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MODIFIED UNSCENTED KALMAN FILTER FOR NONLINEAR SYSTEMS HAVING LINEAR SUBSYSTEMS

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ABSTRACT. The Extended Kalman Filter (EKF) is the often used filtering algorithm for nonlinear systems. But it does not usually produce desirable results. Recently a new nonlinear filtering algorithm named as Unscented Kalman Filter (UKF) is introduced. In this paper, we propose a new modified Unscented Kalman Filter (MUKF) algorithm for nonlinear stochastic systems that are linear in some components. These nonlinear systems can be considered as having linear subsystems with parameters and aim is to estimate the system parameters. In simulation study, performance of the EKF, its known variant Modified Extended Kalman Filter (MEKF), UKF and the proposed MUKF is demonstrated for a nonlinear system that is linear in some components. The results show that MUKF gives the best solution for parameter identification problem.

1. INTRODUCTION

Discrete-time filtering for nonlinear dynamic system is an important research area and attracted considerable interest [1]. The most common way of applying the Kalman Filter to a nonlinear system is in the form of the Extended Kalman Filter (EKF). EKF is based on linearization of the state equations at each time step and on the use of linear estimation theory [2]. However, it has two known drawbacks: (1) the first-order linearization can introduce large errors in mean and covairance of the state vector and (2) the derivation of Jacobian matrices is nontrivial in many applications [2].

Recently, a relatively new nonlinear filtering algorithm named Unscented Kalman Filter (UKF) is proposed as an improvement to EKF [3]. UKF is based on the unscented transformation, which uses a set of appropriately chosen weighted sigma points to estimate the means and covariances of probability distributions. It is not necessary to calculate Jacobians and so the algorithm has superior implementation properties to the EKF [4].

The UKF is widely used in practice: target tracking [3], position determination [5], multi-sensor fusion [6] and training of neural networks [7].

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In [8] it is shown that for a nonlinear system that is linear in some components, a modification of the EKF improves the filter performance. This filter has two parallel algorithms and the modification is achieved by an improved linearization. Algorithm I is a modification of the EKF, in which the real-time linear Taylor approximation is taken at the optimal state estimate which is given by the standard KF of the linear subsystem from Algorithm II. The standard KF is obtained by plugging parameter values estimated by Algorithm I. The MUKF is motivated by the MEKF. In this paper we want to investigate whether similar modification of the UKF improves the filter performance or not.

This paper is organized as follows. In Section 2 nonlinear state-space models are described and brief summary of the UKF algorithm is given. In Section 3 MUKF procedure is introduced. In Section 4 the performance of the EKF, MEKF, UKF and the proposed MUKF is analyzed with a simulation example. Section 5 is the conclusion.

2. UNSCENTED KALMAN FILTER

Consider the following nonlinear discrete-time stochastic system

$$\begin{aligned} x(k) &= f\big(x(k-1)\big) + w(k) \\ z(k) &= h\big(x(k)\big) + v(k) \end{aligned} \tag{1}$$

where $x(k)$ $(n-\text{vector})$ and $z(k)$ $(m-\text{vector})$ denote the state and measurement vectors at time instant k, $w(k)$ and $v(k)$ are uncorrelated zero-mean Gaussian white noise processes with covariance

$$E\big(w(k)w^T(k)\big) = Q(k),\, E\big(v(k)v^T(k)\big) = R(k). \tag{2}$$

UKF is using a minimal set of determinate sample points (sigma points) to completely capture the true mean and covariance of the states via Unscented Transformation (UT). UKF equations are summarized as follows [9]:

A1. Given the state estimate $\hat{x}(k-1)$ and the error covariance matrix $P(k-1)$, the sigma points are formed by

$$\begin{cases} \chi_i(k-1) = \hat{x}(k-1)\;\;,\;\; i=0 \\ \chi_i(k-1) = \hat{x}(k-1) + a\left(\sqrt{nP(k-1)}\right)_i \;,\;\; i=1,\dots,n \\ \chi_i(k-1) = \hat{x}(k-1) - a\left(\sqrt{nP(k-1)}\right)_i \;\;,\;\; i=n+1,\dots,2n \end{cases} \tag{3}$$

where a determines the spread of the sigma points around $\hat{x}(k-1)$ and usually set to a small positive value. $\left(\sqrt{nP(k-1)}\right)_i$ is the $i-$ th row or column of the matrix square root of $nP(k-1)$.

A2. Prediction: These sigma points are instantiated through the process model to yield a set of transformed samples

$$\chi_i(k|k-1) = f(\chi_i(k-1)) \tag{4}$$

The predicted mean and covariance are computed by

$$\hat{x}(k|k-1) = \sum_{i=0}^{2n} w_i \, \chi_i(k|k-1) \tag{5}$$

$$P(k|k-1) = \sum_{i=0}^{2n} w_i \left(\chi_i(k|k-1) - \hat{x}(k|k-1)\right) \times \left(\chi_i(k|k-1) - \hat{x}(k|k-1)\right)^T + Q(k) \tag{6}$$

with weights $w_0, w_1, \dots, w_{2n} \in R^{2n+1}$ satisfying $\sum_{i=0}^{2n} w_i = 1$ given by

$$\begin{cases} w_i = 1 - \frac{1}{a^2} \quad , \quad i = 0 \\ w_i = \frac{1}{2na^2} \quad , \quad i = 1, \dots, 2n \end{cases} . \tag{7}$$

A3. Update: Given the weighted mean of these transformed sigma points $\hat{x}(k|k-1)$ and the prediction covariance matrix $P(k|k-1)$, the new sigma points $\chi_i'(k|k-1)$ are computed as

$$\begin{cases} \chi_i'(k|k-1) = \hat{x}(k|k-1) \ , \ i = 0 \\ \chi_i'(k|k-1) = \hat{x}(k|k-1) + a\left(\sqrt{nP(k|k-1)}\right)_i \quad , i = 1,2, \dots, n \\ \chi_i'(k|k-1) = \hat{x}(k|k-1) - a\left(\sqrt{nP(k|k-1)}\right)_{i-n} \quad , i = n+1, n+2, \dots, 2n \end{cases} \tag{8}$$

The sigma points for the measurements are

$$\mathcal{Z}_i(k) = h\left(\chi_i'(k|k-1)\right). \tag{9}$$

The weighted mean and covariance matrix of the predicted observation is given by

$$\hat{z}(k) = \sum_{i=0}^{2n} w_i \, \mathcal{Z}_i(k) \tag{10}$$

$$P_{zz}(k) = \sum_{i=0}^{2n} w_i \left(\mathcal{Z}_i(k) - \hat{z}(k)\right)\left(\mathcal{Z}_i(k) - \hat{z}(k)\right)^T + R(k) \tag{11}$$

and the covariance matrix between the state and the measurement is computed as follows

$$P_{xz}(k) = \sum_{i=0}^{2n} w_i \left(\chi_i(k|k-1) - \hat{x}(k|k-1)\right) \times \left(\mathcal{Z}_i(k) - \hat{z}(k)\right)^T. \tag{12}$$

Then the state estimate $\hat{x}(k)$ and the corresponding covariance matrix $P(k)$ can be updated by

$$\hat{x}(k) = \hat{x}(k|k-1) + P_{xz}(k)P_{zz}^{-1}(k)\big(z(k) - \hat{z}(k)\big) \quad (13)$$

$$P(k) = P(k|k-1) - P_{xz}(k)P_{zz}^{-1}(k)P'_{xz}(k)\ . \quad (14)$$

A4. Repeat steps 1 to 3 for the next sample.

3. MODIFIED UKF

In this section, with motivation to improve the performance of the UKF for the special form of nonlinear systems having linear subsystems, we will describe a modification of the UKF similar to modification of the EKF which was recommended in [8]. It uses two parallel algorithms (Algorithm I and II). The nonlinear model assumed by MUKF is described as follows. Let $x(k)$ and $y(k)$ be $n-$ vector and $m-$ vector, and the state vector of the system be the $(n+m)-$ vector $[x^T(k)\ \ y^T(k)]^T$ such that it satisfies

$$\begin{aligned} \begin{bmatrix} x(k+1) \\ y(k+1) \end{bmatrix} &= \begin{bmatrix} F_k\big(y(k)\big)x(k) \\ G_k\big(x(k), y(k)\big) \end{bmatrix} + \begin{bmatrix} \xi_1(k) \\ \xi_2(k) \end{bmatrix} \\ z(k) &= \begin{bmatrix} H_k\big(x(k), y(k)\big) & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ y(k) \end{bmatrix} + \eta(k). \end{aligned} \quad (15)$$

$[\xi_1^T(k)\ \ \ \xi_2^T(k)]^T$ and $\eta(k)$ are uncorrelated zero-mean Gaussian white noise sequences with variance matrices

$$Q(k) = Var\left(\begin{bmatrix} \xi_1(k) \\ \xi_2(k) \end{bmatrix}\right), \quad R(k) = Var\big(\eta(k)\big) \quad (16)$$

respectively. F_k, G_k, H_k are nonlinear matrix valued functions.

With motivation to improve the performance, in Algorithm I, the sigma points are evaluated at the optimal state estimation $\hat{x}(k-1)$ which is determined by the standard KF (Algorithm II) of the subsystem

$$\begin{aligned} x(k+1) &= F_k\big(\tilde{y}(k)\big)x(k) + \xi^1(k) \\ z(k) &= H_k\big(\tilde{x}(k), \tilde{y}(k)\big)x(k) + \eta(k) \end{aligned} \quad (17)$$

of (15) evaluated at the estimate $\big(\tilde{x}(k), \tilde{y}(k)\big)$ from Algorithm I. Two algorithms are applied in parallel starting with the same initial estimate. Algorithm I is used yielding the estimate $[\tilde{x}^T(k)\ \ \tilde{y}^T(k)]^T$ with the input $\hat{x}(k-1)$ obtained from Algorithm II (Standard KF for the linear system) and Algorithm II is used for yielding the estimate $\hat{x}(k)$ with the inputs $\tilde{y}(k-1)$ and $[\tilde{x}^T(k|k-1)\ \ \tilde{y}^T(k|k-1)]^T$ obtained from Algorithm I. Algorithm I and Algorithm II are given below.

Algorithm I.

$$\begin{bmatrix}\tilde{x}(0)\\ \tilde{y}(0)\end{bmatrix} = \begin{bmatrix}E\big(x(0)\big)\\ E\big(y(0)\big)\end{bmatrix}, P(0) = Var\left(\begin{bmatrix}x(0)\\ y(0)\end{bmatrix}\right)$$

A1. The sigma points are formed by

$$\begin{cases} \chi_i(\mathrm{k}-1) = \begin{bmatrix}\hat{\mathrm{x}}(\mathrm{k}-1)\\ \tilde{\mathrm{y}}(\mathrm{k}-1)\end{bmatrix} \quad , \qquad i = 0 \\ \chi_i(\mathrm{k}-1) = \begin{bmatrix}\hat{\mathrm{x}}(\mathrm{k}-1)\\ \tilde{\mathrm{y}}(\mathrm{k}-1)\end{bmatrix} + a\left(\sqrt{(n+m)P(k-1)}\right)_i \quad , i = 1,2,\dots,n+m \\ \chi_i(\mathrm{k}-1) = \begin{bmatrix}\hat{\mathrm{x}}(\mathrm{k}-1)\\ \tilde{\mathrm{y}}(\mathrm{k}-1)\end{bmatrix} - a\left(\sqrt{(n+m)P(k-1)}\right)_{i-n+m} \qquad i = n+m+1,\dots,2(n+m) \end{cases} \tag{18}$$

A2.

$$\chi_i(\mathrm{k}|\mathrm{k}-1) = \begin{bmatrix}\mathrm{F}_{\mathrm{k}-1}([\chi_i(\mathrm{k}-1)]_2)[\chi_i(\mathrm{k}-1)]_1\\ \mathrm{G}_{\mathrm{k}-1}\big(\chi_i(\mathrm{k}-1)\big)\end{bmatrix}$$

$$\begin{bmatrix}\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1)\\ \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\end{bmatrix} = \sum_{\mathrm{i}=0}^{2(n+m)} \mathrm{W_i}\,\chi_i(\mathrm{k}|\mathrm{k}-1) \tag{19}$$

$$P(\mathrm{k}|\mathrm{k}-1) = \sum_{\mathrm{i}=0}^{2(n+m)} \mathrm{W_i}\left[\chi_i(\mathrm{k}|\mathrm{k}-1) - \begin{bmatrix}\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1)\\ \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\end{bmatrix}\right] \times \left[\chi_i(\mathrm{k}|\mathrm{k}-1) - \begin{bmatrix}\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1)\\ \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\end{bmatrix}\right]^T + Q(k)$$

where $[\chi_i(.)]_1$ is part of the vector related to $\hat{\mathrm{x}}(.)$, and $[\chi_i(.)]_2$ is part of the vector related to $\tilde{\mathrm{y}}(.)$.

A3.

$$\begin{cases} \chi_i'(\mathrm{k}|\mathrm{k}-1) = \begin{bmatrix}\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1)\\ \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\end{bmatrix} \quad , \qquad i = 0 \\ \chi_i'(\mathrm{k}|\mathrm{k}-1) = \begin{bmatrix}\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1)\\ \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\end{bmatrix} + a\left(\sqrt{(n+m)P(k|k-1)}\right)_i \quad , i = 1,2,\dots,n+m \\ \chi_i'(\mathrm{k}|\mathrm{k}-1) = \begin{bmatrix}\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1)\\ \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\end{bmatrix} - a\left(\sqrt{(n+m)P(k|k-1)}\right)_{i-n+m} \quad , i = n+m+1,\dots,2(n+m) \end{cases}$$

$$\mathcal{Z}_i(k) = H_k\big(\chi_i'(\mathrm{k}|\mathrm{k}-1)\big)$$

$$\hat{z}(k) = \sum_{\mathrm{i}=0}^{2(n+m)} \mathrm{W_i}\mathcal{Z}_i(k)$$

$$P_{zz}(k) = \sum_{\mathrm{i}=0}^{2(n+m)} \mathrm{W_i}\left(Z_i(k) - \hat{z}(k)\right)\left(Z_i(k) - \hat{z}(k)\right)^T + R(k) \tag{20}$$

$$P_{xz}(k) = \sum_{\mathrm{i}=0}^{2(n+m)} \mathrm{W_i}\left(\chi_i(\mathrm{k}|\mathrm{k}-1) - \begin{bmatrix}\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1)\\ \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\end{bmatrix}\right) \times \left(z(k) - \hat{z}(k)\right)^T$$

$$\begin{bmatrix}\tilde{\mathrm{x}}(\mathrm{k})\\ \tilde{\mathrm{y}}(\mathrm{k})\end{bmatrix} = \begin{bmatrix}\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1)\\ \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\end{bmatrix} + P_{xz}(k)P_{zz}^{-1}(k)\left(z(k) - \hat{z}(k)\right)$$

$$P(k) = P(k|k-1) - P_{xz}(k)P_{zz}^{-1}(k)P_{xz}^T(k)$$

Algorithm II.

$$\hat{x}(0) = E\left(x(0)\right), P(0) = Var\left(x(0)\right)$$

$$P(\mathrm{k}|\mathrm{k}-1) = \left[F_{k-1}\left(\tilde{\mathrm{y}}(\mathrm{k}-1)\right)\right]P(k-1) \times \left[F_{k-1}\left(\tilde{\mathrm{y}}(\mathrm{k}-1)\right)\right]^T + Q(k-1)$$

$$\hat{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1) = F_{k-1}\left(\tilde{\mathrm{y}}(\mathrm{k}-1)\right)\hat{\mathrm{x}}(\mathrm{k}-1) \tag{21}$$

$$K(k) = P(\mathrm{k}|\mathrm{k}-1)\left[H_k\left(\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1), \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\right)\right]^T \times \left\{\left[H_k\left(\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1), \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\right)\right]P(\mathrm{k}|\mathrm{k}-1) + R_k\right\}^{-1}$$

$$P(k) = \left\{I - K(k)\left[H_k\left(\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1), \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\right)\right]\right\} \times P(\mathrm{k}|\mathrm{k}-1)$$

$$\hat{\mathrm{x}}(\mathrm{k}) = \hat{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1) + K(k) \times \left\{z(k) - \left[H_k\left(\tilde{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1), \tilde{\mathrm{y}}(\mathrm{k}|\mathrm{k}-1)\right)\hat{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1)\right]\right\}, k = 1,2, \ldots$$

4. SIMULATION STUDY

In this section we present some simulation results to show numerically the performance differences of the EKF, MEKF, UKF and proposed MUKF. Let consider the state-space model given by

$$\begin{bmatrix}x_1(k+1)\\ x_2(k+1)\\ x_3(k+1)\end{bmatrix} = \begin{bmatrix}1 - 0.1x_3(k) & 0.1 & 0\\ -0.1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1(k)\\ x_2(k)\\ x_3(k)\end{bmatrix} + \xi(k) \tag{22}$$

$$z(k) = [1 \quad 0 \quad 0]x(k) + \upsilon(k).$$

Here the system is nonlinear according to state variable x_3 and the system can be described as

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1-0.1x_3(k) & 0.1 \\ -0.1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \\ x_3(k) \end{bmatrix} + \xi(k)$$

$$z(k) = \begin{bmatrix} [1 \quad 0] & 0 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \\ x_3(k) \end{bmatrix} + \upsilon(k)\ .$$

The state variable x_3 is the parameter of the upper subsystem and can be estimated with using EKF, MEKF, UKF and MUKF. $\xi(k)$ and $\upsilon(k)$ are uncorrelated zero-mean Gaussian white noise sequences with $Var(\xi(k)) = 10^{-6}I_3$ and $Var(\upsilon(k)) = 10^{-5}$ for all k (I_3-3×3 idendity matrix).

For simulation study, the values of $x_3(k)$ are given by,

a) $x_3(k) = 1,\ \ k = 1, \dots, 200$
b) $x_3(k) = 1 + 0.01 \times k,\ \ k = 1, \dots, 200$ (23)

Table 1. Initial values and noise covariance

Initial state $x(0)$	$[0.9; 0.9; 0.9\ ]$
Initial error covariance -$P(0)$	$10^{-5}I_3$

and we want to identify these values by using EKF, MEKF, UKF and MUKF.

For simulations initial values and noise covariance are given in Table 1. Scaling parameter a is taken as 0.1. The aim is to compare the performance of the EKF, MEKF, UKF and the performance of the proposed MUKF. Simulation was repeated 100 times. For state variable x_3, simulation results are given in Figures 1-2 and the mean square errors (MSE) of all variables are given Table 2 and Table 3. As, it can be seen, for all state variables the proposed MUKF is giving better results than conventional UKF. For state variable x_3, performance of the proposed MUKF is the best, performance of the MEKF is better than EKF and UKF. But for state variables x_1 and x_2, the EKF demonstrates the best performance.

5. CONCLUSION

In this paper, with the intention to improve the performance of the UKF for special nonlinear signal models, MUKF algorithm is introduced. The MUKF contains two parallel algorithms. Algorithm I is used for yielding the estimates which are used in Algorithm II to implement conventional Kalman Filter Algorithm. Two algorithms are applied in parallel starting with the same initial estimate.

In simulation study, a nonlinear system that contains a linear subsystem is considered. EKF, MEKF, UKF and newly proposal MUKF were applied to obtain systems states estimates and results are compared using mean square error criteria.

It can be seen that, performance of the proposed MUKF is better than UKF in terms of mean square estimation error. As a result, we can say that the proposed MUKF is considered as an alternative method to parameter estimation problem in state-space models.

Table 2. MSE of EKF and MEKF

	MSE	
	EKF	MEKF
$a)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$	$\begin{bmatrix} 0.0001702 \\ 0.0025883 \\ 0.004326 \end{bmatrix}$	$\begin{bmatrix} 0.0001752 \\ 0.0025893 \\ 0.004019 \end{bmatrix}$
$b)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$	$\begin{bmatrix} 0.0001852 \\ 0.0028743 \\ 0.0057051 \end{bmatrix}$	$\begin{bmatrix} 0.000189 \\ 0.002924 \\ 0.005453 \end{bmatrix}$

Table 3. MSE of UKF and MUKF

	MSE	
	UKF	MUKF
$a)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$	$\begin{bmatrix} 0.0001822 \\ 0.0026913 \\ 0.004229 \end{bmatrix}$	$\begin{bmatrix} 0.0001752 \\ 0.0025633 \\ 0.003973 \end{bmatrix}$
$b)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$	$\begin{bmatrix} 0.000205 \\ 0.003013 \\ 0.005513 \end{bmatrix}$	$\begin{bmatrix} 0.0001882 \\ 0.0028963 \\ 0.005344 \end{bmatrix}$

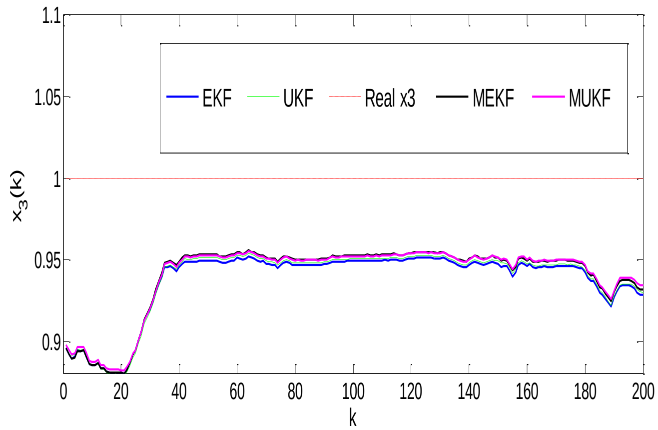

Figure 1. Estimation of state variable $x_3(k)$ for case a

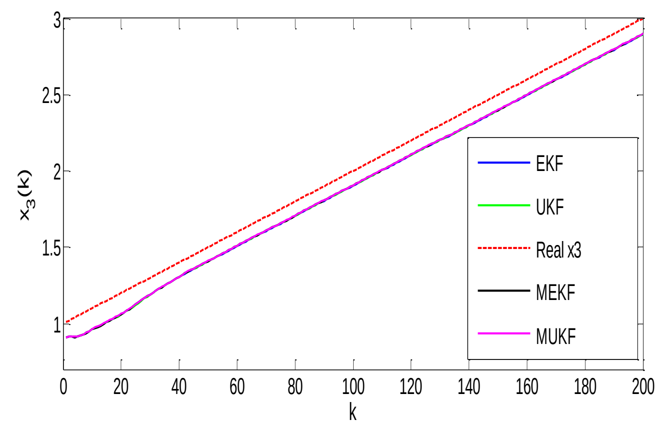

Figure 2. Estimation of state variable $x_3(k)$ for case b

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