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MODELING OF CAPACITY UTILIZATION RATIO WITH FUZZY TIME SERIES BASED ON MARKOV TRANSITION MATRIX

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ABSTRACT. Modeling of time series with fuzzy logic has found an increasingly expanding usage in recent years. One of the most important reasons for this is that fuzzy logic approach doesn't require assumptions needed by the typical time series. Inclusion of the some weightings and probability calculations at the forecast stage into the first studies starting through the modeling of time series with fuzzy logic resulted in further improvement of the forecast quality. Tsaur (2011) achieved better forecasted results by including the Markov transition probabilities matrix. Fuzzy time series is also an approach which can be flexibly used in various model structures as it easily overcomes the difficulties caused by the model structure - linear or non-linear form. In this study, Markov method of Tsaur is applied on the monthly capacity utilization ratio (CUR) of Turkey which has a non-linear structure and free of seasonality belonging to the period between 2007-2015. In this sense, the results are compared to the results of SETAR model and it's seen that Tsaur's approach has provided better results compared to the forecasts of typical time series.

1. INTRODUCTION

Particularly, in cases of time series not showing a regular pattern, it's quite difficult to create an adequate time series model. In such a case, more prior studies should be conducted: to search for the presence of unit root in the series, to look whether there is structural break, examine stationarity of the series, and to determine the approximate behavior of data etc. This kind of prior studies increases the work load in modeling of time series. While all initial studies increase the work load, it makes the modeling difficult that particularly in cases of data not showing regular behavior pattern, forecasting with typical time series approach require satisfying the assumptions. When time series analysis is performed by using fuzzy logic, the series is analyzed based on the information obtained from the behavior of the relevant data in time. Not requiring assumptions regarding the series and its applicability, in particular, small sample size satisfactorily are the most important advantage provided by the fuzzy time series approach.

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Defined by Zadeh for the first time in 1965, the fuzzy set is the set of elements with continuous membership degrees. Fuzzy set is characterized with the membership function that assigns membership degree to each element between 0-1. It's developed against the binary logic system as "An object is whether the element of the set or not" based on the Aristotle's logic. Fuzzy logic is a logic system that determines at which degree the object is an element of the set by assigning membership degrees to the event of object being an element of a set (Zadeh, 1965). Fuzzy systems generally consist of three basic phases as fuzzylising, fuzzy inference and defuzzifying (Aladag and Turksen, 2015). Fuzzy time series analysis is sort of fuzzy system. Fuzzylising means the assignment of time series observations to the fuzzy sets with certain membership degrees. In other words, it corresponds to the determination of lengths of interval. The phase of fuzzy inference is the determination of fuzzy relationships. Defuzzifying is, on the other hand, a process which converts the fuzzy set or fuzzy number into a crisp number.

Song and Chissom (1991) used a fixed length of interval and fuzzy relationship matrix in their first study in which they proposed the first degree fuzzy time series model. Chen (1996) proposed a simpler fuzzy time series approach for the complicated matrix manipulation of Song and Chissom (1991). The biggest difference between these two studies is that Chen does not use membership degree, and makes forecast by classifying the fuzzy relationships. As this method proposed by Chen has ease of application, it has become one of the most preferred methods in the literature. Although Hwang et al. (1998) divided universe of discourse with a fixed length of interval, he fuzzyfied the first degree difference of the data instead of fuzzifying the data. Huarng (2001) discussed that the first degree difference of the series method gives better forecasts comparing with randomly choosing the length of interval. Tsaur et al. (2005) considered the entropy as fuzziness measure, and achieved fuzzy relationship matrix with the help of entropy. Yu (2005-a) claimed that it will provide better forecasted results to give different weights according to the importance levels instead of taking repeated relationships only once. Yu (2005-b) apply fuzzy correction on Chen's model for adjusting fuzzy relationships to propose a new fuzzy time series model. Thus, he achived better forecasted results than Chen's forecasts by additionally taking into account this fuzzy correction instead of centroid method in defuzzify process. Huarng and Yu (2006) defined the length of interval with percentile values by taking relative differences of observation values. They showed that length of interval based on ratio gives better forecasted results than distribution and average based length of interval. In the study in which Cheng et al. (2008) used adaptive expectation model, after making fuzzy time series forecast with weighted model in Yu (2005), they adjusted the forecasted results with adaptive expectation. Yolcu et al. (2009) used single variable constrained optimization to determine the length of interval, and thus, improved the ratio and average based method of Huarng. The proposed method needs less procedures as it does not require relative difference. Eğrioğlu et al. (2009) offered an approach with higher performance by combining SARIMA models with seasonal fuzzy time series models for the analysis of seasonal fuzzy time series. In this method, they used feed-forward artificial neural network to determine the fuzzy relationships. Aladağ et al. (2009) established fuzzy relationships for higher-degree AR models with feed-forward artificial neural network. Aladag et al. (2010), in another study, tried adaptive expectation in higher-degree fuzzy time series models. Tsaur (2011) used Markov chain transition matrix to achieve fuzzy relationship groups. This method that is applied for first degree fuzzy time series calculated observation frequencies of repeated relationships, and subsequently considered these frequencies as repetition probabilities of the relationships, and created transition matrix from state i to state j. at the final stage he corrects the forecasts through this transition matrix according to the Markov transition process. Thus, he showed superiority to many methods available in the literature in terms of forecasting performance.

This study is interested in modeling of capacity utilization ratio (CUR) data with a length of n=97 for Turkey and between years 2007-2015. The data is seasonally adjusted and has a non-linear behavior. The examination of data shows that while there is a decreasing trend from 1st month of 2007 until 7th month of 2008 with a rapid decrease from 8th month of 2008, the series started to increase after the 4th month of 2009, and stabilize following 10th month of 2010. As the data does not behave in a linear form, it will not be adequate to approach by linear time series models. Self-existing threshold autoregressive (SETAR) would be an appropriate modeling approach for the data with this pattern. Although, a non-linearity is apparently seen in this data, only reviewing the diagram may sometimes cause misinterpretation. Thus, a preliminary study for more accurate decision about non-linearity through a statistical tests would be well. In this study, Tsaur‘s first degree fuzzy time series approach with Markov transition matrix is applied on our data. To evaluate the performance of this approach in modeling the considered data, it also has been analyzed with SETAR model complying with the form of data. As a result of the comparison made, it's seen that first degree fuzzy time series approach of Tsaur (2011) based on Markov transition matrix gives better forecast performance. It stands out as an important finding that fuzzy time series approach based on Markov transition matrix that does not require many prior process and statistical tests as typical approaches produces quite good forecasts also for a real life data.

The following part of the study includes basic definitions and refers to essential characteristics of Tsaur's method. In the 3rd part, analysis of the studied data with Tsaur's method is explained in detail. In the traditional time series analysis of data, firstly, some statistical tests are conducted to see that a SETAR model approach is appropriate. Performance comparison of these two methods is made by calculating MAPE (mean absolute percentage error) values of the forecasts.

2. Tsaur's Markov Model

Before mentioning about Tsaur's model, it will be useful to provide basic definitions regarding first-degree fuzzy time series.

Let U be the universe of discourse, $U = \{u_1, u_2, ..., u_n\}$. A fuzzy set A of $U$is defined by

$$A = {}^{f_A(u_1)}\!/_{u_1} + {}^{f_A(u_2)}\!/_{u_2} + ... + {}^{f_A(u_n)}\!/_{u_n} \tag{1}$$

where f_A is the membership function of A, $f_{A_i} : U \rightarrow [0,1]$, and $f_A(u_i)$ indicates the grade of membership of u_i in A, where $f_A(u_i) \in [0,1]$ and $1 \leq i \leq n$. The basic definitions of fuzzy time series can be summarized as follows (Song and Chissom 1991; Song and Chissom 1993).

Definition 1. *Let $Y_t \in R^1 \quad (t = 0, 1, 2, ...)$ be a time series. If $f_i(t)$ a fuzzy set in Y_t and $F(t) = \{f_1(t), f_2(t), ...\}$, then $F(t)$ is called a fuzzy time series in Y_t.*

Definition 2. *Suppose $F(t)$ is caused by $F(t-1)$ only, i.e., $F(t-1) \rightarrow F(t)$. Then this relation can be expressed as $F(t) = F(t-1) \circ R(t, t-1)$ where $R(t, t-1)$ is a fuzzy relationship, and is called the first-order model of $F(t)$.*

Chen (1996) emphasized the complexity of fuzzy matrix processes in his study and proposed a simple algorithm. In the method, first universe of discourse is defined as to cover all data, and, universe of discourse is divided into equal intervals according to the number of linguistic variables. After the data fuzzyfied with the help of fuzzy sets defined on the universe of discourse, fuzzy logical relationships are created and classification is made according to the left sides of these relationships. Then, the fuzzy forecasts are defuzzyfied with average method.

In Tsaur's (2011) method, in addition to Chen's algorithm, Markov chain transition matrix is used when relationship groups are obtained. This method applied for first degree fuzzy time series considers repeated relationships depending on a probability, and has the superiority in forecasted values over the many methods in the literature by making corrections in the forecasts through the transition matrix. In the study, the well-known data of student enrollment in Alabama University is used. In his method, universe of discourse is defined as to cover all data, and then universe of discourse is divided into equal intervals to create fuzzy sets. Following the fuzzification, the process follows two steps of operations: fuzzy logical relationships are defined, and then Markov transition probabilities matrix is created according these relationships. The forecasts obtained by using Markov transition probabilities matrix are corrected with Markov transition process diagram. This calculation process can be summarized algorithmically as follows:

(1) Define the universe of discourse.
(2) Divide universe of discourse into equal intervals, and define fuzzy sets.
(3) Fuzzify the historical data.
(4) Determine fuzzy logical relationships and groups.

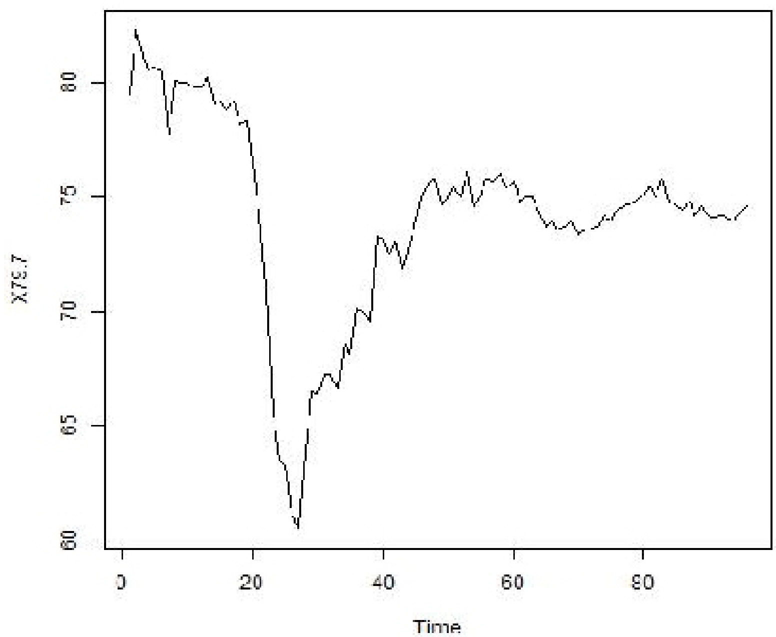

Figure 1. Industry (CUR) and Seasonally Adjusted CUR* (Weighted Average - %.

(5) Establish Markov state transition matrix, and then, draw transition process diagram from fuzzy logical relationships.
(6) Calculate the forecasted output by using Markov state transition matrix.
(7) Adjust the tendency of the forecasted values.

3. Fuzzy Time Series Forecasting of Capacity Utilization Ratio Data

Capacity utilization ratio (CUR) data including 97 observations considered here is published on the website of Central Bank of Republic of Turkey. Non-linear, and interesting structure of the data can be seen in the plot in Figure 1. CUR data shows different behaviors in different period intervals. The estimation process will be conducted at two subsequent stages: forecasting with fuzzy time series method of Tsaur based on Markov matrix, and then SETAR model. Finally, we will evaluate the performances of these two methods are compared via their MAPE's. All calculations are performed in R package program.

For CUR data in Table 1, to calculate first degree fuzzy time series, first frequency of repeated relationships is calculated, and then, these frequencies are employed as repetition probabilities of the relationships. Transition matrix from state i to state$j$ is created by replacing the calculated probabilities. Finally, forecasts made with this matrix are corrected, and, thus, improved by considering the Markov transition process diagram. The required calculations are given through the following steps.

TABLE 1. Utilization Rate of Manufacturing Industry (CUR) and Seasonally Adjusted CUR* (Weighted Average - %)

Years	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2007	79.7	79.5	82.3	81.2	80.6	80.7	80.5	77.8	80.1	80.0	80.0	79.8
2008	79.8	80.2	79.1	79.2	78.9	79.2	78.2	78.3	76.5	74.2	71.0	65.6
2009	63.5	63.3	61.2	60.5	63.9	66.5	66.4	67.2	67.2	66.7	68.7	68.1
2010	70.1	70.0	69.6	73.3	73.1	72.5	73.2	71.9	72.9	74.0	75.0	75.6
2011	75.8	74.7	75.1	75.4	75.0	76.1	74.6	75.2	75.8	75.7	76.0	75.4
2012	75.7	74.8	75.0	75.0	74.3	73.7	74.0	73.6	73.7	74.0	73.4	73.6
2013	73.6	73.7	74.2	74.0	74.5	74.6	74.7	74.9	75.1	75.5	75.0	75.8
2014	74.8	74.7	74.5	74.8	74.2	74.6	74.2	74.1	74.2	74.1	74.0	74.4
2015	74.6											

Step 1. Definition of universe of discourse: The universe of discourse *is* $U = [D_{\min} - D_1, D_{\max} + D_2]$, where $D_{\min} = 60.5$ and $D_{\max} = 82.3$ denote the minimum and maximum values of data set respectively. D_1 and D_2 are selected as to cover all data of universe of discourse of CUR data. Thus, $D_1 = 0.5$ and $D_2 = 0.7$ are arbitrarily selected, and now the universe of discourse can be written as $U = [60, 83]$.

Step 2. Definition of fuzzy sets: Universe of discourse $U = [60, 83]$ is divided into intervals according to the personal experience. Here, class number is supposed 8, and therefore, dividing the universe of discourse into 8 equal intervals. This results in the following sub-intervals:

$$
\begin{aligned}
u_1 &= [60 \ , \ 62.875] \\
u_2 &= [62.875 \ , \ 65.75] \\
u_3 &= [65.75 \ , \ 68.625] \\
u_4 &= [68.625 \ , \ 71.5] \\
u_5 &= [71.5 \ , \ 74.375] \\
u_6 &= [74.375 \ , \ 77.25] \\
u_7 &= [77.25 \ , \ 80.125] \\
u_8 &= [80.125 \ , \ 83]
\end{aligned}
$$

Depending on sub-intervals of universe of discourse U, $A_1, A_2, A_3, ..., A_8$ fuzzy sets can be written as in the following equalities, with membership degrees.

$$
\begin{aligned}
A_1 &= {}^{1}\!/_{u_1} + {}^{0.5}\!/_{u_2} + {}^{0}\!/_{u_3} + {}^{0}\!/_{u_4} + {}^{0}\!/_{u_5} + {}^{0}\!/_{u_6} + {}^{0}\!/_{u_7} + {}^{0}\!/_{u_8} \\
A_2 &= {}^{0.5}\!/_{u_1} + {}^{1}\!/_{u_2} + {}^{0.5}\!/_{u_3} + {}^{0}\!/_{u_4} + {}^{0}\!/_{u_5} + {}^{0}\!/_{u_6} + {}^{0}\!/_{u_7} + {}^{0}\!/_{u_8} \\
A_3 &= {}^{0}\!/_{u_1} + {}^{0.5}\!/_{u_2} + {}^{1}\!/_{u_3} + {}^{0.5}\!/_{u_4} + {}^{0}\!/_{u_5} + {}^{0}\!/_{u_6} + {}^{0}\!/_{u_7} + {}^{0}\!/_{u_8} \\
A_4 &= {}^{0}\!/_{u_1} + {}^{0}\!/_{u_2} + {}^{0.5}\!/_{u_3} + {}^{1}\!/_{u_4} + {}^{0.5}\!/_{u_5} + {}^{0}\!/_{u_6} + {}^{0}\!/_{u_7} + {}^{0}\!/_{u_8} \\
A_5 &= {}^{0}\!/_{u_1} + {}^{0}\!/_{u_2} + {}^{0}\!/_{u_3} + {}^{0.5}\!/_{u_4} + {}^{1}\!/_{u_5} + {}^{0.5}\!/_{u_6} + {}^{0}\!/_{u_7} + {}^{0}\!/_{u_8} \\
A_6 &= {}^{0}\!/_{u_1} + {}^{0}\!/_{u_2} + {}^{0}\!/_{u_3} + {}^{0}\!/_{u_4} + {}^{0.5}\!/_{u_5} + {}^{1}\!/_{u_6} + {}^{0.5}\!/_{u_7} + {}^{0}\!/_{u_8} \\
A_7 &= {}^{0}\!/_{u_1} + {}^{0}\!/_{u_2} + {}^{0}\!/_{u_3} + {}^{0}\!/_{u_4} + {}^{0}\!/_{u_5} + {}^{0.5}\!/_{u_6} + {}^{1}\!/_{u_7} + {}^{0.5}\!/_{u_8} \\
A_8 &= {}^{0}\!/_{u_1} + {}^{0}\!/_{u_2} + {}^{0}\!/_{u_3} + {}^{0}\!/_{u_4} + {}^{0}\!/_{u_5} + {}^{0}\!/_{u_6} + {}^{0.5}\!/_{u_7} + {}^{1}\!/_{u_8}
\end{aligned}
$$

Here, u_i in the denominator of fuzzy sets A_i are sub-intervals, while numbers in nominator denote membership degrees of u_i to A_i, subject to $0 \leq u_i \leq 1$.

Step 3. Fuzzification of data: CUR data should be fuzzyfied according to the biggest membership degree. If the highest membership degree of data appears in fuzzy setA_k, the corresponding data is fuzzyfied asA_k. For example, the data of January 2007, (79.7) is a value falling into sub-interval u_7. As the highest membership degree of the sub-interval $u_7$is in A_7, this value is fuzzyfied as A_7. Set of values that are fuzzyfied in similar manner are given in Table 2.

TABLE 2. Fuzzyfied Utilization Rate of Manufacturing Industry (CUR) data

Years	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2007	79.7	79.5	82.3	81.2	80.6	80.7	80.5	77.8	80.1	80.0	80.0	79.8
Fuzzyfied	A_7	A_7	A_8	A_8	A_8	A_8	A_8	A_7	A_7	A_7	A_7	A_7
2008	79.8	80.2	79.1	79.2	78.9	79.2	78.2	78.3	76.5	74.2	71.0	65.6
Fuzzyfied	A_7	A_8	A_7	A_7	A_7	A_7	A_7	A_7	A_6	A_5	A_4	A_2
2009	63.5	63.3	61.2	60.5	63.9	66.5	66.4	67.2	67.2	66.7	68.7	68.1
Fuzzyfied	A_2	A_2	A_1	A_1	A_2	A_3	A_3	A_3	A_3	A_3	A_4	A_3
2010	70.1	70.0	69.6	73.3	73.1	72.5	73.2	71.9	72.9	74.0	75.0	75.6
Fuzzyfied	A_4	A_4	A_4	A_5	A_5	A_5	A_5	A_5	A_5	A_5	A_6	A_6
2011	75.8	74.7	75.1	75.4	75.0	76.1	74.6	75.2	75.8	75.7	76.0	75.4
Fuzzyfied	A_6	A_6	A_6	A_6	A_6	A_6	A_6	A_6	A_6	A_6	A_6	A_6
2012	75.7	74.8	75.0	75.0	74.3	73.7	74.0	73.6	73.7	74.0	73.4	73.6
Fuzzyfied	A_6	A_6	A_6	A_6	A_5	A_5	A_5	A_5	A_5	A_5	A_5	A_5
2013	73.6	73.7	74.2	74.0	74.5	74.6	74.7	74.9	75.1	75.5	75.0	75.8
Fuzzyfied	A_5	A_5	A_6	A_5	A_6	A_6	A_6	A_6	A_6	A_6	A_5	A_6
2014	74.8	74.7	74.5	74.8	74.2	74.6	74.2	74.1	74.2	74.1	74.0	74.4
Fuzzyfied	A_6	A_6	A_6	A_6	A_5	A_6	A_5	A_5	A_5	A_5	A_5	A_6
2015	74.6											
Fuzzyfied	A_6											

Step 4. Definition of fuzzy logic relationships: In this step, fuzzy logical relationships are defined between the fuzzyfied data, and then, fuzzy relationship groups are formed. Classification process is performed based on the current status of the fuzzy logical relationships. All defined fuzzy relationship groups are listed in Table 3.

Step 5. Establish Markov state transition matrix and transition process: Transition probabilities matrix P is obtained by using from the fuzzy relationships in

TABLE 3. Fuzzy Relationship Groups

$$A_1 \rightarrow A_1, A_2$$
$$A_2 \rightarrow A_1, A_2, A_2, A_3$$
$$A_3 \rightarrow A_3, A_3, A_3, A_3, A_3, A_3, A_3, A_3, A_3, A_5$$
$$A_4 \rightarrow A_2, A_5$$
$$A_5 \rightarrow A_4, A_4, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_5, A_6, A_6, A_6, A_6$$

Step 4. Defining n states (8 states) for each one of the fuzzy sets, nxn (8x8) dimensional matrix is produced. State transition probabilities P_{ij}, from state A_i to state A_j in one step, are calculated with equality

$$P_{ij} = \frac{M_{ij}}{M_i} \quad i, j = 1, 2, ..., n$$

Here, M_{ij} and M_i denote transition time in one step from state A_i to stateA_j, and data amount in state A_i respectively. Thus, Markov transition probabilities matrix P_{ij} becomes

$$P_{ij} = \begin{bmatrix} P_{11} & P_{12} & ... & ... & P_{1n} \\ P_{21} & P_{22} & ... & ... & P_{2n} \\ ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... \\ P_{n1} & P_{n2} & ... & ... & P_{nn} \end{bmatrix}$$

For CUR data, transition probabilities are given below.

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/4 & 2/4 & 1/4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 1/5 & 1/5 & 2/5 & 1/5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/26 & 21/26 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4/33 & 29/33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/14 & 11/14 & 2/14 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 2/3 \end{bmatrix}$$

Markov matrix given above will be used to create Markov transition probabilities diagram in Figure 3.

Step 6. Calculation of forecasted values: Fuzzy forecasting is conducted regarding two cases: one-to-one and one-to-many. Following rules given by Tsaur (2011) are taken into account in these calculations.

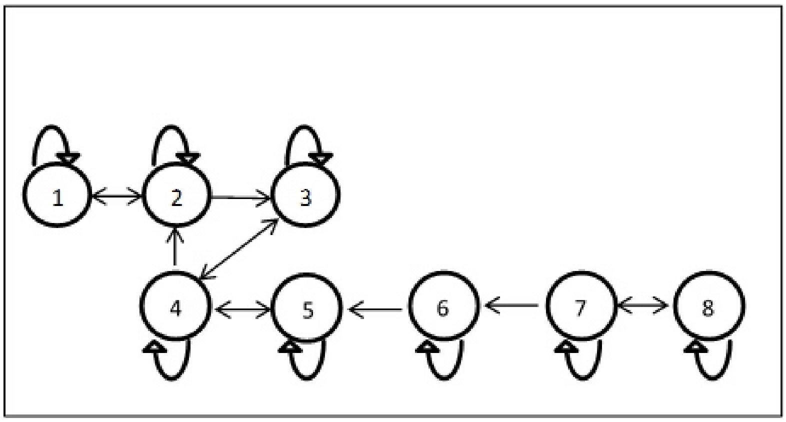

FIGURE 2. Transition process

- One-to-one: If the fuzzy logical relationship group of A_i is one-to-one (i.e., $A_i \to A_k$, with $P_{ik} = 1$ and $P_{ij} = 0$, $j \neq k$), then the forecasting of $F(t)$ is m_k, the midpoint of u_k, according to the equation $F(t) = m_k P_{ik} = m_k$.
- One-to-many: If the fuzzy logical relationship group of A_j is one-to-many (i.e., $A_i \to A_1, A_2, ...A_n, j = 1, 2, ..., n$), when collected data $Y(t-1)$ at time $(t-1)$ is in the state A_j, then the forecasting of $F(t)$ is equal to $F(t) = m_1 P_{j1} + m_2 P_{j2} + ... + m_{j-1} P_{j(j-1)} + Y(t-1) P_{jj} + m_{j+1} P_{j(j+1)} + ... + m_n P_{jn}$, where $m_1, m_2, ..., m_{j-1}, m_j, ..., m_n$ are the midpoint of $u_1, u_2, ..., u_{j-1}$, $u_j, ..., u_n$, and m_j is substituted for $Y(t-1)$ in order to have more information from the state A_j at time $(t-1)$.

For example, the forecasted value for 2nd month in 2007 is calculated as follows:

$$F(2007:2) = \frac{1}{14} m_6 + \frac{11}{14} Y(2007:1) + \frac{2}{14} m_8$$

$$F(2007:2) = \frac{1}{14} 75.8125 + \frac{11}{14} 79.7 + \frac{2}{14} 81.5625 = 79.68839 \approx 79.7$$

Step 7. Adjusting forecasted values: For series with small sample size estimated Markov chain matrix is usually biased, and some adjustments for the forecasts are suggested to revise the forecasting errors for one-to-many cases. First, in a fuzzy logical group where A_i communicates with A_i and A_j for $i \neq j$,$j = 1, 2, ..., n$. If a larger state A_j is accessible from state A_i, then the forecasting value for A_j is usually underestimated because the lower state values are used for forecasting the value of A_j. On the other hand, an overestimated value should be adjusted for the forecasting value A_j because a smaller state A_j is accessible from A_i, $i, j = 1, 2, ..., n$. If the data occurs in the state A_i, and then jumps forward to state A_{i+k} $(k \geq 2)$ or jumps backward to state A_{i-k} $(k \geq 2)$, then it is necessary to adjust the trend of the pre-obtained forecasting value. Thus, we have smoother values of forecasting.

Corrections are made by taking into account some of the following rules. To understand the rules better, first of all necessary definitions are given.

Before giving the correction rules suggested by Tsaur (2011), let us give to necessary definitions.

Definition 3. *If $P_{ij} > 0$, then state A_j is accessible fromA_i, $A_i \rightarrow A_j$.*

Definition 4. *If states A_i and A_j are accessible to each other, then A_i communicates withA_j, $A_i \leftrightarrow A_j$.*

Now, the corrections are made by taking into account the following rules.

- If $A_i \leftrightarrow A_j$, starting in state A_i at time $(t-1)$ as $F(t-1) = A_i$, and makes an increasing transition into state A_j at time t, $(i < j)$, then the adjusted trend value D_t is defined as $D_{t1} = \left({}^{l}\!/_{2} \right)$.
- If $A_i \leftrightarrow A_j$, starting in state A_i at time $(t-1)$ as $F(t-1) = A_i$, and makes an decreasing transition into state A_j at time t, $(i > j)$, then the adjusted D_t is defined as $D_{t1} = -\left({}^{l}\!/_{2} \right)$.
- If the current state is in the state A_i at time $(t-1)$ as $F(t-1) = A_i$, and makes a jump forward transition into state A_{i+s} at time t, $(1 \leq s \leq n-i)$, adjusted D_t is defined as $D_{t2} = \left({}^{l}\!/_{2} \right) s$, $(1 \leq s \leq n-i)$, where l is the length that the universal discourse U must be partitioned into as n equal intervals.
- If the process is defined to be in state A_i at time $(t-1)$ as $F(t-1) = A_i$, and makes a jump-backward transition into state A_{i-v} at time t, $(1 \leq v \leq i)$, the adjusted D_t is defined as $D_{t2} = -\left({}^{l}\!/_{2} \right) v$, $(1 \leq v \leq i)$.

The original data and the adjusted forecasts calculated by the procedure explained above are given in Table 3. In the table, original data are given in the first line and corresponding adjusted forecast values (AV) are given in the following line.

As can be seen in Figure 2, the fuzzy adjusted forecasting values are very close to the real values.

MAPE (Mean Absolute Percentage Error) is used to evaluate the forecast values. $MAPE = \left(\frac{1}{M} \sum_{l=1}^{M} \left| \frac{e_l}{Z_{n+l}} \right| \right) 100\%$ value is calculated for obtained forecast values and achieved as $MAPE = 0.949$.

Although it is seen that CUR values do not show a linear structure in Figure 1, we can use objective approaches testing whether the data is non-linear or not. In this sense, results in Table 4 are obtained by using the tests known as Tsay, Keenan and Jarque Bera. tsDyn package in R is used for non-linearity tests and SETAR model forecasts.

TABLE 4. Fuzzy Adjusted Forecasting Values of CUR

Years	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2007	79.7	79.5	82.3	81.2	80.6	80.7	80.5	77.8	80.1	80.0	80.0	79.8
AV	**-**	**79.7**	**79.5**	**81.1**	**80.4**	**79.9**	**80**	**79.9**	**78.2**	**80**	**79.9**	**79.9**
2008	79.8	80.2	79.1	79.2	78.9	79.2	78.2	78.3	76.5	74.2	71.0	65.6
AV	**79.8**	**79.8**	**79.7**	**79.2**	**79.3**	**79.1**	**79.3**	**78.5**	**78.6**	**76.1**	**74.3**	**69.3**
2009	63.5	63.3	61.2	60.5	63.9	66.5	66.4	67.2	67.2	66.7	68.7	68.1
AV	**64.9**	**63.9**	**63.8**	**62.8**	**62.4**	**64.1**	**67.7**	**67.6**	**68.2**	**68.1**	**67.8**	**68.4**
2010	70.1	70.0	69.6	73.3	73.1	72.5	73.2	71.9	72.9	74.0	75.0	75.6
AV	**68.8**	**68.9**	**68.9**	**68.7**	**73.6**	**73.4**	**72.9**	**73.5**	**72.4**	**73.2**	**74.1**	**74.7**
2011	75.8	74.7	75.1	75.4	75.0	76.1	74.6	75.2	75.8	75.7	76.0	75.4
AV	**75.3**	**75.5**	**74.5**	**74.8**	**75.1**	**74.8**	**75.7**	**74.4**	**74.9**	**75.5**	**75.4**	**75.6**
2012	75.7	74.8	75.0	75.0	74.3	73.7	74.0	73.6	73.7	74.0	73.4	73.6
AV	**75.1**	**75.4**	**74.6**	**74.6**	**74.8**	**74.4**	**73.9**	**74.1**	**73.8**	**73.9**	**74.1**	**73.6**
2013	73.6	73.7	74.2	74.0	74.5	74.6	74.7	74.9	75.1	75.5	75.0	75.8
AV	**73.8**	**73.8**	**73.9**	**74.3**	**74.1**	**74.3**	**74.4**	**74.5**	**74.7**	**74.8**	**75.2**	**74.8**
2014	74.8	74.7	74.5	74.8	74.2	74.6	74.2	74.1	74.2	74.1	74.0	74.4
AV	**75.5**	**74.6**	**74.5**	**74.3**	**74.6**	**74.3**	**74.4**	**74.3**	**74.2**	**74.3**	**74.2**	**74.1**
2015	74.6											
AV	**74.2**											

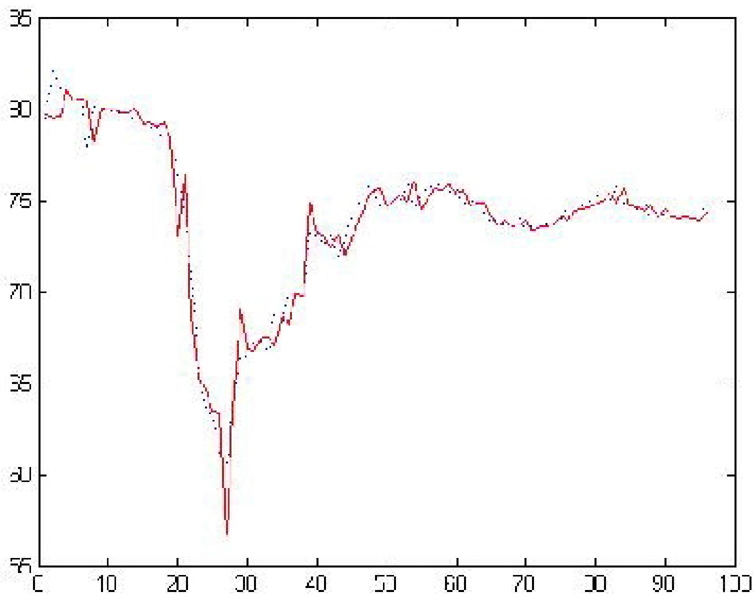

FIGURE 3. **CUR and adjusted forecast values**

Table 5. Outcome for Non-Linearity Tests

	Critical-value	**p-value**
Tsay	4.68	0.0000
Keenan	7.26	0.0084
Jarque Bera	19.54	0.0000

Since p-values are < 0.05 for all tests, it can be interpreted that the series does not exhibit a linear structure.

tsDyn package has found the most suitable SETAR structure of data as 2-regime SETAR (2,2,2) model with the threshold value 79.1. Thus, 85.11% of 97 data is assigned to low regime and 14.89% is assigned to high regime, and SETAR(2,2,2) model with two regimes can be written as follows:

$$Y_t = \left\{ \begin{array}{l} 6.0314 + 1.2554\mathrm{Y}_{\text{t-1}} - 0.3388Y_{t-2} + \varepsilon_t \leq 79.1 \\ 36.4197 + 0.3542Y_{t-1} + 0.1872Y_{t-2} + \varepsilon_t > 79.1 \end{array} \right\} \quad (2)$$

For forecast values through this model, $MAPE$ is 1.066%. It's concluded that MAPE value (0.949%) obtained with first degree fuzzy time series analysis based on Markov transition probabilities matrix is better.

4. Conclusion

There are relatively easy and well serving approaches for obtaining linear time series forecasting models, but it is not easy to tell the same for nonlinear time series. When it is desired to model the non-linear series with typical time series approaches, there occurs many test procedures and calculation loads. It is also encountered with conflicting comments in the literature for non-linear methods. Chen's (1996) fuzzy time series method, suggested as an alternative to clasical time series anaylsis, shows often good performance in linear time series. CUR data considered in the study has not a linear structure and even, if we do not include its result in this study, method of Chen (1996) for forecast of the capacity utilization ratios has shown worse performance. Since Tsaur (2011) included Markov transition probabilities matrix into the method of Chen (1996), by taking into account the repetition of relationships, better forecasted values are achieved by weighting the fuzzy sets in a sense. Monthly seasonally adjusted CUR values of Turkey belonging to years 2007-2015 show quite different behavior pattern by periods. Therefore, nonlinearity tests performed regarding the CUR series have supported the presence of a non-linearity with two regime behavior model. Thus, this series with irregular behavior is modeled with first degree fuzzy time series approach of Tsaur based on Markov matrix that is quite flexible and easy to apply. Through this method, from the point of information of the repeated relationships in CUR values similar to Chen (1996), forecast at time t is made considering time *t-1* and also with the inclusion of Markov matrix given in Tsaur (2011) achieved better result. As a final point, the problem of encountering with biased forecast in sudden decrease and increase in the data is achieved with adjusting forecasts. As a forecast performance of Tsaur's method in modeling CUR data, MAPE=0.949 is found satisfactorily a low value.

CUR series displaying non-linear behavior is then modeled with SETAR to compare with the performance of Tsaur (2011) approach. 2-regime SETAR(2,2,2) model which includes 1 threshold is determined as a suitable model. Forecast performance of this model is again evaluated as MAPE and it's found as MAPE=1.066

Fuzzy time series analysis approach based on Markov matrix of Tsaur (2011) does not require a number of prior and diagnostic tests whereas typical time series modelling does. The method acts according to the behavior of the data. Considering all these facts, fuzzy time series analysis method proposed by Tsaur can be seen as an alternative to determine behavior of non-linear time series with the advantages of easy-to-use.

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