclusion $X_S \subset X$ strictly holds for $X \in \{\ell_\infty, c, c_0\}$. Further, one can deduce that the inclusion $X \subset X_\Delta$ also strictly holds for $X \in \{\ell_\infty, c, c_0, \ell_p\}$, where $0 < p < \infty$. However, if we define $X = c_0 \oplus span\{z\}$ with $z = \{(-1)^k\}$, i.e., $x \in X$ if and only if $x = s + \alpha z$ for some $s \in c_0$ and some $\alpha \in \mathbb{C}$, and consider the matrix A with the rows A_n defined by $A_n = (-1)^n e^{(n)}$ for all $n \in \mathbb{N}$, we have $Ae = z \in X$ but $Az = e \notin X$ which lead us to the consequences that $z \in X \setminus X_A$ and $e \in X_A \setminus X$. That is to say that the sequence spaces X_A and X are overlap but neither contains the other. This approach was employed by number of researchers.

Let $\lambda = (\lambda_k)_{k=0}^{\infty}$ be a strictly increasing sequence of positive reals tending to infinity, that is

$$0 < \lambda_0 < \lambda_1 < \lambda_2 < \cdots \quad \text{and} \quad \lim_{k\to\infty} \lambda_k = \infty$$

and $(\mu_n)_{n=0}^{\infty}$ be defined by $\mu_n = \sum_{k=0}^{n} \lambda_k$ for all $n \in \mathbb{N}$. Following Mursaleen and Noman [36], we define the matrix $\Lambda = (\lambda_{nk})$ of weighted mean relative to the sequence λ by

$$\lambda_{nk} := \begin{cases} \frac{\lambda_k - \lambda_{k-1}}{\lambda_n} & , \quad 0 \le k \le n \\ 0 & , \quad k > n \end{cases} \tag{1.3}$$

for all $k, n \in \mathbb{N}$. Introducing the concept of Λ-*strong convergence* several results on Λ-strong convergence of numerical sequences and Fourier series were given by Móricz [34].

In this study, following Başar [14], we summarize some knowledge in the existing literature on the normed and paranormed sequence spaces derived by the domain of the triangle matrix Λ, defined by (1.3), above. Additionally, we note some new developments concerning with the applications of Λ summability.

The rest of this paper is organized, as follows:

In section 2, we emphasize on the sequence spaces obtained by the domain of the matrix Λ in some normed spaces. We begin with the spaces of lambda-bounded, lambda-convergent, lambda-null and lambda-absolutely $p-$summable sequences which are the domain of the matrix Λ in the classical spaces ℓ_∞, c, c_0 and ℓ_p. Additionally, we present some results on the difference and generalized difference spaces of lambda-convergent and lambda-null sequences. Section 2 terminates by the lines about the spaces f_0^λ and f^λ of almost lambda-null and almost lambda-convergent sequences. Section 3 is devoted to the paranormed sequence spaces derived by the matrix Λ from some Maddox's spaces. In the final section of the paper; after summarizing the consequences related to the results in the existing literature, open problems and further suggestions are noted.

2. Domain of the Matrix Λ in the Normed Sequence Spaces

In this section, we shortly give the knowledge on the sequence spaces derived by the matrix Λ from some well-known normed sequence spaces. For the concerning literature about the domain μ_A of an infinite matrix A in a sequence space μ, the following table may be useful:

μ	A	μ_A	refer to:
$c_0,\ c,\ \ell_\infty$	Λ	$c_0^\lambda,\ c^\lambda,\ \ell_\infty^\lambda$	[36]
$c_0^\lambda,\ c^\lambda$	Δ	$c_0^\lambda(\Delta),\ c^\lambda(\Delta)$	[37]
$\ell_p,\ (0 < p \leq \infty)$	Λ	$\ell_p^\lambda,\ \ell_\infty^\lambda$	[40]
$\ell_1,\ \ell_p,\ (1 < p \leq \infty)$	Λ	$\ell_1^\lambda,\ \ell_p^\lambda$	[41]
$c_0^\lambda,\ c^\lambda$	$B(r,s)$	$c_0^\lambda(B),\ c^\lambda(B)$	[46]
$c_0^\lambda,\ c^\lambda,\ \ell_\infty^\lambda$	$\Lambda(u)$	$c_0^\lambda(u),\ c^\lambda(u),\ \ell_\infty^\lambda(u)$	[51]
$f_0,\ f$	Λ	$f_0^\lambda,\ f^\lambda$	[50]
$c_0(p),\ c(p),\ \ell_\infty(p)$	Λ	$c_0(\lambda,p),\ c(\lambda,p),\ \ell_\infty(\lambda,p)$	[23]

Table 1: The domains of Λ in certain sequence spaces.

2.1. **The Sequence Spaces ℓ_∞^λ, c^λ, c_0^λ and ℓ_p^λ.** In this subsection, we give some results about the spaces ℓ_∞^λ, c^λ, c_0^λ and ℓ_p^λ of lambda-bounded, lambda-convergent, lambda-null and lambda-absolutely $p-$summable sequences which are introduced by Mursaleen and Noman [36, 38]. In other words, we emphasize on the spaces ℓ_∞^λ, c^λ, c_0^λ and ℓ_p^λ of $\Lambda-$bounded, $\Lambda-$convergent, $\Lambda-$null and $\Lambda-$absolutely $p-$summable sequences, respectively, that is to say that

$$\ell_\infty^\lambda := \left\{ x = (x_k) \in \omega : \sup_{n\in\mathbb{N}} \left| \frac{1}{\lambda_n} \sum_{k=0}^{n} (\lambda_k - \lambda_{k-1})\, x_k \right| < \infty \right\},$$

$$c^\lambda := \left\{ x = (x_k) \in \omega : \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k=0}^{n} (\lambda_k - \lambda_{k-1})\, x_k \text{ exists} \right\},$$

$$c_0^\lambda := \left\{ x = (x_k) \in \omega : \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k=0}^{n} (\lambda_k - \lambda_{k-1})\, x_k = 0 \right\},$$

$$\ell_p^\lambda := \left\{ x = (x_k) \in \omega : \sum_{n=0}^{\infty} \left| \frac{1}{\lambda_n} \sum_{k=0}^{n} (\lambda_k - \lambda_{k-1})\, x_k \right|^p < \infty \right\},\ (1 \leq p < \infty).$$

It is trivial that the sequence spaces ℓ_∞^λ, c^λ, c_0^λ and ℓ_p^λ are the domain of the matrix Λ in the classical sequence spaces ℓ_∞, c, c_0 and ℓ_p, respectively. Thus, with the notation of (1.2) we can redefine the spaces ℓ_∞^λ, c^λ, c_0^λ and ℓ_p^λ by

$$\ell_\infty^\lambda = \{\ell_\infty\}_\Lambda,\ c^\lambda = c_\Lambda,\ c_0^\lambda = \{c_0\}_\Lambda \ \text{ and } \ \ell_p^\lambda = \{\ell_p\}_\Lambda.$$

Define the sequence $y = (y_k)$ by the $\Lambda-$transform of a sequence $x = (x_k)$, i.e.,

$$y_k := (\Lambda x)_k = \frac{1}{\lambda_k} \sum_{j=0}^{k} (\lambda_j - \lambda_{j-1}) x_j \quad \text{for all} \quad k \in \mathbb{N}. \tag{2.1}$$

Since the matrix Λ is triangle, one can easily observe that $x = (x_k) \in X^\lambda$ if and only if $y = (y_k) \in X$, where the sequences $x = (x_k)$ and $y = (y_k)$ are connected with the relation (2.1), and X denotes any of the classical sequence spaces ℓ_∞, c, c_0 and ℓ_p. Therefore, one can easily see that the linear operator $T : X^\lambda \longrightarrow X$, $Tx = y = \Lambda x$ which maps every sequence x in X^λ to the associated sequence y in X, is bijective and norm preserving, where $\|x\|_{X^\lambda} = \|\Lambda x\|_X$. This gives the fact that X^λ and X are norm isomorphic.

Define the sequence $S(x) = \{S_n(x)\}$ by

$$S_n(x) := \begin{cases} \frac{1}{\lambda_n} \sum_{k=1}^{n} \lambda_{k-1}(x_k - x_{k-1}) & , \quad n \geq 1, \\ 0 & , \quad n = 0. \end{cases}$$

Mursaleen and Noman [36, 40, 41] prove the following theorem concerning the inclusion relations between these spaces and the classical sequence spaces ℓ_∞, c and c_0:

Theorem 2.1. *The following relations hold:*

(i) [36, Lemma 2.3] *The inclusion $c^\lambda \subset c$ holds if and only if $S(x) \in c_0$.*
(ii) [36, Lemma 2.5] *The inclusion $\ell_\infty^\lambda \subset \ell_\infty$ holds if and only if $S(x) \in \ell_\infty$.*
(iii) [40, Theorem 4.3] *The inclusion $\ell_p^\lambda \subset \ell_q^\lambda$ strictly holds, if $0 < p < q < \infty$.*
(iv) [40, Theorem 4.4] *The inclusions $\ell_p^\lambda \subset c_0^\lambda \subset c^\lambda \subset \ell_\infty^\lambda$ strictly hold.*
(v) [40, Lemma 4.5] *The inclusion $\ell_p^\lambda \subset \ell_p$ holds if and only if $S(x) \in \ell_p$ for every $x \in \ell_p^\lambda$, where $0 < p \leq \infty$.*
(vi) [36, Theorem 4.6] *The inclusions $c_0 \subset c_0^\lambda$, $c \subset c^\lambda$ and $\ell_\infty \subset \ell_\infty^\lambda$ strictly hold if and only if $\liminf_{n\to\infty} \frac{\lambda_{n+1}}{\lambda_n} = 1$.*
(vii) [40, Theorem 4.7] *The inclusion $\ell_\infty \subset \ell_\infty^\lambda$ strictly holds if and only if $\liminf_{n\to\infty} \frac{\lambda_{n+1}}{\lambda_n} = 1$.*
(viii) [40, Corollary 4.8] *The equality $\ell_\infty^\lambda = \ell_\infty$ strictly holds if and only if $\liminf_{n\to\infty} \frac{\lambda_{n+1}}{\lambda_n} > 1$.*
(ix) [40, Lemma 4.9] *The spaces ℓ_p and ℓ_p^λ are overlap. Additionally, if $1/\lambda \notin \ell_p$ then neither of them includes the other one, where $0 < p < \infty$.*
(x) [40, Lemma 4.10] *If the inclusion $\ell_p \subset \ell_p^\lambda$ holds, then $1/\lambda \in \ell_p$, where $0 < p < \infty$.*

The alpha-, beta- and gamma-duals of the spaces ℓ_∞^λ, c^λ and c_0^λ are determined. Some matrix transformations on these spaces are also characterized.

Now, because of the transformation T defined from c_0^λ or ℓ_p^λ to c_0 or ℓ_p; $Tx = \Lambda x$ is an isomorphism; the inverse image of the basis $\{e^{(k)}\}_{k=0}^{\infty}$ of the space c_0 and ℓ_p is the basis for the new spaces c_0^λ and ℓ_p^λ. Therefore, we have:

Theorem 2.2. *Define the sequence* $e_\lambda^{(n)} := \left\{\left(e_\lambda^{(n)}\right)_k\right\}_{k\in\mathbb{N}}$ *of the elements of the space* ℓ_p^λ *by*

$$\left(e_\lambda^{(n)}\right)_k := \begin{cases} (-1)^{k-n}\frac{\lambda_n}{\lambda_k-\lambda_{k-1}} & , \quad n \le k \le n+1, \\ 0 & , \quad \text{otherwise}, \end{cases}$$

for every fixed $n \in \mathbb{N}$. *Then, the following statements hold:*

(i) [36, Part (a) of Corollary 3.4], [40, Theorem 5.1] *The sequence* $\left\{e_\lambda^{(0)}, e_\lambda^{(1)}, e_\lambda^{(2)}, \ldots\right\}$ *is a Schauder basis for the spaces* c_0^λ *and* ℓ_p^λ, *and every* $x \in c_0^\lambda$ *or* $\in \ell_p^\lambda$ *has a unique representation of the form* $x := \sum_{n=0}^{\infty}(\Lambda x)_n e_\lambda^{(n)}$.

(ii) [36, Part (b) of Corollary 3.4] *The sequence* $\left\{e, e_\lambda^{(0)}, e_\lambda^{(1)}, e_\lambda^{(2)}, \ldots\right\}$ *is a Schauder basis for the space* c^λ *and every* $x \in c^\lambda$ *has a unique representation of the form* $x := le + \sum_{n=0}^{\infty}[(\Lambda x)_n - l]e_\lambda^{(n)}$, *where* $l = \lim_{n\to\infty}(\Lambda x)_n$.

2.2. **Difference Spaces of Lambda-null and Lambda-convergent Sequences.** In this subsection, we give some results about difference spaces of lambda-null and lambda-convergent sequences. Following Mursaleen and Noman [42], define the matrix $\overline{\Lambda} = (\overline{\lambda}_{nk})$ by

$$\overline{\lambda}_{nk} := \begin{cases} \frac{2\lambda_k - \lambda_{k-1} - \lambda_{k+1}}{\lambda_n} & , \quad k < n \\ \frac{\lambda_n - \lambda_{n-1}}{\lambda_n} & , \quad k = n \\ 0 & , \quad k > n \end{cases}$$

for all $k, n \in \mathbb{N}$. Then, Mursaleen and Noman [36, 37, 42] define the difference sequence spaces $c_0^\lambda(\Delta)$, $c^\lambda(\Delta)$ and $\ell_\infty^\lambda(\Delta)$ as the matrix domain of the triangle matrix $\overline{\Lambda}$ in the spaces c_0, c and ℓ_∞, respectively. They prove some estimates for the operator norms and the Hausdorff measure of noncompactness of certain matrix operators on the spaces $c_0^\lambda(\Delta)$ and $\ell_\infty^\lambda(\Delta)$. Moreover, necessary and sufficient conditions for such operators to be compact are derived in this paper. Recently, Mursaleen and Noman introduced the difference sequence spaces $c_0^\lambda(\Delta)$, $c^\lambda(\Delta)$ and

$\ell_\infty^\lambda(\Delta)$ in [37] of non-absolute type as follows:

$$c_0^\lambda(\Delta) := \left\{ x = (x_k) \in \omega : \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k=0}^{n} (\lambda_k - \lambda_{k-1})(x_k - x_{k-1}) = 0 \right\},$$

$$c^\lambda(\Delta) := \left\{ x = (x_k) \in \omega : \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k=0}^{n} (\lambda_k - \lambda_{k-1})(x_k - x_{k-1}) \ \text{ exists} \right\},$$

$$\ell_\infty^\lambda(\Delta) := \left\{ x = (x_k) \in \omega : \sup_{n\in\mathbb{N}} \frac{1}{\lambda_n} \left| \sum_{k=0}^{n} (\lambda_k - \lambda_{k-1})(x_k - x_{k-1}) \right| < \infty \right\}.$$

Here and after, we use the convention that any term with a negative subscript is equal to zero, e.g., $\lambda_{-1} = 0$ and $x_{-1} = 0$. With the notation of (1.2) we can redefine the spaces $c_0^\lambda(\Delta)$, $c^\lambda(\Delta)$ and $\ell_\infty^\lambda(\Delta)$ by

$$c_0^\lambda(\Delta) = \{c_0^\lambda\}_\Delta, \ c^\lambda(\Delta) = \{c^\lambda\}_\Delta \ \text{ and } \ \ell_\infty^\lambda(\Delta) = \{\ell_\infty^\lambda\}_\Delta.$$

They show that these spaces are $BK-$spaces of non-absolute type and prove that these are linearly isomorphic to the spaces c_0 and c in Theorem 2.1 and Theorem 2.2, respectively.

Theorem 2.3. *The following relations hold:*

- (i) [36, Theorems 3.1 and 3.2] *The inclusions $c_0^\lambda(\Delta) \subset c^\lambda(\Delta)$ and $c \subset c_0^\lambda(\Delta)$ hold.*
- (ii) [36, Corollaries 3.3 and 3.4] *The inclusions $c_0 \subset c_0^\lambda(\Delta)$ and $c \subset c^\lambda(\Delta)$ strictly hold, and the spaces ℓ_∞ and $c_0^\lambda(\Delta)$ are overlap.*
- (iii) [36, Theorem 3.6] *The inclusion $\ell_\infty \subset c_0^\lambda(\Delta)$ strictly holds if and only if $z \in c_0^\lambda$.*
- (iv) [36, Corollary 3.7] *The inclusion $\ell_\infty \subset c_0^\lambda(\Delta)$ holds if $\lim_{n\to\infty} \frac{\lambda_{n+1}-\lambda_n}{\lambda_n-\lambda_{n-1}} = 1$.*

Theorem 2.4. *Define the sequence $b^{(k)}(\lambda) := \left\{ b_n^{(k)}(\lambda) \right\}_{n\in\mathbb{N}}$ of the elements of the space $c_0^\lambda(\Delta)$ by*

$$b_n^{(k)}(\lambda) := \begin{cases} 0 & , \quad n < k, \\ \frac{\lambda_k}{\lambda_k-\lambda_{k-1}} & , \quad n = k, \\ \frac{\lambda_k}{\lambda_k-\lambda_{k-1}} - \frac{\lambda_k}{\lambda_{k+1}-\lambda_k} & , \quad n > k, \end{cases}$$

for every fixed $k \in \mathbb{N}$. Then, the following statements hold:

- (i) [36, Theorem 4.1] *The sequence $\left\{ b_n^{(k)}(\lambda) \right\}_{n\in\mathbb{N}}$ is a Schauder basis for the spaces $c_0^\lambda(\Delta)$ and every $x \in c_0^\lambda(\Delta)$ has a unique representation of the form $x := \sum_{k=0}^\infty (\overline{\Lambda}x)_n b^{(k)}(\lambda)$.*
- (ii) [36, Theorem 4.2] *The sequence $\left\{ b, b_\lambda^{(0)}, b_\lambda^{(1)}, b_\lambda^{(2)}, \ldots \right\}$ is a Schauder basis for the space $c^\lambda(\Delta)$ and every $x \in c^\lambda(\Delta)$ has a unique representation of the form $x := lb + \sum_{n=0}^\infty [(\overline{\Lambda}x)_n - l] b_\lambda^{(k)}$, where $l = \lim_{k\to\infty} (\overline{\Lambda}x)_n$.*

The authors also determine the alpha-, beta- and gamma-duals of those spaces and finally, characterize the classes $(c_0^\lambda(\Delta):\ell_p)$, $(c_0^\lambda(\Delta):\ell_\infty)$, $(c^\lambda(\Delta):c)$, $(c^\lambda(\Delta):c_0)$, $(c_0^\lambda(\Delta):c)$, $(c_0^\lambda(\Delta):c_0)$, $(c_0:c_0^\lambda(\Delta))$, $(c:c_0^\lambda(\Delta))$, $(\ell_p:c_0^\lambda(\Delta))$, $(c_0:c^\lambda(\Delta))$, $(c:c^\lambda(\Delta))$ and $(\ell_p:c^\lambda(\Delta)$ of matrix mappings, where $1\le p<\infty$.

2.3. **Domain of the Generalized Difference Matrix $B(r,s)$ In the Spaces of $\Lambda-$null and $\Lambda-$convergent Sequences.** In this subsection, following Sönmez and Başar [46], we introduce the domain of the generalized difference matrix $B(r,s)$ in the spaces c_0^λ and c^λ.

Let r and s be non–zero real numbers, and define the generalized difference matrix $B(r,s)=\{b_{nk}(r,s)\}$ by

$$b_{nk}(r,s):=\begin{cases} r & , \quad k=n,\\ s & , \quad k=n-1,\\ 0 & , \quad \text{otherwise,}\end{cases} \tag{2.2}$$

for all $k,n\in\mathbb{N}$. The $B(r,s)-$transform of a sequence $x=(x_k)$ is

$$\{B(r,s)(x)\}_k=rx_k+sx_{k-1} \quad \text{for all} \quad k\in\mathbb{N}.$$

We note that the matrix $B(r,s)$ is reduced to the backward difference matrix Δ in the case $r=1$ and $s=-1$. So, the results related to the domain of the matrix $B(r,s)$ are more general and more comprehensive than the consequences of the domain of the matrix Δ, and include them.

Now, following Sönmez and Başar [46] which is the continuation of Başar and Altay [15], and Aydın and Başar [11], we proceed essentially different than Kızmaz [24] and the other authors following him, and employ a technique of obtaining a new sequence space by means of the matrix domain of a triangle matrix.

Quite recently, Sönmez and Başar [46] have introduced the difference sequence spaces $c_0^\lambda(B)$ and $c^\lambda(B)$, which are the generalization of the spaces $c_0^\lambda(\Delta)$ and $c^\lambda(\Delta)$ introduced by Mursaleen and Noman [37], as follows:

$$c_0^\lambda(B):=\left\{x=(x_k)\in\omega:\lim_{n\to\infty}\frac{1}{\lambda_n}\sum_{k=0}^{n}\left(\lambda_k-\lambda_{k-1}\right)\left(rx_k+sx_{k-1}\right)=0\right\},$$

$$c^\lambda(B):=\left\{x=(x_k)\in\omega:\lim_{n\to\infty}\frac{1}{\lambda_n}\sum_{k=0}^{n}\left(\lambda_k-\lambda_{k-1}\right)\left(rx_k+sx_{k-1}\right)\text{ exists}\right\}.$$

With the notation of (1.2), we can redefine the spaces $c_0^\lambda(B)$ and $c^\lambda(B)$ as

$$c_0^\lambda(B)=\{c_0^\lambda\}_B \quad \text{and} \quad c^\lambda(B)=\{c^\lambda\}_B, \tag{2.3}$$

where B denotes the generalized difference matrix $B(r,s)=\{b_{nk}(r,s)\}$ defined by (2.2).

It is immediate by (2.3) that the sets $c_0^\lambda(B)$ and $c^\lambda(B)$ are linear spaces with coordinatewise addition and scalar multiplication, that is $c_0^\lambda(B)$ and $c^\lambda(B)$ are the spaces of generalized difference sequences. Sönmez and Başar [46] have proved that these spaces are the $BK-$spaces of non-absolute type and norm isomorphic to the spaces c_0 and c, respectively.

Theorem 2.5. *The following relations hold:*

(i) [46, Theorem 3.1] *The inclusion $c_0^\lambda(B) \subset c^\lambda(B)$ strictly holds.*
(ii) [46, Theorem 3.2] *If $s + r = 0$, then the inclusion $c \subset c_0^\lambda(B)$ strictly holds.*
(iii) [46, Corollary 3.3] *The inclusions $c_0 \subset c_0^\lambda(B)$ and $c \subset c^\lambda(B)$ strictly hold.*
(iv) [46, Corollary 3.4] *Although the spaces ℓ_∞ and $c_0^\lambda(B)$ overlap, the space ℓ_∞ does not include the space $c_0^\lambda(B)$.*
(v) [46, Theorem 3.6] *The inclusion $\ell_\infty \subset c_0^\lambda(B)$ strictly holds if and only if $z \in c_0^\lambda$, where the sequence $z = (z_k)$ is defined by*

$$z_k := \left| \frac{r(\lambda_k - \lambda_{k-1}) + s(\lambda_{k+1} - \lambda_k)}{\lambda_k - \lambda_{k-1}} \right| \quad \text{for all} \quad k \in \mathbb{N}.$$

Prior to giving the theorem constructing the Schauder bases of the spaces $c_0^\lambda(B)$ and $c^\lambda(B)$, define the triangle matrix $\widehat{\Lambda} = (\widehat{\lambda}_{nk})$ by

$$\widehat{\lambda}_{nk} := \begin{cases} \frac{r(\lambda_k - \lambda_{k-1}) + s(\lambda_{k+1} - \lambda_k)}{\lambda_n} & , \quad k < n, \\ r\frac{(\lambda_n - \lambda_{n-1})}{\lambda_n} & , \quad k = n, \\ 0 & , \quad k > n \end{cases}$$

for all $n, k \in \mathbb{N}$.

Theorem 2.6. *Let $\alpha_k(\lambda) = \widehat{\Lambda}_k(x)$ for all $k \in \mathbb{N}$ and $l = \lim_{k\to\infty} \widehat{\Lambda}_k(x)$. Define the sequence $b^{(k)}(\lambda) = \left\{ b_n^{(k)}(\lambda) \right\}_{k=0}^{\infty}$ for every fixed $k \in \mathbb{N}$ by*

$$b_n^{(k)}(\lambda) := \begin{cases} \left(\frac{-s}{r}\right)^{n-k} \left[\frac{\lambda_k}{r(\lambda_k - \lambda_{k-1})} + \frac{\lambda_k}{s(\lambda_{k+1} - \lambda_k)} \right] & , \quad k < n, \\ \frac{1}{r}\frac{\lambda_k}{(\lambda_k - \lambda_{k-1})} & , \quad k = n, \\ 0 & , \quad k > n. \end{cases}$$

Then, the following statements hold:

(i) [46, Part (a) of Theorem 4.1] *The sequence $\left\{ b^{(k)}(\lambda) \right\}_{k=0}^{\infty}$ is a basis for the space $c_0^\lambda(B)$ and any $x \in c_0^\lambda(B)$ has a unique representation of the form $x := \sum_k \alpha_k(\lambda) b^{(k)}(\lambda)$.*
(ii) [46, Part (b) of Theorem 4.1] *The sequence $\left\{ b, b^{(0)}(\lambda), b^{(1)}(\lambda), \ldots \right\}$ is a basis for the space $c^\lambda(B)$ and any $x \in c^\lambda(B)$ has a unique representation of the form $x := lb + \sum_k \left[\alpha_k(\lambda) - l\right] b^{(k)}(\lambda)$, where $b = (b_k) = \left\{ \sum_{j=0}^{k} (-s/r)^j / r \right\}_{k=0}^{\infty}$.*

Furthermore, they have determined the $\alpha-$, $\beta-$ and $\gamma-$ duals of those spaces and finally, characterized some matrix classes from the spaces $c_0^\lambda(B)$ and $c^\lambda(B)$ to the spaces ℓ_p, c_0 and c.

2.4. **Spaces of Almost Lambda-Null and Almost Lambda-Convergent Sequences.** Following Yeşilkayagil and Başar [50], in this subsection we introduce the spaces f_0^λ and f^λ of almost lambda-null and almost lambda-convergent sequences.

Quite recently, Yeşilkayagil and Başar [50] have studied the sequence spaces f_0^λ and f^λ as the sets of all almost lambda-null and almost lambda-convergent sequences, respectively. That is,

$$f_0^\lambda := \left\{ x = (x_k) \in \omega : \lim_{m\to\infty} \frac{1}{m+1} \sum_{k=0}^{m} (\Lambda x)_{n+k} = 0 \;\text{ uniformly in }\; n \right\},$$

$$f^\lambda := \left\{ x = (x_k) \in \omega : \exists l \in \mathbb{C} \;\ni\; \lim_{m\to\infty} \frac{1}{m+1} \sum_{k=0}^{m} (\Lambda x)_{n+k} = l \;\text{ uniformly in }\; n \right\}.$$

With the notation of (1.2), we can restate the spaces f_0^λ and f^λ by the matrix domain of triangle Λ in the spaces f_0 and f, respectively, as follows:

$$f_0^\lambda = (f_0)_\Lambda \quad \text{and} \quad f^\lambda = f_\Lambda.$$

Now, we may give the following theorems on some inclusion relations and the alpha-, beta- and gamma-duals of the spaces f_0^λ and f^λ:

Theorem 2.7. *The following relations hold:*

(i) [50, Theorem 3.5] *The inclusions $f_0 \subset f_0^\lambda$ and $f \subset f^\lambda$ strictly hold. Furthermore, the equalities $f_0 = f_0^\lambda$ and $f = f^\lambda$ hold if and only if $Sx \in f_0$ for every x in the spaces f^λ and f_0^λ, respectively.*

(ii) [50, Theorem 3.6] *The inclusion $f_0^\lambda \subset f^\lambda$ strictly holds.*

(iii) [50, Theorem 3.7] *The inclusions $c^\lambda \subset f^\lambda \subset \ell_\infty^\lambda$ strictly hold.*

Theorem 2.8. *The following relations hold:*

(i) [50, Theorem 4.2] *The $\alpha-$dual of the space f^λ is the set a_1^λ defined by*

$$a_1^\lambda = \left\{ a = (a_k) \in \omega : \sum_{k=0}^{\infty} \frac{\lambda_k}{\lambda_k - \lambda_{k-1}} |a_k| < \infty \right\}.$$

(ii) [50, Theorem 4.4] *The γ-dual of the space f^λ is the set $d_1 \cap d_2$, where*

$$d_1 := \left\{ a = (a_k) \in \omega : \sup_{n\in\mathbb{N}} \sum_{k=0}^{n-1} \left| \Delta\left(\frac{a_k}{\lambda_k - \lambda_{k-1}} \right) \lambda_k \right| < \infty \right\},$$

$$d_2 := \left\{ a = (a_k) \in \omega : \left(\frac{a_n \lambda_n}{\lambda_n - \lambda_{n-1}} \right) \in \ell_\infty \right\}.$$

(iii) [50, Theorem 4.6] *Let $d_3 = cs$ and define the sets d_4 and d_5 by*

$$d_4 := \left\{ a = (a_k) \in \omega : \left\{ \Delta\left(\frac{a_k}{\lambda_k - \lambda_{k-1}} \right) \lambda_k \right\} \in c \right\},$$

$$d_5 := \left\{ a = (a_k) \in \omega : \left\{ \left| \Delta\left[\Delta\left(\frac{a_k}{\lambda_k - \lambda_{k-1}} \right) \lambda_k \right] \right| \right\} \in cs \right\}.$$

Then, $\{f^{\lambda}\}^{\beta} = \bigcap_{i=1}^{5} d_i$.

Finally, Yeşilkayagil and Başar [50] they have proven two basic results on the space f of almost convergent sequences and characterize the classes $(f^{\lambda} : \mu)$ and $(\mu : f^{\lambda})$ of infinite matrices, and also gave the characterizations of some other classes as an application of those main results, where μ is any given sequence space.

3. Domain of the Matrix Λ In the Paranormed Sequence Spaces

In this section, we shortly give the knowledge on the paranormed sequence spaces derived by the matrix Λ from some Maddox's spaces.

Quite recently, Karakaya et al. [23] have introduced the paranormed sequence spaces $c_0(\lambda,p)$, $c(\lambda,p)$ and $\ell_{\infty}(\lambda,p)$, as follows:

$$c_0(\lambda,p) := \left\{ x=(x_k)\in\omega : \lim_{n\to\infty}\frac{1}{\lambda_n}\left|\sum_{k=0}^{n}(\lambda_k-\lambda_{k-1})\,x_k\right|^{p_n}=0 \right\},$$

$$c(\lambda,p) := \left\{ x=(x_k)\in\omega : \exists l\in\mathbb{C}\ \ni\ \lim_{n\to\infty}\frac{1}{\lambda_n}\left|\sum_{k=0}^{n}(\lambda_k-\lambda_{k-1})\,(x_k-l)\right|^{p_n}=0 \right\},$$

$$\ell_{\infty}(\lambda,p) := \left\{ x=(x_k)\in\omega : \sup_{n\in\mathbb{N}}\frac{1}{\lambda_n}\left|\sum_{k=0}^{n}(\lambda_k-\lambda_{k-1})\,x_k\right|^{p_n}<\infty \right\}.$$

With the notation of (1.2), we can redefine the spaces $c_0(\lambda,p)$, $c(\lambda,p)$ and $\ell_{\infty}(\lambda,p)$ by the domain of the matrix Λ in the spaces $c_0(p)$, $c(p)$ and $\ell_{\infty}(p)$, respectively, as

$$c_0(\lambda,p)=\{c_0(p)\}_{\Lambda},\quad c(\lambda,p)=\{c(p)\}_{\Lambda}\quad\text{and}\quad \ell_{\infty}(\lambda,p)=\{\ell_{\infty}(p)\}_{\Lambda}.$$

Theorem 3.1. *The following relations hold:*

(i) [23, Theorem 3] *The inclusions* $c_0(\lambda,p)\subset c(\lambda,p)\subset \ell_{\infty}(\lambda,p)$ *strictly hold.*

(ii) [23, Theorem 4] *If* $1\le p_n\le p_{n+1}$ *for all* $n\in\mathbb{N}$, *then the inclusions* $c_0(p)\subset c_0(\lambda,p)$, $c(p)\subset c(\lambda,p)$ *and* $\ell_{\infty}(p)\subset \ell_{\infty}(\lambda,p)$ *hold.*

(iii) [23, Parts (i) and (ii) of Theorem 5] *Let* μ *denotes any of the spaces* c_0, c *and* ℓ_{∞}. *Then, the inclusion* $\mu^{\lambda}\subset\mu(\lambda,p)$ *holds if* $p_n>1$ *for all* $n\in\mathbb{N}$ *and the inclusion* $\mu(\lambda,p)\subset\mu^{\lambda}$ *holds if* $p_n<1$ *for all* $n\in\mathbb{N}$.

Karakaya et al. [23] have investigated some topological properties and additionally, computed the alpha-, beta- and gamma-duals of the spaces $\ell_{\infty}(\lambda,p)$, $c(\lambda,p)$ and $c_0(\lambda,p)$. Finally, they have characterized the classes $(c_0(\lambda,p):\mu)$, $(c(\lambda,p):\mu)$ and $(\ell_{\infty}(p):\mu)$ of matrix transformations, where $\mu\in\{c_0(q),c(q),\ell_{\infty}(q)\}$ and $q=(q_k)$ is the bounded sequence of strictly positive reals.

4. Conclusion

Malkowsky and Rakočević [31] characterized some matrix classes and studied related compact operators involving c_0^{λ}, c^{λ} and ℓ_{∞}^{λ}. Malkowsky and Savaş [33] determined the beta-dual of ℓ_p^λ and characterized some matrix classes involving ℓ_p^λ, where $1 \leq p < \infty$. Mursaleen and Noman [38] apply the Hausdorff measure of noncompactness to characterize some matrix classes of compact operators on the sequence space ℓ_p^λ, where $1 \leq p < \infty$.

Mursaleen and Alotaibi [35] introduced statistical λ-convergence and strong λ_q-convergence and established some relations between λ-statistical convergence, statistical λ-convergence and strong λ_q-convergence, by using the generalized de la Vallée-Poussin mean, where $0 < q < \infty$. Also, they proved an analogue of the classical Korovkin theorem by using the concept of statistical λ-convergence.

Mursaleen and Noman [39] established some identities or estimates for the operator norms and the Hausdorff measures of noncompactness of certain matrix operators on the spaces c_0^λ and ℓ_∞^λ which have recently been introduced [36]. Further, by using the Hausdorff measure of noncompactness, the authors characterized some classes of compact operators on the spaces c_0^λ and ℓ_∞^λ.

Quite recently, Yeşilkayagil and Başar [49] have determined the fine spectrum with respect to Goldberg's classification of the operator defined by the matrix Λ acting on the sequence spaces c_0 and c. As a new development, they have given the approximate point spectrum, defect spectrum and compression spectrum of the matrix operator Λ on the sequence spaces c_0 and c. As a natural continuation of this study, one can work on the fine spectrum with respect to Goldberg's classification of the operator defined by the matrix Λ over the sequence spaces cs, ℓ_p and bv_p, where bv_p denotes the space of all sequences whose $\Delta-$transforms are in the space ℓ_p and is recently studied in the case $1 \leq p \leq \infty$ by Başar and Altay [15], and in the case $0 < p < 1$ by Altay and Başar [8]. Of course, determination the fine spectrum of some triangle matrices over the sequence space X will be very interesting, where X is any of the spaces c^λ, c_0^λ, ℓ_p^λ, $c_0^\lambda(\Delta)$, $c^\lambda(\Delta)$, $c_0^\lambda(B)$ and $c^\lambda(B)$.

We should record that to investigate the domain of the matrix Λ in the Maddox's sequence space $\ell(p)$ and to examine its algebraic and topological properties will be meaningful. Finally, we note that the investigation of the Hausdorff measures of noncompactness of the matrix operators defined by some triangle matrices on the spaces c^λ and ℓ_p^λ is still an open problem.

Acknowledgement

The author has benefited a lot from the constructive report of the anonymous referee. So, he is thankful for his/her valuable comments and corrections on the first draft of this paper which improved the presentation and readability.

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