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H_∞ CONTROL AND INPUT-TO-STATE STABILIZATION FOR HYBRID SYSTEMS WITH TIME DELAY

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Abstract. This paper addresses the problem of designing a robust reliable H_∞ control and a switching law to guarantee input-to-state stabilization (ISS) for a class of uncertain switched control systems with time delay not only when all the actuators are operational, but also when some of them experience failure. The output of faulty actuators are treated as a disturbance signal that is augmented with the system disturbance input. Multiple Lyapunov function with Razumikhin technique, and average dwell-time switching signal are used to establish the ISS property.

1. Introduction

A switched system is a special class of hybrid systems that consists of a family of continuous- or discrete-time dynamical subsystems, and a switching rule that controls the switchings among the subsystems. One may refer to [7, 8, 12] and the references therein.

The reliable control is the controller that tolerates failures in the control components. In reality, such failures are frequently encountered, yet the immediate repair may be impossible in some critical cases. Consequently, it is necessary to design a reliable controller that guarantees an acceptable level of performance [3, 11, 15, 18].

In practice, most of the real control systems are subject to some disturbance inputs. ISS notion, introduced in [13] which addresses the system response to a bounded disturbance when the unforced system is asymptotically stable, is an efficient tool to deal with these disturbances. In [1], a robust reliable H_∞ controller was designed to guarantee ISS with a desired level of performance for stochastic systems with time delay. In this work, the nonzero output of faulty actuators was augmented with the system disturbance. Furthermore, Lyapunov function with the Razumikhin approach were used in that work.

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The novelty of this work is to develop new sufficient conditions that guarantee the input-to-state stabilization and H_∞ performance of the switched system in the presence of the disturbance, state uncertainties, time delay, and nonlinear lumped perturbation not only when all the actuators are operational, but also when some of them experience failure. While in most of the available literature on reliable controls, the faulty actuators are modeled as outages (i.e., zero output), in this work the output signal of these actuators is treated as a disturbance signal that is augmented with the system disturbance input. The latter case is more practical because most of the control component failures occur unexpectedly. The method of multiple Lyapunov functions is utilized to analyze the ISS. It is well known that the stability of a switched system is not guaranteed by the stability of each individual mode unless the switching among them is ruled by a logic-based switching signal, where in this work the average dwell-time condition is used. To the best of our knowledge, these results have not been studied in the available literature.

This paper is organized as follows. Section 2 involves the problem description, definitions, and a useful lemma. The main results and proofs are stated in Section 3. A numerical example with simulations is presented in Section 4. The conclusion is given in Section 5.

2. Problem formulation

Consider a class of uncertain switched systems with time delay given by

$$\begin{cases} \dot{x} = (A_{\varrho(t)} + \Delta A_{\varrho(t)})x + (\bar{A}_{\varrho(t)} + \Delta\bar{A}_{\varrho(t)})x(t-r) + B_{\varrho(t)}u + G_{\varrho(t)}w + f_{\varrho(t)}(x(t-r)), \\ z = C_{\varrho(t)}x + F_{\varrho(t)}u, \\ x_{t_0} = \phi(t),\ t \in [-r, 0],\ r > 0, \end{cases}$$

where $x \in \mathbb{R}^n$ is the system state, , $u \in \mathbb{R}^l$ is the control input, and $w \in \mathbb{R}^p$ is an input disturbance, which is assumed to be in $L_2[t_0, \infty)$ that is $||w||_2^2 = \int_{t_0}^{\infty} ||w(t)||^2\, dt < \infty$, and $z \in \mathbb{R}^r$ is the controlled output. ϱ is the switching rule which is a piecewise constant function defined by $\varrho : [t_0, \infty) \to \mathcal{S} = \{1, 2, \cdots, N\}$. For $r > 0$, let $\mathcal{C}_r$ be the space of all continuous functions that are defined from $[-r, 0]$ to $\mathbb{R}^n$. For any $t \in \mathbb{R}_+$, let $x(t)$ be a function defined on $[t_0, \infty]$. Then, we define the functional $x_t : [-r, 0] \to \mathbb{R}^n$ by $x_t(s) = x(t+s)$ for all $s \in [-r, 0]$, and its norm by $||x_t||_r = \sup_{t-r\le\theta\le t} ||x(\theta)||$, where $r > 0$ is the time delay. For each $i \in \mathcal{S}$, A_i is a non Hurwitz matrix, $K_i \in \mathbb{R}^{l\times n}$ is the control gain matrix such that $u = K_i x$, where (A_i, B_i) is assumed to be stabilizable, $f_i(\cdot) \in \mathbb{R}^n$ is some nonlinearity, A_i, B_i, G_i, C_i and F_i are known real constant matrices, and ΔA_i, $\Delta\bar{A}_i$ are piecewise continuous functions representing system parameter uncertainties. For any $i \in \mathcal{S}$ the closed-loop system is

$$\begin{cases} \dot{x} = (A_i + \Delta A_i + B_i K_i)x + (\bar{A}_i + \Delta\bar{A}_i)x(t-r) + G_i w + f_i(x(t-r)), \\ z = C_{ic}x, \qquad C_{ic} = C_i + F_i K_i \\ x_{t_0} = \phi(t),\ t \in [-r, 0],\ r > 0, \end{cases}$$

To analyze the reliable stabilization with respect to actuator failures, for any $i \in \mathcal{S}$, we write $B_i = B_{i\Sigma} + B_{i\bar{\Sigma}}$, where $\Sigma \subseteq \{1, 2, ..., l\}$ the set of actuators that are susceptible to failure, and $\bar{\Sigma} \subseteq \{1, 2, ..., l\} - \Sigma$ the other set of actuators which are robust to failures and essential to stabilize the given system, moreover, the matrices $B_{i\Sigma}$, $B_{i\bar{\Sigma}}$ are the control matrices associated with Σ, $\bar{\Sigma}$ respectively, and are generated by zeroing out the columns corresponding to $\bar{\Sigma}$ and Σ, respectively. The pair $(A_i, B_{i\bar{\Sigma}})$ is assumed to be stabilizable. For a fixed $i \in \mathcal{S}$, let $\sigma \subseteq \Sigma$ corresponds to some of the actuators that experience failure, and assume that the output of faulty actuators is any arbitrary energy-bounded signal which belongs to $L_2[t_0, \infty)$. Then, the decomposition becomes $B_i = B_{i\sigma} + B_{i\bar{\sigma}}$, where $B_{i\sigma}$ and $B_{i\bar{\sigma}}$ have the same definition of $B_{i\Sigma}$ and $B_{i\bar{\Sigma}}$, respectively. Furthermore, the augmented disturbance input to the system becomes $w_\sigma^F = (w^T \ (u_\sigma^F)^T)^T$, where $u_\sigma^F \in \mathbb{R}^l$ is the failure vector whose elements corresponding to the set of faulty actuators σ, and F here stands for "failure". Thus, the closed-loop system becomes

$$\begin{cases} \dot{x} = (A_i + \Delta A_i + B_{i\bar{\sigma}} K_i)x + (\bar{A}_i + \Delta \bar{A}_i)x(t-r) + G_{ic} w_\sigma^F + f_i(x(t-r)), \\ z = C_{ic} x, \\ x_{t_0} = \phi(t), \ t \in [-r, 0], \ r > 0, \end{cases} \tag{1}$$

where $G_{ic} = (G_i \ B_{i\sigma})$.

Definition 1. System (2) is said to be robustly globally exponentially ISS if there exist $\lambda > 0$, $\bar{\lambda} > 0$ and a function $\rho \in \mathcal{K}$ such that the solution $x(t)$ exists $\forall t \geq t_0$ and satisfies

$$||x|| \leq \bar{\lambda} ||x_{t_0}||_r e^{-\lambda(t-t_0)} + \rho\big(\sup_{t_0 \leq \tau \leq t} ||w(\tau)||\big).$$

Definition 2. Given a constant $\gamma > 0$, system (2) is said to be ISS-H_∞ if there exists a state feedback law $u(t) = K_i x(t)$, such that, for any admissible parameter uncertainties ΔA_i and $\Delta \bar{A}_i$, the closed-loop system (2) is globally exponentially ISS, and the controlled output z satisfies

$$||z||_2^2 = \int_{t_0}^{\infty} ||z||^2 \, dt \leq \gamma^2 ||w||_2^2 + m_0, \qquad \text{for some} \quad m_0 > 0.$$

Assumption A. For any $i \in \mathcal{S}$ and $\forall \, t \in \mathbb{R}_+$, $\Delta A_i(t) = D_i \mathcal{U}_i(t) H_i$ and $\Delta \bar{A}_i(t) = \bar{D}_i \bar{\mathcal{U}}_i(t) \bar{H}_i$, with D_i, $H_i, \bar{D}_i$, $\bar{H}_i$ being known real matrices with appropriate dimensions, and $\mathcal{U}_i(t)$,$\bar{\mathcal{U}}_i(t)$ being unknown real time-varying matrices and satisfying $||\mathcal{U}_i(t)|| \leq 1$ and $||\bar{\mathcal{U}}_i(t)|| \leq 1$.

Lemma. For any $\xi_j > 0$ $(j = 1, \cdots, 5)$, and a positive- definite matrix P, we have

(i) $2x^T P(\Delta A)x \leq x^T(\xi_1 P D D^T P + \frac{1}{\xi_1} H^T H)x$.

(ii) $2x^T P G w \leq x^T(\xi_2 P G G^T P)x + \frac{1}{\xi_2} w^T w$.

Moreover, for $x \in \mathcal{C}_r$, if $||x(t-r)||_r^2 \leq q||x||^2$ with $q > 1$, then

(iii) $2x^T P \bar{A} x(t-r) \leq x^T(\xi_3 P \bar{A} (\bar{A})^T P + \frac{q}{\xi_3} I)x$.

(iv) $2x^T P(\Delta \bar{A})x(t-r) \leq x^T(\xi_4 P \bar{D} \bar{D}^T P + \frac{q}{\xi_4} ||\bar{H}||^2)x$.

(v) $2x^T Pf(x(t-r)) \leq x^T(\xi_5 P^2 + \frac{1}{\xi_5}\delta qI)x$, where $\delta > 0$ such that $||f(x(t-r))||^2 \leq \delta ||x(t-r)||_r^2$.

Average Dwell Time Condition. The number of switches $N(t_0, t)$ in the interval (t_0, t) for a finite t satisfies $N(t_0, t) \leq N_0 + \frac{t-t_0}{\tau_a}$, where N_0 is the chatter bound, and τ_a is the average dwell time.

3. Main results

In this section, we shall state and prove our main results. The following theorem gives robust global exponential ISS property of the system.

Theorem 1. For any $i \in \mathcal{S}$, let K_i and $\gamma_i > 0$ be given. Assume that Assumption A holds and there exist positive constants $\xi_{ji} > 0$ $(j = 1, \cdots, 5)$, a positive-definite matrix P_i, and $V_i : \mathbb{R}^n \to \mathbb{R}_+$ such that

(i) $\dot{V}_i(\psi(0)) \leq 0$ whenever $V_i(\psi) \leq q_i V_i(\psi(0))$, and $\gamma(\sup_{t_{k-1} \leq s \leq t_k} |w(s)|) \leq V_i(\psi(0))$ for $\psi \in \mathcal{C}_r$ and $t \in [t_{k-1}, t_k)$, where $q_i > 1$;

(ii) for all k, $\tau_a \leq t_k - t_{k-1} \leq \beta$ holds where τ_a is the average dwell time, and $\beta > 0$;

(iii) the following algebraic Riccati-like equation holds

$$(A_i + B_iK_i)^T P_i + P_i(A_i + B_iK_i) + P_i(\xi_{1i} D_i D_i^T + \xi_{2i} G_i G_i^T + \xi_{3i} \bar{A}_i(\bar{A}_i)^T + \xi_{4i} \bar{D}_i(\bar{D}_i)^T + \xi_{5i} I)P_i + \frac{1}{\xi_{1i}} H_i^T H_i + (\frac{q_i}{\xi_{3i}} + \frac{q_i}{\xi_{4i}} ||\bar{H}_i||^2 + \frac{\delta_i q_i}{\xi_{5i}})I + C_{ic}^T C_{ic} + \alpha_i P_i = 0 \tag{2}$$

where $\delta_i > 0$ such that $||f_i(\psi)||^2 \leq \delta_i ||\psi||_r^2$.

Then, system (2) is robustly globally exponentially ISS-H_∞.

Proof. For all $t \in [-r, \infty)$, let $x(t) = x(t, t_0, \phi)$ be the solution of system (2). For any $i \in \mathcal{S}$, define $V_i(x) = x^T P_i x$. Then, $\lambda_{\min}(P_i)||x||^2 \leq V_i(x) \leq \lambda_{\max}(P_i)||x||^2$ and

$$\dot{V}_i(x) = x^T[(A_i + B_iK_i)^T P_i + P_i(A_i + B_iK_i)]x + 2x^T P_i(\Delta A_i)x + 2x^T P_i G_i w + 2x^T P_i f_i((t-r)) + 2x^T P_i(\Delta\bar{A}_i)x(t-r) + 2x^T P_i \bar{A}_i x(t-r).$$

Claim. For any $i \in \mathcal{S}$, and $k \in \mathbb{N}$, $t \in [t_{k-1}, t_k)$, conditions (i) and (ii) imply that

$$||x(t)||^2 \leq M_i ||\phi||_r^2 e^{-\lambda_i(t-t_{k-1})} + \gamma(\sup_{t_{k-1} \leq s \leq t} ||w(s)||), \qquad \lambda_i > 0, \quad M_i > 1.$$

Proof of the claim. Choose $M_i > 1$ such that

$$c_{2i}||x_{t_{k-1}}||_r^2 \leq M_i c_{2i}||x_{t_{k-1}}||_r^2 e^{-\lambda_i(t_k - t_{k-1})} + \bar{\gamma}(t) \tag{3}$$

where $\bar{\gamma}(t) = \gamma(\sup_{t_{k-1} \leq s \leq t}(||w(s)||))$ and $c_{2i} = \lambda_{\max}(P_i)$. Let $v(t) = V_i(x(t))$, for all $t \in [t_{k-1} - r, t_k)$, $k = 1, 2, \cdots$. From (3), we have for $t \in [t_{k-1} - r, t_k)$,

$$v(t) \leq c_{2i}||x(t)||^2 \leq c_{2i}||x_{t_{k-1}}||_r^2 \leq M_i c_{2i}||x_{t_{k-1}}||_r^2 e^{-\lambda_i(t_k - t_{k-1})} + \bar{\gamma}(t)$$

If (3) is not true, then there exists $\bar{t} \in (t_{k-1}, t_k)$ such that

$$v(\bar{t}) > M_i c_{2i}||x_{t_{k-1}}||_r^2 e^{-\lambda_i(t_k - t_{k-1})} + \bar{\gamma}(t) \geq c_{2i}||x_{t_{k-1}}||_r^2 \geq v(t+s), \ s \in [-r, 0]$$

From the continuity, there exists $t^* \in (t_{k-1}, \bar{t})$ such that

$$v(t^*) = M_i c_{2i} ||x_{t_{k-1}}||_r^2 e^{-\lambda_i (t_k - t_{k-1})} + \bar{\gamma}(t^*)$$

and for all $t \in [t_{k-1} - r, t^*]$, we have

$$v(t) \le M_i c_{2i} ||x_{t_{k-1}}||_r^2 e^{-\lambda_i (t_k - t_{k-1})} + \bar{\gamma}(t)$$

Also, there exists $t^{**} \in [t_{k-1}, t^*)$ such that $v(t^{**}) = c_{2i}||x_{t_{k-1}}||_r^2$ and for $t \in [t^{**}, t^*]$, $v(t) \ge c_{2i}||x_{t_{k-1}}||_r^2$. Hence, from (**??**), for all $t \in [t^{**}, t^*]$, and $s \in [-r, 0]$, we have

$$\begin{aligned}
v(t+s) &\le M_i c_{2i} ||x_{t_{k-1}}||_r^2 e^{-\lambda_i (t_k - t_{k-2})} e^{\lambda_i (t_{k-1} - t_{k-2})} + \bar{\gamma}(t) \\
&\le M_i v(t) e^{\lambda_i (t_{k-1} - t_{k-2})} + \gamma(\sup_{t_{k-1} \le s \le t} (||w(s)||)) \\
&\le (M_i e^{\lambda_i (t_{k-1} - t_{k-2})} + 1) v(t) \le (M_i e^{\lambda_i \beta} + 1) v(t) = q_i v(t)
\end{aligned}$$

where we used condition (i) to get the second last inequality. Therefore, we have $\dot{v}(t) \le 0$ for $t \in [t^{**}, t^*]$ which is a contradiction. Now by the claim, the lemma, and condition (2), we have

$$\begin{aligned}
\dot{V}_i(x) \le\ & x^T[(A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) + P_i (\xi_{1i} D_i D_i^T + \xi_{2i} G_i G_i^T + \xi_{3i} \bar{A}_i (\bar{A}_i)^T \\
& + \xi_{4i} \bar{D}_i (\bar{D}_i)^T + \xi_{5i} I) P_i + \frac{1}{\xi_{1i}} H_i^T H_i + (\frac{q_i}{\xi_{3i}} + \frac{q_i}{\xi_{4i}} ||\bar{H}_i||^2 + \frac{\delta_i q_i}{\xi_{5i}}) I] x + \frac{1}{\xi_{2i}} w^T w \\
\le\ & -\alpha_i V_i(x) + \frac{1}{\xi_{2i}} w^T w,
\end{aligned}$$

Hence, $\forall\, t \in [t_{k-1}, t_k)$, we have $\dot{V}_i(x) \le -\bar{\alpha}_i V_i(x) - \theta_i V_i(x) + \frac{1}{\xi_{2i}} w^T w$, where $\bar{\alpha}_i = \alpha_i - \theta_i$ and $0 < \theta_i < \alpha_i$. The foregoing inequality implies that, $\forall t \in [t_{k-1}, t_k)$, $\dot{V}_i(x) \le -\bar{\alpha}_i V_i(x)$, provided that $V_i(x) > \frac{1}{\theta_i \xi_{2i}} ||w||^2$, or $||x|| > \frac{||w||}{\sqrt{\theta_i c_2 \xi_{2i}}} =: \rho_i(||w||)$, where $c_2 = \max_{i \in \mathcal{S}} \lambda_{\max}(P_i)$. Then, for all $t \in [t_{k-1}, t_k)$,

$$V_i(x(t)) \le V_i(x(t_{k-1})) e^{-\bar{\alpha}_i (t - t_{k-1})}$$

provided that $||x|| > \rho(||w||)$, where $\rho(||w||) = \max_{i \in \mathcal{S}} \{\rho_i(||w||)\}$. As for the switching, we have for any $i, j \in \mathcal{S}$,

$$V_j(x(t)) \le \mu V_i(x(t)),\ \mu = \frac{c_2}{c_1},$$

where $c_1 = \min_{i \in \mathcal{S}} \{\lambda_{\min}(P_i)\}$ and $c_2 = \max_{i \in \mathcal{S}} \{\lambda_{\max}(P_i)\}$. Then, for $i \in \mathcal{S}$ and $t \in [t_{k-1}, t_k)$,

$$V_i(x(t)) \le \mu^{k-1} e^{-\bar{\alpha}_i (t - t_{k-1})} e^{-\bar{\alpha}_{i-1} (t_{k-1} - t_{k-2})} \cdots e^{-\bar{\alpha}_1 (t_1 - t_0)} V_1(x_0)$$

provided that $||x|| > \rho(||w||)$. Letting $\alpha^* = \min\{\bar{\alpha}_i;\, i \in \mathcal{S}\}$, one may get

$$V_i(x(t)) \le \mu^{k-1} e^{-\alpha^* (t - t_0)} V_1(x_0) = e^{(k-1)\ln\mu - \alpha^*(t - t_0)} V_1(x_0),$$

provided that $||x|| > \rho(||w||)$.

Using ADTC with $N_0 = \frac{\eta}{\ln \mu}$, $\tau_a = \frac{\ln \mu}{\alpha^* - \nu}$, $(\nu < \alpha^*)$, for some $\eta > 0$, we get

$$V_i(x(t)) \leq e^{\eta - \nu(t-t_0)} V_1(x_0), \qquad \text{provided that } \ ||x|| > \rho(||w||).$$

This implies that [5] $||x|| \leq b||x_{t_0}||_r e^{-\nu(t-t_0)/2} + \gamma(\sup_{t_0 \leq \tau \leq t} ||w(\tau)||)$, $\forall\, t \geq t_0$, where $b = \sqrt{e^{\eta} c_2/c_1}$, and $\gamma(s) = \sqrt{\frac{c_2}{c_1}} \rho(s)$. This completes the proof of exponential ISS.

To prove the upper bound on $||z||$, for any $i \in \mathcal{S}$, let $J_i = \int_{t_0}^{\infty} (z^T z - \gamma_i^2 w^T w) dt$. Then,

$$\begin{aligned}
J_i &= \int_{t_0}^{\infty} (z^T z - \gamma_i^2 w^T w)\, dt + \int_{t_0}^{\infty} \dot{V}_i\, dt - V_i(\infty) + V_i(x_0) \\
&\leq \int_{t_0}^{\infty} (z^T z - \gamma_i^2 w^T w)\, dt + V_i(x_0) + \int_{t_0}^{\infty} \{x^T[(A_i + B_i K_i)^T P_i + P_i(A_i + B_i K_i) \\
&\quad + P_i(\xi_{1i} D_i D_i^T + \xi_{3i} \bar{A}_i (\bar{A}_i)^T + \xi_{4i} \bar{D}_i (\bar{D}_i)^T + \xi_{5i} I) P_i + \frac{1}{\xi_{1i}} H_i^T H_i \\
&\quad + (\frac{q_i}{\xi_{3i}} + \frac{q_i}{\xi_{4i}} ||\bar{H}_i||^2 + \frac{\delta_i q_i}{\xi_{5i}}) I + \gamma_i^{-2} P_i G_i G_i^T P_i - \gamma_i^{-2} P_i G_i G_i^T P_i] x + 2x^T P_i G_i w\}\, dt \\
&= V_i(x_0) + \int_{t_0}^{\infty} \{x^T[(A_i + B_i K_i)^T P_i + P_i(A_i + B_i K_i) + P_i(\xi_{1i} D_i D_i^T + \xi_{3i} \bar{A}_i (\bar{A}_i)^T \\
&\quad + \xi_{4i} \bar{D}_i (\bar{D}_i)^T + \xi_{5i} I) P_i + \frac{1}{\xi_{1i}} H_i^T H_i + (\frac{q_i}{\xi_{3i}} + \frac{q_i}{\xi_{4i}} ||\bar{H}_i||^2 + \frac{\delta_i q_i}{\xi_{5i}}) I + \gamma_i^{-2} P_i G_i G_i^T P_i \\
&\quad + C_{ic}^T C_{ic}] x\}\, dt - \int_{t_0}^{\infty} \gamma_i^2 (w - \gamma_i^{-2} G_i^T P_i x)^T (w - \gamma_i^{-2} G_i^T P_i x)\, dt.
\end{aligned}$$

The last term is strictly negative, using (2) with $\gamma_i^{-2} = \xi_{2i}$, we get $J_i \leq V_i(x_0)$ which leads to $||z||_2^2 \leq \gamma^2 ||w||_2^2 + m_0$, where $m_0 = \max_{i \in \mathcal{S}} \{V_i(x_0)\}$, and $\gamma = \max_{i \in \mathcal{S}} \{\gamma_i\}$.

Remark. Theorem 1 provides sufficient conditions to ensure the robust global exponential ISS property. The algebraic Riccati-like equation in (2) is to guarantee the existence of the positive-definite matrix P_i (for all $i \in \mathcal{S}$), which implies that the solution trajectories of the subsystems are decreasing outside a certain neighbourhood of the disturbance $w(t)$. The role of the average dwell time condition is to organize the switching among the system modes. ξ_1, ξ_2 are tuning parameters to reduce the conservativeness of the Riccati-like equation.

Theorem 2. **(Reliability)** For any $i \in \mathcal{S}$, let the constant $\gamma_i > 0$ be given, and assume that there exist positive constants $\xi_{ji}, (j = 1, \cdots, 5)$, ϵ_i, α_i, a positive-definite matrix P_i, and $V_i : \mathbb{R}^n \to \mathbb{R}_+$ such that conditions (i)-(iii) from Theorem 1 hold, $K_i = -\frac{1}{2}\epsilon_i B_{i\bar{\sigma}}^T P_i$, and the following algebraic Riccati-like equation holds

$$A_i^T P_i + P_i A_i + P_i(\xi_{1i} D_i D_i^T + \xi_{2i} G_{ic} G_{ic}^T - \epsilon_i B_{i\bar{\Sigma}} B_{i\bar{\Sigma}}^T + \xi_{3i} \bar{A}_i (\bar{A}_i)^T + \xi_{4i} \bar{D}_i (\bar{D}_i)^T$$
$$+ \xi_{5i} I) P_i + (\frac{q_i}{\xi_{3i}} + \frac{q_i}{\xi_{4i}} ||\bar{H}_i||^2 + \frac{\delta_i q_i}{\xi_{5i}}) I + \frac{1}{\xi_{1i}} H_i^T H_i + C_{ic}^T C_{ic} + \alpha_i P_i = 0, \tag{4}$$

where $\delta_i > 0$ such that $||f_i(\psi)||^2 \leq \delta_i ||\psi||_r^2$. Then, system (1) is robustly globally exponentially ISS-H_∞.

Proof. Let $x(t) = x(t, t_0, \phi)$ be the solution of (1). $\forall i \in \mathcal{S}$, define $V_i(x) = x^T P_i x$. Then,

$$\begin{aligned}
\dot{V}_i(x) \leq\, & x^T[A_i^T P_i + P_i A_i + P_i(\xi_{1i} D_i D_i^T + \xi_{2i} G_{ic} G_{ic}^T + \xi_{3i} \bar{A}_i (\bar{A}_i)^T \\
& + \xi_{4i} \bar{D}_i (\bar{D}_i)^T - \epsilon_i B_{i\bar{\Sigma}} B_{i\bar{\Sigma}}^T + \xi_{5i} I) P_i + \frac{1}{\xi_{1i}} H_i^T H_i + (\frac{q_i}{\xi_{3i}} + \frac{q_i}{\xi_{4i}} ||\bar{H}_i||^2 \\
& + \frac{\delta_i q_i}{\xi_{5i}}) I] x + \frac{1}{\xi_{2i}} (w_\sigma^F)^T w_\sigma^F \\
\leq\, & -\alpha_i V_i(x) + \frac{1}{\xi_{2i}} (w_\sigma^F)^T w_\sigma^F,
\end{aligned}$$

where we used the claim proved in Theorem 1, the lemma, the fact that [11] $B_{i\bar{\Sigma}} B_{i\bar{\Sigma}}^T \leq B_{i\bar{\sigma}} B_{i\bar{\sigma}}^T$, and condition (4). Then, for any $i \in \mathcal{S}$, $\dot{V}_i(x) \leq -\bar{\alpha}_i V_i(x) - \theta_i V_i(x) + \frac{1}{\xi_{2i}} (w_\sigma^F)^T w_\sigma^F$, where $\bar{\alpha}_i = \alpha_i - \theta_i$ and $0 < \theta_i < \alpha_i$. Then, for all $t \in [t_{k-1}, t_k)$, $\dot{V}_i(x) \leq -\bar{\alpha}_i V_i(x)$, provided that $||x|| > \frac{||w_\sigma^F||}{\sqrt{\theta_i c_2 \xi_{2i}}} =: \rho_i(||w_\sigma^F||)$, which implies that $V_i(x(t)) \leq e^{\eta - \nu(t-t_0)} V_1(x_0)$ provided that $||x|| > \rho(||w||)$, where $\rho(||w||) = \max_{i \in \mathcal{S}} \{\rho_i(||w||)\}$. This also implies that [5]

$$||x|| \leq b||x_{t_0}||_r e^{-\nu(t-t_0)/2} + \gamma\big(\sup_{t_0 \leq \tau \leq t} ||w_\sigma^F(\tau)||\big),\ t \geq t_0,$$

where $b = \sqrt{e^\eta c_2/c_1}$, $\gamma(s) = \sqrt{\frac{c_2}{c_1}} \rho(s)$. As for the upper bound $||z||$, one can follow the same steps in Theorem 1, where $J_i = \int_{t_0}^\infty (z^T z - \gamma_i^2 (w_\sigma^F)^T w_\sigma^F) dt$.

4. Numerical examples

Consider system (2) with $\mathcal{S} = \{1, 2\}$,

$$A_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & -6 \end{bmatrix}, B_1 = \begin{bmatrix} -3 & 1 \\ 0.1 & 0.2 \end{bmatrix}, C_1 = \begin{bmatrix} 2 & 0.1 \\ 0 & 2 \end{bmatrix}, F_1 = \begin{bmatrix} 0.1 & -2 \\ 0.1 & 0 \end{bmatrix},$$

$$\bar{A}_1 = \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 1 \end{bmatrix}, D_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, H_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \bar{D}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \bar{H}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, f_1 = 0.1 \begin{bmatrix} \sin(x_1(t-1)) \\ \sin(x_2(t-1)) \end{bmatrix}, \mathcal{U}_1 = \sin(t),$$

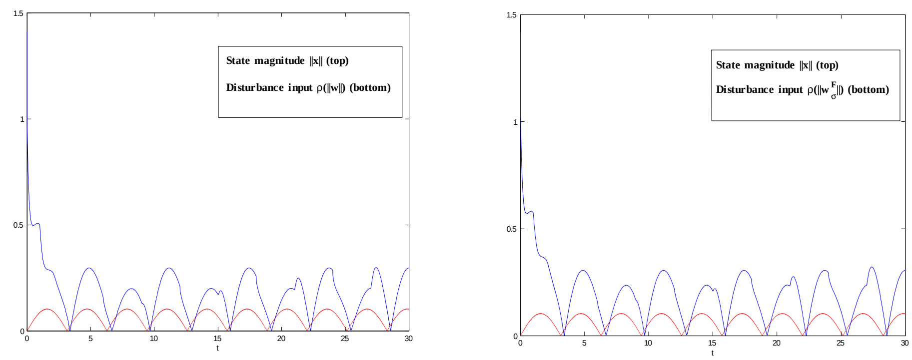
$\epsilon_1 = 2,\ \xi_{11} = 0.2,\ \gamma_1 = 0.1,\ \alpha_1 = 2,\ \xi_{21} = \gamma_1^{-2},\ \xi_{31} = 0.1,\ \xi_{41} = 0.3,\ \xi_{51} = 0.2,\ M_1 = 2,\ \beta = 3,\ \theta_1 = 0.05$, and $\delta_1 = 0.1$. As for the second mode,

$$A_2 = \begin{bmatrix} -9 & 0.2 \\ 0 & 0.1 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 & 0.5 \\ 0.1 & -1 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, F_2 = \begin{bmatrix} 0.1 & 0 \\ -3 & 0.1 \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.1 \end{bmatrix}, D_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, H_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \bar{D}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \bar{H}_2 = \begin{bmatrix} 0 & 1 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}, f_2 = 0.01 \begin{bmatrix} \sin(x_1(t-1)) \\ \sin(x_2(t-1)) \end{bmatrix}, \mathcal{U}_2 = \sin(t),$$

$\epsilon_2 = 0.5,\ \xi_{12} = 0.3,\ \gamma_2 = 0.15,\ \alpha_2 = 2.5,\ \xi_{22} = \gamma_2^{-2},\ \xi_{32} = 0.2,\ \xi_{42} = 0.09,\ \xi_{52} = 0.1,\ M_2 = 1.1,\ \theta_2 = 0.15$, and $\delta_2 = 0.01$. The disturbance $w^T(t) = 1.2[\sin(t)\ \ \sin(t)]$.



(a) Operational actuators. (b) Faulty actuators.

Figure 1. Input-to-statestabilization, $\phi(s) = 1 - s,\ s \in [-1, 0]$.

Case 1. When all actuators are operational, we have $P_1 = \begin{bmatrix} 0.7234 & -0.0157 \\ -0.0157 & 0.5559 \end{bmatrix}$, $P_2 = \begin{bmatrix} 11.6224 & -1.2007 \\ -1.2007 & 10.6159 \end{bmatrix}$, with $c_{11} = \lambda_{\min}(P_1) = 9.8173,\ c_{12} = \lambda_{\max}(P_1) = 12.4211,\ c_{21} = \lambda_{\min}(P_2) = 26.6962,\ c_{22} = \lambda_{\max}(P_2) = 54.1990$, so, $c_1 = 9.8173,\ c_2 = 54.1990$, and $K_1 = \begin{bmatrix} 34.9874 & -4.6636 \\ -11.3823 & -0.9225 \end{bmatrix}$, $K_2 = \begin{bmatrix} -1.2381 & -0.5812 \\ -7.7135 & 7.3350 \end{bmatrix}$. Thus, the matrices $A_i + B_i K_i$ $(i = 1, 2)$ are Hurwitz and $\tau_a = \frac{\ln \mu}{\alpha^* - \nu} = 1.1783$, with $\nu = 0.5$, the upper bound of the disturbance magnitude is 0.1031, and the cheater bound $N_0 = 0.5853$.

Case 2. When there is a failure in the first actuator, i.e., $B_{1\Sigma} = \{1\}$ and $B_{1\bar{\Sigma}} = \begin{bmatrix} 0 & 1 \\ 0 & 0.2 \end{bmatrix}$, and $B_{2\Sigma} = \{2\}$ and $B_{2\bar{\Sigma}} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0 \end{bmatrix}$, we have $P_1 = \begin{bmatrix} 11.7139 & -3.1981 \\ -3.1981 & 11.5155 \end{bmatrix}$, $P_2 = \begin{bmatrix} 53.1251 & -4.8927 \\ -4.8927 & 27.3562 \end{bmatrix}$, with $c_{11} = \lambda_{\min}(P_1) = 8.4151$, $c_{12} = \lambda_{\max}(P_1) = 14.8144$, $c_{21} = 26.4585$, $c_{22} = 54.0228$, so $c_1 = 8.4151$, $c_2 = 54.0228$, and the control gain matrices $K_1 = \begin{bmatrix} 35.4616 & -10.7459 \\ 0 & 0 \end{bmatrix}$, $K_2 = \begin{bmatrix} 0 & 0 \\ -7.8638 & 7.4506 \end{bmatrix}$. Thus, the matrices $A_i + B_iK_i$ $(i = 1, 2)$ are Hurwitz and $\tau_a = 1.2823$, the upper bound of the disturbance magnitude is 0.1033, and the cheater bound $N_0 = 0.5378$.

Figure 1 shows the simulation results of $||x||$ (top) and $\rho(s)$ (bottom) for both cases, where $\rho(s) = \max\{\rho_1(s), \rho_2(s)\}$ and $\rho_i(s) = s/\sqrt{c_2\theta_i\xi_{2i}}$, $\tau_a = 3$. The figure shows the input-to-state stability of the system where the state magnitude $||x||$ is bounded below by the system disturbance magnitude.

5. Conclusion

The system under investigation has been exponentially stabilized by state feedback robust reliable controllers. The Razumikhin technique along with average dwell time approach by multiple Lyapunov functions has been utilized to fulfill our purpose, which implies that the results are delay independent. The output of the faulty actuators has been treated as a disturbing signal that has been augmented with the system disturbance.

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[0]Başlık: Zaman gecikmeli melez sistemler için H_∞ kontrol ve girdi-konum kararlılığı

Anahtar Kelimeler: Çevirmeli sistemler, ISS, güçlü güvenilir H_∞ kontrol, çoklu Lyapunov fonksiyonları, ortalama bekleme zamanı, gecikme