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α-γ-CONVERGENCE, α-γ-ACCUMULATION AND α-γ-COMPACTNESS

ALIAS B. KHALAF AND HARIWAN Z. IBRAHIM

Abstract. The aim of the present paper is to introduce the concept of α-γ-compactness by means of γ-operation defined on the family of α-open sets of a topological space. We define the α-γ-convergence, α-γ-accumulation points of a filterbase and give some of their properties. Some characterizations and properties of α-γ-compact spaces are obtained.

1. Introduction

The notion of compactness is useful and fundamental notion of not only general topology but also of other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness. The productivity and fruitfulness of this notion of compactness motivated mathematicians to generalize this notion. Njastad [5] introduced a new class of generalized open sets in a topological space called α-open sets. The concept of operation γ was initiated by Ibrahim [6]. He also introduced the concept of α_γ-open sets. The aim of this paper is to introduce the concept of α-γ-compactness in topological spaces and is to give some characterizations of α-γ-compact spaces. The notion of α-γ-convergence and α-γ-accumulation are defined and are used to characterize α-γ-compactness.

2. Preliminaries

Throughout the present paper (X, τ) (or simply X) denotes a topological space. Let A be a subset of X. We denote the interior and the closure of a set A by $Int(A)$ and $Cl(A)$, respectively. A subset A of X is said to be α-open [5] if $A \subseteq Int(Cl(Int(A)))$. The complement of an α-open set is said to be α-closed. The family of all α-open sets in a topological space (X, τ) is denoted by $\alpha O(X, \tau)$. An operation $\gamma : \alpha O(X, \tau) \to P(X)$ [6] is a mapping satisfying the condition, $V \subseteq V^\gamma$ for each $V \in \alpha O(X, \tau)$. We call the mapping γ an operation on $\alpha O(X, \tau)$. A subset

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A of X is called an α_γ-open set [6] if for each point $x \in A$, there exists an α-open set U of X containing x such that $U^\gamma \subseteq A$. The complement of an α_γ-open set is said to be α_γ-closed. We denote the set of all α_γ-open (resp., α_γ-closed) sets of (X, τ) by $\alpha O(X, \tau)_\gamma$ (resp., $\alpha C(X, \tau)_\gamma$). A point $x \in X$ is in αCl_γ-closure [6] of a set $A \subseteq X$, if $U^\gamma \cap A \neq \phi$ for each α-open set U containing x. The αCl_γ-closure of A is denoted by $\alpha Cl_\gamma(A)$. An operation γ on $\alpha O(X, \tau)$ is said to be α-regular [6] if for every α-open sets U and V containing a point $x \in X$, there exists an α-open set W containing x such that $W^\gamma \subseteq U^\gamma \cap V^\gamma$. An operation γ on $\alpha O(X, \tau)$ is said to be α-open [6] if for every α-open set U containing $x \in X$, there, exists an α_γ-open set V of X such that $x \in V$ and $V \subseteq U^\gamma$. An operation $\gamma : \alpha O(X) \to P(X)$ is said to be α-monotone [3] if for all $A, B \in \alpha O(X)$, $A \subseteq B$ implies $A^\gamma \subseteq B^\gamma$. The operation $id : \alpha O(X, \tau) \to P(X)$ is defined by $id(V) = V$ for any set $V \in \alpha O(X, \tau)$ this operation is called the identity operation on $\alpha O(X, \tau)$ [6]. An operation $\gamma : \alpha O(X) \to P(X)$ is said to be α-additive [3] if $(A \cup B)^\gamma = A^\gamma \cup B^\gamma$ for all $A, B \in \alpha O(X)$.

Definition 1. [1] *A topological space (X, τ) is said to be α_γ-regular if for each $x \in X$ and for each α-open set V in X containing x, there exists an α-open set U in X containing x such that $U^\gamma \subseteq V$.*

Definition 2. [2] *Let H be any subset of X, $\alpha|H = \{V = U \cap H : U \in \alpha O(X)\}$. An operation γ from $\alpha O(X)$ to $P(X)$ is said to be α-stable with respect to H if γ induces an operation $\gamma_H : \alpha|H \to P(H)$ satisfying the following two properties:*

1. *$(U \cap H)^{\gamma_H} = U^\gamma \cap H$ for every $U \in \alpha O(X)$ and*
2. *$W \cap H = S \cap H$ implies that $W^\gamma \cap H = S^\gamma \cap H$ for every $W, S \in \alpha O(X)$.*

Definition 3. [4] *A space X with an operation γ on $\alpha O(X)$ is called α-γ-T_2 space if for any two distinct points $x, y \in X$, there exist two α-open sets U and V containing x and y, respectively, such that $U^\gamma \cap V^\gamma = \phi$.*

Definition 4. [8] *A filterbase $\mathcal{F}$ is said to be α-converges to a point $x \in X$ if for each α-open set V containing x, there exists an $F \in \mathcal{F}$ such that $F \subseteq V$.*

Definition 5. [7] *A space X is said to be α-compact iff every α-open cover of X has a finite subcover*

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Definition 6. *A filterbase $\mathcal{F}$ in a topological space (X, τ) with an operation γ on $\alpha O(X)$ is said to be:*

1. *α-γ-converges to a point $x \in X$ if for every α-open set V containing x, there exists $F \in \mathcal{F}$ such that $F \subseteq V^\gamma$.*
2. *α-γ-accumulates to a point $x \in X$ if $F \cap V^\gamma \neq \phi$, for every α-open set V containing x and every $F \in \mathcal{F}$.*

Remark 1. *If a filterbase $\mathcal{F}$ in a topological space (X, τ) is α-converges to a point $x \in X$, then $\mathcal{F}$ is α-γ-converges to a point $x \in X$*

The converse of above remark is not true in general as it is shown in the following example.

Example 1. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ be a topology on X and $\mathcal{F} = \{\{b\}, \{b, c\}\}$ be a filterbase. For each $A \in \alpha O(X)$ we define γ on $\alpha O(X)$ by $A^\gamma = Cl(A)$. Then, $\mathcal{F}$ is α-γ-converges to a point $a \in X$, but $\mathcal{F}$ is not α-converges to a.*

Corollary 1. *Let (X, τ) be a topological space and γ be an operation on $\alpha O(X)$. If a filterbase $\mathcal{F}$ in X, α-γ-converges to a point $x \in X$, then $\mathcal{F}$ α-γ-accumulates to x.*

The converse of above corollary is not true in general as it is shown in the following example.

Example 2. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ be a topology on X and $\mathcal{F} = \{\{a, b\}\}$ be a filterbase. For each $A \in \alpha O(X)$ we define γ on $\alpha O(X)$ by*

$$A^\gamma = \begin{cases} Cl(A) & \text{if } A = \{b\} \\ X & \text{if } A \neq \{b\}. \end{cases}$$

Then, $\mathcal{F}$ is α-γ-accumulates to a point $b \in X$, but $\mathcal{F}$ is not α-γ-converges to b.

Proposition 1. *Let $\mathcal{F}$ be a filterbase in a topological space (X, τ), γ is α-open and E is any α_γ-open set containing x. If there exists an $F \in \mathcal{F}$ such that $F \subseteq E$, then $\mathcal{F}$ is α-γ-converges to a point $x \in X$.*

Proof. Obvious. □

Proposition 2. *Let $\mathcal{F}$ be a filterbase in a topological space (X, τ), γ is α-open and E is any α_γ-open set containing x such that $F \cap E \neq \phi$ for each $F \in \mathcal{F}$, then $\mathcal{F}$ is α-γ-accumulation to a point $x \in X$.*

Proof. The proof is similar to Proposition 1. □

Theorem 1. *If a filterbase $\mathcal{F}$ in X is contained in a filterbase which is α-γ-accumulate to $x \in X$, then $\mathcal{F}$ is α-γ-accumulate to x.*

Proof. Obvious. □

Theorem 2. *Let (X, τ) be a topological space and γ an α-regular operation on $\alpha O(X)$. If a filterbase $\mathcal{F}$ in X is α-γ-accumulates to x, then there exists a filterbase $\mathcal{H}$ in X such that $\mathcal{F} \subseteq \mathcal{H}$ and $\mathcal{H}$ is α-γ-converges to x.*

Proof. Let the filterbase $\mathcal{F}$ be α-γ-accumulate to x. Hence for every α-open set U containing x and for each $A \in \mathcal{F}$, $A \cap U^\gamma \neq \phi$. Then $x \in \alpha Cl_\gamma(A)$, for every $A \in \mathcal{F}$. Consider the set $\mathcal{G} = \{A \cap U^\gamma : U$ is α-open set of X containing x and $A \in \mathcal{F}\}$. Suppose that $G_1, G_2 \in \mathcal{G}$. Then $G_1 \cap G_2 = (A_1 \cap U_1^\gamma) \cap (A_2 \cap U_2^\gamma) = (A_1 \cap A_2) \cap (U_1^\gamma \cap U_2^\gamma)$ for every $A_1, A_2 \in \mathcal{F}$ and U_1, U_2 are α-open sets of X containing x. Since γ is α-regular, then there exists an α-open set U_3 of X containing x such that $U_3^\gamma \subseteq U_1^\gamma \cap U_2^\gamma$. Since $\mathcal{F}$ is a filterbase, then there exists $A_3 \in \mathcal{F}$ such that

$A_3 \subseteq A_1 \cap A_2$. Hence $A_3 \cap U_3^\gamma \subseteq G_1 \cap G_2$. Thus we see that $\mathcal{G}$ is a filterbase. Now the set $\mathcal{H} = \{B : \exists C \in \mathcal{G}$ with $C \subseteq B\}$ is filter generated by $\mathcal{G}$. For each α-open set U containing x and for each $A \in \mathcal{F}$, $U^\gamma \supseteq A \cap U^\gamma \in \mathcal{H}$, where $A \cap U^\gamma \in \mathcal{G}$. So $\mathcal{H}$ is α-γ-converges to x. Also for each $A \in \mathcal{F}$, $A = X^\gamma \cap A \in \mathcal{G}$, and thus $A \in \mathcal{H}$. Hence, $\mathcal{F} \subseteq \mathcal{H}$. □

Corollary 2. *Let (X, τ) be a topological space and γ be an α-monotone operation on $\alpha O(X)$. If a maximal filterbase in X, α-γ-accumulates to a point $x \in X$ then it α-γ-converges to x.*

Proof. Similar to the proof of Theorem 2. □

Definition 7. *A topological space (X, τ) is said to be α-γ-compact if for every α-open cover $\{V_i : i \in I\}$ of X, there exists a finite subset I_0 of I such that $X = \bigcup\{V_i^\gamma : i \in I_0\}$.*

Theorem 3. *Every α-compact space is α-γ-compact.*

Proof. Obvious. □

Remark 2. *The converse of above theorem is not true in general as it is shown in the following example.*

Example 3. *Let N be the set of all natural numbers with the discrete topology τ. For a topological space (N, τ), we have, $\alpha O(N, \tau) = \tau$. Let $\gamma : \alpha O(N, \tau) \to P(N)$ be an operation defined by $A^\gamma = N$ for every set $A \in \alpha O(N, \tau)$. Then, N is α-γ-compact but not α-compact.*

Remark 3. *If γ is α-identity operation, then α-γ-compactness coincides with α-compact.*

Theorem 4. *If a topological space (X, τ) is α-γ-compact for some operation γ on $\alpha O(X)$ such that (X, τ) is α_γ-regular, then (X, τ) is α-compact.*

Proof. Let $\mathcal{U} = \{U_i : i \in I\}$ be an α-open cover of X. Since X is α_γ-regular, then for each $i \in I$, $V_i^\gamma \subseteq U_i$. Since V_i is α-open set, therefore the set $\{V_i : i \in I\}$ is an α-open cover of X. Since X is α-γ-compact, there is a finite subset I_0 of I such that $X = \bigcup\{V_i^\gamma : i \in I_0\}$. For each $i \in I_0$, there exists U_i such that $V_i^\gamma \subseteq U_i$, therefore we have $X = \bigcup\{U_i : i \in I_0\}$ and so X is α-compact. □

Theorem 5. *Let (X, τ) be a topological space, and γ an α-monotone operation on $\alpha O(X)$. Then, the following conditions are equivalent:*

1. *(X, τ) is α-γ-compact.*
2. *Each maximal filterbase in X α-γ-converges to some point of X.*
3. *Each filterbase in X α-γ-accumulates to some point of X.*

Proof. (1) $\Rightarrow$ (2): Suppose that X is α-γ-compact space and let $\mathcal{F} = \{F_i : i \in I\}$ be a maximal filterbase. Suppose that $\mathcal{F}$ does not α-γ-converges to any point of X. Since $\mathcal{F}$ is maximal, by Corollary 2, $\mathcal{F}$ does not α-γ-accumulates to any point of X.

This implies that for every $x \in X$, there exists an α-open set V_x and an $F_{i(x)} \in \mathcal{F}$ such that $F_{i(x)} \cap V_x^\gamma = \phi$. The family $\{V_x : x \in X\}$ is an α-open cover of X and by hypothesis, there exists a finite number of points $x_1, x_2, ..., x_n$ of X such that $X = \cup\{V_{(xj)}^\gamma : j = 1, 2, ..., n\}$. Since $\mathcal{F}$ is a filterbase on X, there exists an $F_0 \in \mathcal{F}$ such that $F_0 \subseteq \cap\{F_{i(xj)} : j = 1, 2, ..., n\}$. Hence $F_0 \cap V_{(xj)}^\gamma = \phi$ for $j = 1, 2, ..., n$. Which implies that $F_0 \cap \{\cup V_{(xj)}^\gamma : j = 1, 2, ..., n\} = F_0 \cap X = \phi$. Therefore, we obtain $F_0 = \phi$. Contracting the fact that $F_0 \neq \phi$.

(2) $\Rightarrow$ (3): Let $\mathcal{F}$ be any filterbase on X. Then, there exists a maximal filterbase $\mathcal{F}_0$ such that $\mathcal{F} \subseteq \mathcal{F}_0$. By hypothesis, $\mathcal{F}_0$ α-γ-converges to some point $x \in X$. For every $F \in \mathcal{F}$ and every α-open set V containing x, there exists an $F_0 \in \mathcal{F}_0$ such that $F_0 \subseteq V^\gamma$, hence $\phi \neq F_0 \cap F \subseteq V^\gamma \cap F$. This shows that $\mathcal{F}$ α-γ-accumulates at x.

(3) $\Rightarrow$ (1): Let $\mathcal{U} = \{U_i : i \in I\}$ be an α-open cover of X such that $X \neq \cup_{i=1}^n U_i^\gamma$. Let $\mathcal{B}$ denote the set of all sets of the form $\bigcap_{i=1}^n (U_i^\gamma)^c$. Since $\bigcap (U_i^\gamma)^c \neq \phi$, $\mathcal{B}$ is filterbase in X and so by our assumption, it α-γ-accumulates to some point $x \in X$. But then x does belong to some $U \in \mathcal{U}$ and so $(U^\gamma)^c \in \mathcal{B}$ yields a contradiction $(U^\gamma)^c \cap U \neq \phi$. This completes the proof. □

Proposition 3. *Let X be an α-γ-compact space. Then, for every regular open cover $\{F_i : i \in I\}$ of X, there exists a finite subset I_0 of I such that $X = \bigcup\{F_i^\gamma : i \in I_0\}$.*

Proof. Let $\{F_i : i \in I\}$ be any regular open cover of X. Since, $F_i \in \alpha O(X)$ for each $i \in I$, then the family $\{F_i : i \in I\}$ forms α-open of X, since X is α-γ-compact space, there exists a finite subset I_0 of I such that $X = \bigcup\{F_i^\gamma : i \in I0\}$. □

Proposition 4. *Let X be an α-γ-compact space. Then, for every family $\{X \setminus V_i : i \in I\}$ of regular closed subsets of X such that $\bigcap\{X \setminus V_i : i \in I\} = \phi$, there exists a finite subset I_0 of I such that $\bigcap\{X \setminus V_i^\gamma : i \in I\} = \phi$.*

Proof. let $\{X \setminus V_i : i \in I\}$ be a family of regular closed such that $\bigcap\{X \setminus V_i : i \in I\} = \phi$. So $\{V_i : i \in I\}$ is a family of regular open and $X = \bigcup\{V_i : i \in I\}$, by Proposition 3, there exists a finite subset I_0 of I such that $X = \bigcup\{V_i^\gamma : i \in I_0\}$ implies that $\phi = \bigcap\{X \setminus V_i^\gamma : i \in I_0\}$. □

Definition 8. *A subset A of a topological space (X, τ) is said to be α-γ-compact of X if for every α-open cover $\{V_i : i \in I\}$ of A, there exists a finite subset I_0 of I such that $A \subseteq \bigcup\{V_i^\gamma : i \in I_0\}$.*

Theorem 6. *Let (X, τ) be a topological space, K be a subset of X and γ be operation on $\alpha O(X)$ which is α-stable with respect to K. If X is α-γ-compact and K is α_γ-closed, then K is α-γ_K-compact.*

Proof. Let $\mathcal{U} = \{U_\beta\}_{\beta \in I}$ be an α-open cover of K by $\alpha|K$. Lets $\mathcal{U}^*$ be the family of all α-open sets such that for each $V \in \mathcal{U}^*$, $V \cap K \in \mathcal{U}$. Since $X \setminus K$ is α_γ-open, we can take an α-open cover of $X \setminus K$ say $\mathcal{W} = \{W_x \in \alpha O(X) : W_x^\gamma \subseteq X \setminus K, x \in X \setminus K\}$. Then the collection $\mathcal{U}^* \cup \mathcal{W}$ is an α-open cover of X. Since X is α-γ-compact, we have two finite subcollections $\{V_1, ..., V_n\} \subseteq \mathcal{U}^*$ and $\{W_1, ..., W_m\} \subseteq \mathcal{W}$ such that $X = \{\cup_{i=1}^n V_i^\gamma\} \cup \{\cup_{j=1}^m W_j^\gamma\}$. Then $K = \{\cup_{i=1}^n V_i^\gamma \cap K\} \cup \{\cup_{j=1}^m W_j^\gamma \cap K\} = \cup_{i=1}^n (V_i \cap K)^{\gamma_K} = \cup_{i=1}^n (U_i)^{\gamma_K}$, since $W_j^\gamma \cap K = \phi$ for $j = 1, 2, ..., m$ and γ is α-stable with respect to K. Therefore K is α-γ_K-compact. □

Theorem 7. *Let (X, τ) be a topological space and K be a subset of X. Let γ be an operation on $\alpha O(X)$ and α-stable with respect to K. Then K is α-γ-compact if and only if K is α-γ_K-compact.*

Proof. Suppose that $K \subseteq X$ is α-γ-compact and let $\mathcal{C}$ be an α-open cover of K by $\alpha|K$. Then the set $\mathcal{C}$ of all $G \in \alpha O(X)$ with $G \cap K \in \mathcal{C}$ is an α-open cover of K, and hence we can find a subfamily $\{G_1, G_2, ..., G_n\}$ of $\mathcal{C}$ such that $K \subseteq \cup_{i=1}^n G_i^\gamma$. Therefore we have

$K = (\cup_{i=1}^n G_i^\gamma) \cap K = \cup_{i=1}^n (G_i^\gamma \cap K) = \cup_{i=1}^n (G_i \cap K)^{\gamma_K}$.

Conversely, suppose that K is α-γ_K-compact. If $\mathcal{C}$ is an α-open cover of K, then $\{G \cap K : G \in \alpha O(X)\} \subseteq \alpha|K$ is a cover of K, and so there exists a finite subfamily $\{G_1, G_2, ..., G_n\}$ of $\mathcal{C}$ such that

$K = \cup_{i=1}^n (G_i \cap K)^{\gamma_K} = \cup_{i=1}^n (G_i^\gamma \cap K) \subseteq \cup_{i=1}^n G_i^\gamma$. □

Theorem 8. *Let K be a subset of X, $\gamma : \alpha O(X) \to P(X)$ and $\gamma_K : \alpha|K \to P(K)$ be operations satisfying the following properties, $(V \cap K)^{\gamma_K} \subseteq V^\gamma \cap K$ for any α-open set V of X such that $V \cap K \neq \phi$. If K is α-γ_K-compact in $(K, \alpha|K)$, then K is α-γ-compact.*

Proof. Let $\mathcal{C}$ be an α-open cover of K. Then $\{G \cap K : G \in C\} \subseteq \alpha|K$ is a cover of K and so there exists a finite subfamily $\{G_1, G_2, ..., G_n\}$ of $\mathcal{C}$ such that $K = \cup_{i=1}^n (G_i \cap K)^{\gamma_K} \subseteq \cup_{i=1}^n G_i^\gamma \cap K \subseteq \cup_{i=1}^n G_i^\gamma$. Therefore, K is α-γ-compact. □

Theorem 9. *A space X is α-γ-compact if and only if every proper α-closed subset of X is α-γ-compact.*

Proof. Let F be any proper α-closed subset of X. Let $\{V_i : i \in I\}$ be an α-open cover of F. Since F is α-closed set, then $X \setminus F$ is α-open set. So the family $\{V_i : i \in I\} \cup X \setminus F$ is an α-open cover of X. Since X is α-γ-compact, there exists a finite subset I_0 of I such that $X = \bigcup\{V_i^\gamma : i \in I_0\} \cup (X \setminus F)^\gamma$. Therefore, we obtain $F \subseteq \bigcup\{V_i^\gamma : i \in I_0\} \cup (X \setminus F)^\gamma$. Hence F is α-γ-compact of X.

Conversely, let $\{V_i : i \in I\}$ be an α-open cover of X. Suppose that $X \neq V_{i_0} \neq \phi$ for every $i_0 \in I$. Then $X \setminus V_{i_0}$ is a proper α-closed subset of X. Therefore, by hypothesis, there exists a finite subset I_0 of I such that $X \setminus V_{i_0} \subseteq \cup\{V_i^\gamma : i \in I_0\}$. Therefore, we obtain $X = \bigcup\{V_i^\gamma : i \in I_0\} \cup V_{i_0}^\gamma$. Which shows that X is α-γ-compact. □

Theorem 10. *Let A and B be subsets of a space X such that $A \cap B \neq \phi$. If A is α-γ-compact subset of X and B is α-closed set, then $A \cap B$ is α-γ-compact subset of X.*

Proof. Let $\{V_i : i \in I\}$ be any α-open cover of $A \cap B$. Since B is α-closed set, then $X \setminus B$ is α-open. So the family $\{V_i : i \in I\} \cup X \setminus B$ is an α-open cover of A. Since A is α-γ-compact subset of X, then there exists a finite subset I_0 of I such that $A \subseteq \{V_i^\gamma : i \in I_0\} \cup (X \setminus B)^\gamma$. Therefore, we obtain $A \cap B \subseteq \{V_i^\gamma : i \in I_0\} \cup (X \setminus B)^\gamma$. Hence $A \cap B$ is α-γ-compact subset of X. □

Corollary 3. *Let A be an α-γ-compact subset of X. If B is an α-closed set of X and $B \subseteq A$, then B is α-γ-compact subset of X.*

Theorem 11. *Let A be any subset of a topological space (X, τ) such that A and $X \setminus A$ are α-γ-compact subsets of X, then X is α-γ-compact.*

Proof. Let $\{V_i : i \in I\}$ be any α-open cover of $X = A \cup X \setminus A$. Then $\{V_i : i \in I\}$ is an α-open cover of A and $X \setminus A$. Therefore, there exists finite subfamilies I_0 and I_1 of I such that $A \subseteq \bigcup\{V_i^\gamma : i \in I_0\}$ and $X \setminus A \subseteq \bigcup\{V_i^\gamma : i \in I_1\}$. Thus, $X = A \cup X \setminus A \subseteq \bigcup\{V_i^\gamma : i \in I_0 \cup I_1\}$. This completes the proof. □

Corollary 4. *The finite union of α-γ-compact subsets of X is α-γ-compact.*

Proof. Straightforward. □

Theorem 12. *Let B be α-γ-compact subset of X and G be α-open subset of a space X such that $G \subseteq B$. Then $B \setminus G$ is α-γ-compact subset of X.*

Proof. Obvious. □

Theorem 13. *Let (X, τ) be a topological space and γ be α-regular operation on $\alpha O(X)$. If X is α-γ-T_2 and $K \subseteq X$ is α-γ-compact, then K is α_γ-closed.*

Proof. We need to prove that $X \setminus K$ is α_γ-open. So let $x_0 \in X \setminus K$. For each $y \in K$, there exists α-open sets U_y and V_y such that $x_0 \in U_y$, $y \in V_y$ and $U_y^\gamma \cap V_y^\gamma = \phi$. In this way we construct an α-open cover $\mathcal{U} = \{V_y : y \in K\}$ of K. Since K is α-γ-compact, there exists a finite collection $\{V_{y_1}, ..., V_{y_n}\}$ of $\mathcal{U}$ such that $K \subseteq \cup_{i=1}^n V_{y_i}^\gamma$. Let $U = \cap_{i=1}^n U_{y_i}$. We can see that U is an α-open set containing x_0, but it does not have to happen that $U^\gamma \subseteq X \setminus K$. Here we need the α-regularity of γ to achieve our purpose. Since $U_{y_1}, ..., U_{y_n}$ are α-open sets containing x_0, then using the α-regularity of γ there exists an α-open set W containing x_0, such that $W \subseteq W^\gamma \subseteq X \setminus K$. This implies that $X \setminus K$ is α_γ-open, and hence K is α_γ-closed. □

Theorem 14. *Let (X, τ) be a topological space and γ an α-additive operation on $\alpha O(X)$. If $Y \subseteq X$ is α-γ-compact, $x \in X \setminus Y$ and (X, τ) is α-γ-T_2, then there exist α-open sets U and V with $x \in U$, $Y \subseteq V^\gamma$ and $U^\gamma \cap V^\gamma = \phi$.*

Proof. For each $y \in Y$, let V_y and V_x^y be α-open sets such that $V_y^\gamma \cap V_x^{y\gamma} = \phi$, with $y \in V_y$ and $x \in V_x^y$. The collection $\mathcal{V} = \{V_y : y \in Y\}$ is an α-open cover of Y. Now, since Y is α-γ-compact, there exists a finite subcollection $\{V_{y_1}, ..., V_{y_n}\}$ of $\mathcal{V}$ such that $Y \subseteq \cup_{i=1}^n V_{y_i}^\gamma$. Let $U = \cap_{i=1}^n V_x^{y_i}$ and $V = \cup_{i=1}^n V_{y_i}$. Since $U \subseteq V_x^{y_i}$ for every

$i \in \{1, 2, ..., n\}$, then $U^\gamma \cap V^\gamma_{y_i} = \phi$ for every $i \in \{1, 2, ..., n\}$. Then $U^\gamma \cap V^\gamma = \phi$ as γ is an α-additive operation on $\alpha O(X)$ and $Y \subseteq V^\gamma$. $\square$

Theorem 15. *Let (X, τ) be a topological space and γ an α-additive operation on $\alpha O(X)$. Suppose that $\bigcap_{i=1}^{k} K_i^\gamma \subseteq (\bigcap_{i=1}^{k} K_i)^\gamma$ for any collection $\{K_1, K_2, ..., K_k\} \subseteq \alpha O(X)$. If A and B are disjoint α-γ-compact subsets of X and (X, τ) is α-γ-T_2, then there exist disjoint α-open sets U and V such that $A \subseteq U^\gamma$ and $B \subseteq V^\gamma$.*

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Current address: Alias B. Khalaf: Department of Mathematics, Faculty of Science, University of Duhok, Iraq.
E-mail address: aliasbkhalaf@gmail.com
Current address: Hariwan Z. Ibrahim: Department of Mathematics, Faculty of Science, University of Zakho, Iraq
E-mail address: hariwan_math@yahoo.com