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SURVIVAL PROBABILITIES FOR COMPOUND BINOMIAL RISK MODEL WITH DISCRETE PHASE-TYPE CLAIMS

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Abstract. Due to having useful properties in approximating to the other distributions and mathematically tractable, phase type distributions are commonly used in actuarial risk theory. Claim occurrence time and individual claim size distributions are modelled by phase type distributions in literature.

This paper aims to calculate the survival probabilities of an insurance company under the assumption that compound binomial risk model where the individual claim sizes are distributed as discrete Phase Type distribution.

1. Introduction

Compound binomial risk model is first proposed by Gerber [8] to describe the surplus process of an insurance company. Compound binomial risk model can be described as a special case of discrete time version of the risk model. This model was studied by Shiu [16], Willmot [20] and Dickson [5]. Recently, Liu et. al. [11], Liu and Zhao [12], Eryilmaz [7], Li and Sendova [10], Tuncel and Tank [19] and Tank and Tuncel [17] have studied some extensions of compound binomial risk model. Stanford and Stroinski [15] calculated finite time ruin probabilities for phase type claim size by recursive methods. Wu and Li [21] studied on discrete time Sparre-Anderson risk model for phase type claims.

The surplus process of an insurance company $\{U_t, t \in \mathbb{N}\}$ is defined as

$$U_t = u + ct - \sum_{i=1}^{t} Y_i,\ t = 0, 1, ... \tag{1.1}$$

with $U_0 = u$ (initial surplus), the periodic premium is c and Y_i is the claim amount in related period. Suppose that I_i be a indicator function which represents the claim occurrence where I_i's are independent and identically distributed (i.i.d.). That is $I_i = 1$ with probability p if a claim occurs in period i and $I_i = 0$ with probability q, otherwise. For $i \geq 1$, define

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$$Y_i = \begin{cases} X_i & , \ I_i = 1 \\ 0 & , \ I_i = 0 \end{cases} \tag{1.2}$$

Here, the random variable X_i strictly positive and $\{X_i, i \geq 1\}$ forms a sequence of *i.i.d.* random variables with probability mass function (p.m.f) $f(x) = P(X = x)$.Under these assumptions Eq. (1.1) can be rewritten as

$$U_n = u + n - \sum_{i=1}^{Nn} I_i X_i, \ n = 0, 1, ... \tag{1.3}$$

where N_n is the total claim number up to time n-th. The compound binomial model can be orally defined as a binomial processes with independent increments for claim occurrences. Then the distribution of N_n random variables is assumed binomially distributed. That is

$$P(N_n = k) = \binom{n}{k} p^k q^{n-k}, k = 0, 1, ..., n$$

Let W_1 denote the time until first claim appearance, W_2 denote the time between first and second claims and more generally W_n denote the time between $(n-1)$st and n-th claims. Thus $W_1, W_2, ..., W_n$ can be thought as sequence of i.i.d. random variables with geometric distribution

$$P(W_n = t) = P(I_1 = 0, ..., I_{t-1} = 0, I_t = 1) = pq^{t-1}, \ t = 1, 2, ... \tag{1.4}$$

For an insurance company, ruin occurs at the first time that the surplus reaches to zero or below to zero. Thus the time of ruin, the ultimate probability of ruin and the finite time probability of ruin are defined as follows

$$T = \inf\{U_t \leq 0, t = 1, 2, ...\}. \tag{1.5}$$

$$\psi(u) = P(T < \infty | U_0 = u) \tag{1.6}$$

$$\psi(u, n) = P(T \leq n | U_0 = u) \tag{1.7}$$

respectively, where the initial surplus U_0 is u. Complement of Eq. (1.7) is described as follows

$$\begin{aligned} \phi(u, n) &= 1 - \psi(u, n) \\ &= P(U_t > 0, t = 1, 2, ..., n) \end{aligned} \tag{1.8}$$

and interpreted as the finite time survival (non-ruin) probability. It is clear that, for $n_0 > 0$

$$\phi(u_1, n_0) \leq \phi(u_2, n_0)$$

where $u_1 \leq u_2$.

Supposing that net profit condition is $p\mu_X < 1$.Under this condition is not certain to occur eventually Eryilmaz [7] .

Recursive formula for survival (non-ruin) probability when the claim occurrences are nonhomogeneous in the compound binomial risk model is given by Tuncel and

Tank [19]. Survival (non-ruin) probabilities after a definite time period of an insurance company in a discrete time model based on non-homogenous claim occurrences is studied by Tank and Tuncel [17]. In their studies, the distribution of N_n given as

$$P\left(N_n=k\right)=\begin{cases} p_n P\left(N_{n-1}=k-1\right)+q_n P\left(N_{n-1}=k\right) & ,1\leq k\leq n-1 \\ \prod\limits_{i=1}^{n} q_i & ,k=0 \\ \prod\limits_{i=1}^{n} p_i & ,k=n \end{cases} \tag{1.9}$$

where $P(I_i=1)=p_i$ and $P(I_i=0)=1-p_i=q_i$ for $i\geq 1$. Claim occurrence probabilities may subject to be different between each other under the model assumption which has been given in Eq. (1.3). The distribution of the N_n random variable can be stated by using the recursive formulas as given in Eq (1.9). Chen et. al. [4] discussed another recursive formula for computing $P(N_n=k)$. But this formula is occasionally not stability of the distribution when p_i's are close to 1 and n is large. The reader is refered to Chen et. al. [4] for the details.

Tuncel and Tank [19] have proposed the distribution of T random variable as

$$\phi^{(1,n)}\left(u\right)=\begin{cases} 1 & ,n=0 \\ \sum\limits_{t=1}^{n} p_t \prod\limits_{i=1}^{t-1} q_i \sum\limits_{x=1}^{u+t-1} f\left(x\right)\phi^{(t+1,n-t)}\left(u+t-x\right)+\prod\limits_{i=1}^{n} q_i & ,n>0 \end{cases} \tag{1.10}$$

with non-homogenous claim occurrence probabilities for compound binomial risk model in related time periods. In here $\phi^{(t+1,n-t)}$ represent ruin time after the t-th period.

2. Discrete Phase-type distributions

The first studies of Phase-Type distributions is appeared at the first decade of 20th century. However the first modern study on the Phase-Type distributions, shortly written as PH distribution, was introduced by Neuts([13] , [14]). After that, the PH distributions has become popular in different areas such as applied probability and actuarial risk theory. PH distributions can be either continuous or discrete. In this paper discrete version of the PH distribution is studied.

Discrete PH distributions may describe the time until absorption in a discrete time Markov chain with a finite number of transient states and one absorbing state. Consider an $(m+1)$ absorbing discrete time Markov Chain with state space $\{0,1,...,m\}$ and let state "0" be the absorbing state. Namely the first m state are transient and the last state is absorbing. In this case transition probability matrix is

$$\mathbf{P}=\begin{bmatrix} \mathbf{T} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}$$

where $\mathbf{T}$ is a square sub-stochastic matrix of dimension m and all elements are between 0 and 1. $\mathbf{t}' = (t_{10}, ..., t_{m0})$ is a column vector and $\mathbf{0}$ is a row vector. Let initial state distribution be $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, ..., \alpha_m)$ and $\sum_{i=1}^{m} \alpha_i = 1$. We denote by X random variable of the time to reach to absorbing state $m+1$. In this case the distribution of X is called a discrete PH distribution which is represented by $(\boldsymbol{\alpha}, \mathbf{T})$. Even if the Markov chain starts from the absorption state "0", it is also possible to apply the discrete PH distribution for positive individual claim size X_i $i = 1, 2, ...$ Furthermore it is also known that $\mathbf{t} = (\mathbf{I} - \mathbf{T})\mathbf{1}$, where $\mathbf{I}$ is identity matrix of dimension $m \times m$ and $\mathbf{1}' = (1, ..., 1)$.

The cumulative distribution function of X is then given by

$$F(x) = P(X \leq x) = 1 - \boldsymbol{\alpha}\mathbf{T}^x\mathbf{1}' \text{ for } x = 0, 1, 2, ... \tag{2.1}$$

and its probability mass function is

$$f(x) = P(X = x) = \boldsymbol{\alpha}\mathbf{T}^{x-1}\mathbf{t} \text{ for } x = 1, 2, ... \tag{2.2}$$

The expected value of X can be computed the form of

$$E(X) = \boldsymbol{\alpha}(\mathbf{I} - \mathbf{T})^{-1}\mathbf{1}'$$

The family of discrete PH distributions is closed under convolution. Note that, some useful properties of the discrete PH distributions make it attractive for risk modelling studies in actuarial sciences.

Suppose that X and Y are two independent discrete random variables that have phase type distributions with representations $(\boldsymbol{\alpha}, \mathbf{T})$ and $(\boldsymbol{\pi}, \mathbf{D})$ respectively. Then, the distribution of $X + Y$ turns into $PH(\boldsymbol{\gamma}, \mathbf{C})$ where

$$\boldsymbol{\gamma} = \begin{bmatrix} \boldsymbol{\alpha} & \alpha_0\boldsymbol{\pi} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{T} & \mathbf{t}\boldsymbol{\pi} \\ \mathbf{0} & \mathbf{D} \end{bmatrix}$$

More details on discrete PH distributions may be found in Asmussen [1], Bladt [2], Breuer and Baum [3], Drekic [6], Latouche and Ramaswami [9], Tank and Eryilmaz [18].

3. Numerical illustration

In this section, we present numerical illustration when individual claim sizes are arisen from zero-truncated geometric distribution which is PH distribution for $m = 2$. Let $\boldsymbol{\alpha} = (1, 0)$, $\mathbf{t} = (0, 1 - \alpha)$.

$$\mathbf{T} = \begin{bmatrix} \alpha & 1 - \alpha \\ \alpha & 0 \end{bmatrix}$$

In this case from Eq. (2.2), probability mass function is

$$P(X = x) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha & 1 - \alpha \\ \alpha & 0 \end{bmatrix}^{x-1} \begin{bmatrix} 0 \\ 1 - \alpha \end{bmatrix}, \; x = 1, 2, \ldots \tag{3.1}$$

This distribution given by Eq. (3.1) is known to be geometric distribution of order "2" Eryilmaz [7]. Let define cases with non-homogenous claim occurrence probabilities in each periods as follows:

CASE 1	$p_i = \begin{cases} 0.1 * i & ,i = 1, ..., 6 \\ 0.1 * [(12-i)+1] & ,i = 7, ..., 12 \end{cases}$
CASE 2	$p_i = 0.01 * i \ , \ i = 1, ..., 12$
CASE 3	$p_i = \begin{cases} 0.1 * [(6-i)+1] & ,i = 1, ..., 6 \\ 0.1 * [(i-7)+1] & ,i = 7, ..., 12 \end{cases}$
CASE 4	$p_i = 0.01 * [(12-i)+1] \ , \ i = 1, ..., 12$

Table 1. Claim occurence probabilities

In Case 1, it can be seen that the probabilities of claim occurrences are increasing for first 6 periods from 0.1 to 0.6 and after that it is decreasing for last 6 periods from 0.6 to 0.1. In Case 2, it can be seen that the probabilities of claim occurrences are increasing from 0.01 to 0.12 for 12 periods. In Case 3, probabilities of claim occurrences are decreasing for first 6 periods from 0.6 to 0.1 and after that it is increasing for last 6 periods from 0.1 to 0.6. Finally, in Case 4, the probabilities of claim occurrences are decreasing from 0.12 to 0.01 for 12 periods.

So, survival probabilities are calculated and presented in Table 2 - Table 13 for different α values, which are probability of claims, by Eq. (1.10) when $P(I_i = 1) = p_i$ for $i = 1, 2, \ldots, 12$.

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.9000	0.9640	0.9768	0.9896	0.9942	0.9972
2	0.8352	0.9094	0.9509	0.9727	0.9854	0.9920
3	0.7505	0.8621	0.9139	0.9512	0.9710	0.9835
4	0.6870	0.7966	0.8720	0.9187	0.9502	0.9692
5	0.5995	0.7321	0.8124	0.8766	0.9177	0.9470
6	0.5238	0.6431	0.7435	0.8152	0.8720	0.9114
7	0.4363	0.5661	0.6624	0.7490	0.8140	0.8664
8	0.3845	0.4990	0.6021	0.6887	0.7629	0.8219
9	0.3464	0.4598	0.5576	0.6475	0.7234	0.7877
10	0.3271	0.4347	0.5323	0.6209	0.6986	0.7646
11	0.3168	0.4229	0.5189	0.6075	0.6853	0.7523
12	0.3134	0.4186	0.5142	0.6025	0.6805	0.7477

Table 2. Survival probabilities for $\alpha = 1/5$ in Case 1

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.9000	0.9160	0.9256	0.9352	0.9433	0.9504
2	0.7488	0.7789	0.8044	0.8272	0.8473	0.8651
3	0.5795	0.6232	0.6609	0.6954	0.7264	0.7544
4	0.4221	0.4698	0.5133	0.5539	0.5915	0.6264
5	0.2874	0.3315	0.3733	0.4137	0.4524	0.4894
6	0.1803	0.2153	0.2502	0.2851	0.3197	0.3539
7	0.1128	0.1385	0.1650	0.1925	0.2206	0.2493
8	0.0779	0.0973	0.1177	0.1394	0.1621	0.1857
9	0.0594	0.0749	0.0915	0.1094	0.1283	0.1483
10	0.0494	0.0626	0.0770	0.0925	0.1091	0.1267
11	0.0442	0.0562	0.0693	0.0835	0.0988	0.1151
12	0.0421	0.0536	0.0661	0.0798	0.0945	0.1103

Table 3. Survival probabilities for $\alpha = 3/5$ in Case 1

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.9000	0.9040	0.9072	0.9104	0.9135	0.9164
2	0.7272	0.7362	0.7445	0.7527	0.7605	0.7682
3	0.5246	0.5379	0.5506	0.5630	0.5750	0.5868
4	0.3359	0.3505	0.3647	0.3786	0.3923	0.4058
5	0.1891	0.2017	0.2142	0.2265	0.2388	0.2510
6	0.0921	0.1008	0.1095	0.1184	0.1273	0.1363
7	0.0454	0.0508	0.0562	0.0618	0.0675	0.0734
8	0.0265	0.0299	0.0335	0.0372	0.0411	0.0451
9	0.0178	0.0203	0.0228	0.0255	0.0283	0.0313
10	0.0135	0.0154	0.0175	0.0196	0.0219	0.0242
11	0.0114	0.0131	0.0148	0.0167	0.0186	0.0206
12	0.0105	0.0121	0.0137	0.0154	0.0172	0.0191

Table 4. Survival probabilities for $\alpha = 4/5$ in Case 1

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.9900	0.9964	0.9977	0.9990	0.9994	0.9997
2	0.9829	0.9917	0.9955	0.9977	0.9988	0.9994
3	0.9757	0.9884	0.9936	0.9968	0.9983	0.9991
4	0.9712	0.9856	0.9922	0.9959	0.9978	0.9958
5	0.9676	0.9836	0.9910	0.9953	0.9975	0.9987
6	0.9650	0.9820	0.9900	0.9947	0.9971	0.9985
7	0.9629	0.9807	0.9893	0.9943	0.9969	0.9983
8	0.9612	0.9796	0.9886	0.9938	0.9966	0.9981
9	0.9598	0.9787	0.9880	0.9935	0.9964	0.9980
10	0.9586	0.9779	0.9875	0.9931	0.9962	0.9979
11	0.9525	0.9772	0.9870	0.9928	0.9960	0.9977
12	0.9566	0.9765	0.9865	0.9925	0.9958	0.9976

Table 5. Survival probabilities for $\alpha = 1/5$ in Case 2

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.9900	0.9916	0.9926	0.9935	0.9943	0.9950
2	0.9734	0.9768	0.9797	0.9822	0.9844	0.9864
3	0.9516	0.9578	0.9629	0.9675	0.9715	0.9750
4	0.9269	0.9359	0.9436	0.9505	0.9565	0.9617
5	0.9003	0.9123	0.9226	0.9318	0.9399	0.9470
6	0.8729	0.8877	0.9007	0.9122	0.9223	0.9313
7	0.8452	0.8628	0.8782	0.8919	0.9040	0.9148
8	0.8178	0.8377	0.8553	0.8711	0.8852	0.8977
9	0.7906	0.8128	0.8324	0.8500	0.8658	0.8800
10	0.7639	0.7879	0.8093	0.8287	0.8461	0.8617
11	0.7376	0.7632	0.7862	0.8070	0.8259	0.8430
12	0.7117	0.7386	0.7629	0.7851	0.8053	0.8236

Table 6. Survival probabilities for $\alpha = 3/5$ in Case 2

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.9900	0.9904	0.9907	0.9910	0.9913	0.9916
2	0.9710	0.9720	0.9730	0.9739	0.9748	0.9756
3	0.9440	0.9459	0.9477	0.9495	0.9512	0.9528
4	0.9101	0.9131	0.9160	0.9188	0.9215	0.9241
5	0.8702	0.8749	0.8790	0.8829	0.8867	0.8904
6	0.8270	0.8325	0.8377	0.8428	0.8478	0.8526
7	0.7801	0.7869	0.7933	0.7996	0.8058	0.8117
8	0.7312	0.7391	0.7468	0.7542	0.7614	0.7684
9	0.6813	0.6903	0.6989	0.7074	0.7156	0.7236
10	0.6311	0.6410	0.6506	0.6599	0.6690	0.6779
11	0.5814	0.5921	0.6024	0.6125	0.6223	0.6319
12	0.5329	0.5440	0.5549	0.5655	0.5759	0.5861

Table 7. Survival probabilities for $\alpha = 4/5$ in Case 2

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.4000	0.7840	0.8608	0.9376	0.9652	0.9831
2	0.3280	0.5456	0.7325	0.8292	0.9031	0.9422
3	0.2582	0.4810	0.6295	0.7575	0.8401	0.9000
4	0.2376	0.4315	0.5870	0.7083	0.8020	0.8676
5	0.2247	0.4145	0.5628	0.6866	0.7806	0.8504
6	0.2213	0.4076	0.5554	0.6784	0.7734	0.8438
7	0.2188	0.4038	0.5505	0.6735	0.7685	0.8397
8	0.2158	0.3983	0.5442	0.6666	0.7621	0.8338
9	0.2116	0.3914	0.5354	0.6575	0.7531	0.8258
10	0.2062	0.3815	0.5236	0.6444	0.7406	0.8142
11	0.1983	0.3682	0.5065	0.6263	0.7222	0.7974
12	0.1878	0.3490	0.4831	0.5998	0.6960	0.7723

Table 8. Survival probabilities for $\alpha = 1/5$ in Case3

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.4000	0.4960	0.5536	0.6112	0.6596	0.7024
2	0.2320	0.2992	0.3549	0.4090	0.4596	0.5066
3	0.1597	0.2128	0.2588	0.3055	0.3505	0.3940
4	0.1260	0.1701	0.2099	0.2509	0.2912	0.3309
5	0.1098	0.1493	0.1855	0.2231	0.2605	0.2977
6	0.1034	0.1410	0.1757	0.2118	0.2479	0.2840
7	0.0980	0.1340	0.1674	0.2022	0.2371	0.2722
8	0.0889	0.1220	0.1530	0.1856	0.2184	0.2516
9	0.0772	0.1066	0.1344	0.1638	0.1938	0.2243
10	0.0639	0.0888	0.1127	0.1383	0.1646	0.1917
11	0.0498	0.0697	0.0893	0.1104	0.1324	0.1553
12	0.0360	0.0509	0.0657	0.0820	0.0993	0.1175

Table 9. Survival probabilities for $\alpha = 3/5$ in Case 3

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.4000	0.4240	0.4432	0.4624	0.4808	0.4986
2	0.2080	0.2264	0.2429	0.2594	0.2756	0.2916
3	0.1306	0.1445	0.1573	0.1703	0.1833	0.1962
4	0.0952	0.1064	0.1169	0.1275	0.1383	0.1490
5	0.0786	0.0882	0.0974	0.1067	0.1162	0.1257
6	0.0719	0.0809	0.0895	0.0983	0.1071	0.1161
7	0.0660	0.0744	0.0825	0.0907	0.0991	0.1075
8	0.0555	0.0629	0.0699	0.0772	0.0846	0.0921
9	0.0426	0.0486	0.0544	0.0603	0.0664	0.0726
10	0.0297	0.0341	0.0385	0.0430	0.0476	0.0524
11	0.0185	0.0215	0.0245	0.0276	0.0308	0.0342
12	0.0102	0.0119	0.0137	0.0137	0.0176	0.0197

Table 10. Survival probabilities for $\alpha = 4/5$ in Case 3

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.8800	0.9568	0.9722	0.9875	0.9930	0.9966
2	0.8452	0.9259	0.9574	0.9777	0.9878	0.9935
3	0.8208	0.9118	0.9471	0.9717	0.9840	0.9912
4	0.8099	0.9023	0.9413	0.9676	0.9815	0.9896
5	0.8027	0.8973	0.9375	0.9651	0.9798	0.9886
6	0.7990	0.8941	0.9354	0.9636	0.9787	0.9879
7	0.7968	0.8924	0.9341	0.9627	0.9781	0.9875
8	0.7957	0.8914	0.9334	0.9622	0.9778	0.9872
9	0.7951	0.8909	0.9330	0.9619	0.9776	0.9871
10	0.7948	0.8907	0.9328	0.9618	0.9775	0.9870
11	0.7947	0.8906	0.9328	0.9617	0.9775	0.9870
12	0.7946	0.8906	0.9327	0.9617	0.9774	0.9870

Table 11. Survival probabilities for $\alpha = 1/5$ in Case 4

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.8800	0.8992	0.9107	0.9222	0.9319	0.9405
2	0.7987	0.8251	0.8450	0.8634	0.8795	0.8937
3	0.7389	0.7707	0.7955	0.8185	0.8387	0.8568
4	0.6951	0.7299	0.7580	0.7840	0.8070	0.8277
5	0.6625	0.6994	0.7295	0.7575	0.7824	0.8049
6	0.6384	0.6774	0.7079	0.7372	0.7634	0.7871
7	0.6206	0.6594	0.6918	0.7219	0.7489	0.7735
8	0.6077	0.6469	0.6799	0.7106	0.7382	0.7634
9	0.5987	0.6382	0.6715	0.7026	0.7306	0.7561
10	0.5927	0.6324	0.6659	0.6972	0.7255	0.7512
11	0.5892	0.6290	0.6626	0.6940	0.7224	0.7483
12	0.5877	0.6275	0.6612	0.6926	0.7211	0.7471

Table 12. Survival probabilities for $\alpha = 3/5$ in Case 4

n	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$
1	0.8800	0.8848	0.8886	0.8925	0.8962	0.8997
2	0.7871	0.7944	0.8010	0.8074	0.8136	0.8196
3	0.7140	0.7232	0.7315	0.7397	0.7476	0.7553
4	0.6564	0.6667	0.6763	0.6857	0.6947	0.7036
5	0.6108	0.6220	0.6324	0.6425	0.6524	0.6620
6	0.5750	0.5866	0.5976	0.6083	0.6187	0.6289
7	0.5470	0.5590	0.5703	0.5813	0.5921	0.6026
8	0.5255	0.5377	0.5492	0.5605	0.5716	0.5823
9	0.5096	0.5219	0.5335	0.5450	0.5562	0.5671
10	0.4983	0.5107	0.5225	0.5340	0.5453	0.5563
11	0.4912	0.5037	0.5155	0.5271	0.5384	0.5495
12	0.4879	0.5003	0.5121	0.5238	0.5351	0.5462

Table 13. Survival probabilities for $\alpha = 4/5$ in Case 4

In case of nonhomogenous claim occurence probabilities, it is obvious to say that the survival probabilities are increasing when the u initial values are increasing and survival probabilities are decreasing for same u initial reserve level in later periods. It is also possible to see that the survival probabilities are decreasing for higher values of α for each cases. Survival probabilities are decreasing for higher probablities of claim occurence with same α level which can be seen by comparing the Tables are given for Case 2 and Case 4. Similar interpretations can be made for Case 1 and Case 3.

4. Conclusion

The theoretical assumptions of this study are basically taken from Tank and Tuncel [17]. In this study, survival exact probabilities in compound binomial risk model are calculated with nonhomogeneous probabilities where the individual claim sizes are discrete Phase Type distribution instead of geometric distribution. Probabilites are calculated by MATLAB software where the individual claim size distribution is discrete phase type distribution and presented in Tables 2-13. By using the given probablities it is easy to calculate the ruin probabilities for an insurance company with respect to the parameters which are assumed.

As a possible future work, nonruin (survival) probabilities for dependent case of I_i $(i \geq 1)$ and continuous time compound binomial risk model can also be studied.

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