Available online: June 15, 2017

Commun.Fac.Sci.Univ.Ank.Series A1
Volume 67, Number 1, Pages 68–79 (2018)
DOI: 10.1501/Commua1_0000000831
ISSN 1303–5991
http://communications.science.ankara.edu.tr/index.php?series=A1

ANKARA UNIVERSITY FACULTY OF SCIENCES 1948
COMMUNICATIONS SERIES A1

ON THE WAVE SOLUTIONS OF CONFORMABLE FRACTIONAL EVOLUTION EQUATIONS

ALPER KORKMAZ

Abstract. The exact solutions in the wave form are derived for the time fractional KdV and the time fractional Burgers' equations in conformable fractional derivative sense. The fractional variable change using the fundamental properties of the conformable derivative reduces both equations to some nonlinear ODEs. The predicted solution is assumed to be in a finite series form of a function satisfying a particular first-order ODE whose solution contains an exponential function in the denominator. The solutions are represented in explicit forms and illustrated by some choices of the parameters for various fractional orders of the equations. The solutions are illustrated for various values of parameters covering derivative order α.

1. Introduction

Recent developments in symbolic programming and computer algebra have enabled to solve more complicated problems in many fields covering engineering, physics, mathematics and the related fields. Moreover, many new techniques have been derived to solve different problems in various forms. The reflections of all stimulate the applied mathematicians to suggest new techniques for solutions of PDEs, particularly the nonlinear ones.

In the last several decades, we all have witnessed that the number of the studies dealing with many problems described by the nonlinear PDEs increases rapidly. Many new methods from the tanh method to different types of expansion methods and the others such as the methods based on ansatzes, exponential rational functions, trial equation, extended equation or first integrals are implemented to nonlinear PDEs covering fractional forms [1–11]. The expansion methods class is a special family of these techniques. There are numerous practical techniques covering the Jacobi elliptic, the exp-function, the hyperbolic tangent expansions

Received by the editors: December 23, 2016, Accepted: March 14 2017.

2010 *Mathematics Subject Classification.* 35C07;35R11;35Q53.

Key words and phrases. Time fractional KdV equation, time fractional Burgers equation, Kudryashov method, wave solution.

and their variations, modifications or generalizations in the literature. The F-expansion method in the generalized form, for example, is used to develop some Jacobi elliptic-type exact solutions, soliton-like and trigonometric type solutions for the Konopelchenko-Dubrovsky equation in two space dimension [12]. The method of (G'/G)-expansion is also a widely used method to derive the solutions to the nonlinear PDEs. In this method, G is chosen as a solution of a second-order ODE. The coupled KdV-mKdV, the KdV-Burgers' and the reaction-diffusion equation have exact solutions represented in the finite series [13]. Sub-equation approach has also been implemented to express some solutions to the generalized Kuramoto-Sivashinsky equation in the conformable fractional sense [14]. Another expansion based on Jacobi elliptic functions has been implemented in a recent study to derive solutions to Boussinesq equation and Kdv-modified KdV equations in the conformable fractional forms [15]. Kurt *et al.* have derived approximate analytical solution of Burgers'-Korteweg-de Vries equation in the conformable fractional form by homotopy analysis method [16].

The variations of the Kudryashov method can also be classified in the expansion methods. The method, briefly, predicts a solution in a finite series form of a function solving a particular first-order ODE. The determination of the coefficients used in the series are determined by forcing the solution to satisfy the equation. Kudryashov, himself, describes the method as one of old methods to solve nonlinear differential equations exactly [17]. That study focuses on exact solutions of the Fisher and a higher order nonlinear PDE and proposes exact solutions in a finite series. Kabir' s study suggests some solitary wave solutions in traveling form for some higher order nonlinear PDEs [18]. Some exact solutions in series of rational functions with exponential components form are derived by Tandogan *et al.* to the power non-linear Rosenau-Kawahara equation [19]. Hosseini *et al.* deal with various nonlinear conformal time fractional Klein-Gordon equations by using the modified form of Kudryashov method [20, 21].

The present study aims to determine some explicit wave type exact solutions of the conformable time fractional Burgers' equation (ctfBE) of the form

$$D_t^{\alpha}(u) + \varepsilon u u_x - \nu u_{xx} = 0,\ t > 0 \tag{1.1}$$

and the conformable time fractional KdV equation (ctfKdVE)

$$D_t^{\alpha}(u) + \varepsilon u u_x + \beta u_{xxx} = 0,\ t > 0 \tag{1.2}$$

where $D_t^{\alpha}(u)$ stands for α fractional derivative of the function u with respect to the variable t by implementing the Kudryashov method in modified form. Before starting to describe the method, some significant properties of the conformable derivative are explained in the next section. The following sections involve the implement of the method to the ctfBE and to the ctfKdVE.

2. Conformable Fractional Derivative

Consider $f = f(t)$ defined in the positive semi half space $t > 0$. The conformable derivative of order α of f is defined as

$$D_t^\alpha(f(t)) = \lim_{h\to 0} \frac{f(t + ht^{1-\alpha}) - f(t)}{h},\ t > 0,\ \alpha \in (0, 1] \tag{2.1}$$

for $f : [0, \infty) \to \mathbb{R}$ [22]. The conformable fractional derivative defined above has properties given in the Theorem 1.

Theorem 1. *Assume that $\alpha \in (0, 1]$ is the derivative order, and suppose that $v = v(t)$ and $w = w(t)$ are α-differentiable for all positive t. Then,*

- $D_t^\alpha(av + bw) = aD_t^\alpha(v) + bD_t^\alpha(w)$
- $D_t^\alpha(t^p) = pt^{p-\alpha}, \forall p \in \mathbb{R}$
- $D_t^\alpha(v(t)) = 0$, *for all constant function* $v(t) = \lambda$
- $D_t^\alpha(vw) = vD_t^\alpha(w) + wD_t^\alpha(v)$
- $D_t^\alpha(\frac{v}{w}) = \dfrac{wD_t^\alpha(v) - vD_t^\alpha(w)}{w^2}$
- $D_t^\alpha(v)(t) = t^{1-\alpha}\frac{dv}{dt}$

for all real a, b [23, 24].

The conformable derivative defined in (2.1) has significant properties like the chain rule and Gronwall's inequality [25]. A useful one is the relation between the conformable derivative and the classical integer ordered derivative in the definition of the composite function.

Theorem 2. *Let v be a differentiable and α-conformable differentiable function and w also be defined defined in the range of the function v and be differentiable. Thus,*

$$D_t^\alpha(v \circ w) = t^{1-\alpha} D_t^\alpha(w)(t) D_t^\alpha v(w(t)) \tag{2.2}$$

where $'$ denotes the derivative with respect to t.

3. Description of the Modified Kudryashov Method

Consider a nonlinear PDE of the form

$$P(u, u_t^\alpha, u_x, u_t^{2\alpha}, u_{xx}, ...) = 0 \tag{3.1}$$

where $u = u(x, t)$ and the fractional derivative order $\alpha \in (0, 1]$. The classical transformation

$$u(x, t) = u(\xi), \xi = x - \frac{c}{\alpha} t^\alpha \tag{3.2}$$

gives an ODE of the form

$$R(u, u', u'', \ldots) = 0 \tag{3.3}$$

where the prime ($'$) shows the derivative of u w.r.t. the transformation variable ξ [26].

Consider the equation (3.3) has a solution of the form

$$u(\xi) = a_0 + a_1 Q(\xi) + a_2 Q^2(\xi) + \dots a_n Q^n(\xi) \tag{3.4}$$

for a finite n where $a_n \neq 0$ and all $a_i, 0 \leq i \leq n$ are constants. This polynomial of $Q(\xi)$ is assumed to satisfy the first-order differential equation

$$Q^{'}(\xi) = Q(\xi)(Q(\xi) - 1) \ln A \tag{3.5}$$

Thus, one can determine it as

$$Q(\xi) = \frac{1}{1 + dA^{\xi}}$$

where d and A are nonzero constants with $A > 0$ and $A \neq 1$. The balance between the nonlinear term and the term having the highest order derivative in (3.3) gives the degree n of the power series (3.4). Since (3.4) is a solution, it must satisfy (3.3). Substituting it into (3.3) and rearranging the resultant equation for the powers of $Q(\xi)$ leads a polynomial for $Q(\xi)$. The obtained polynomial equality is solved by equating the coefficients to zero. Thus, the coefficients $a_0, a_1, a_2, \dots a_n$ are determined algebraically in terms of other parameters originated from the regarding equation, the transformation and the other operations if exist for nonzero a_n.

4. The solution of the ctfBE

The transformation (3.2) decreases the dimension of the cftBE(1.1) to one as

$$-cu^{'} + \varepsilon u u^{'} - \nu u^{''} = 0 \tag{4.1}$$

where $(^{'})$ stands for $\frac{d}{d\xi}$. Integrating (4.1) once gives

$$-cu + \varepsilon \frac{1}{2} u^2 - \nu u^{'} = K \tag{4.2}$$

where K is integral constant. The balance of u^2 and $u^{'}$ gives $n = 1$. Thus, the solution should be expressed as

$$u(\xi) = a_0 + a_1 Q(\xi) \tag{4.3}$$

for a nonzero a_1. Substituting the solution (4.3) and its derivative into (4.2) gives

$$\left(\frac{1}{2}\, \varepsilon\, {a_1}^2 - \nu\, a_1 \ln(A)\right) Q^2(\xi) + \left(\varepsilon\, a_0 a_1 - c a_1 + \nu\, a_1 \ln(A)\right) Q(\xi) - c a_0 + \frac{1}{2}\, \varepsilon\, {a_0}^2 - K = 0 \tag{4.4}$$

Equating the coefficients of each power of $Q(\xi)$ and the constant term to zero yields the algebraic system of equations

$$\begin{aligned} -K - c a_0 + \frac{1}{2}\, \varepsilon\, {a_0}^2 &= 0 \\ \varepsilon\, a_0 a_1 - c a_1 + \nu\, a_1 \ln(A) &= 0 \\ \frac{1}{2}\, \varepsilon\, {a_1}^2 - \nu\, a_1 \ln(A) &= 0 \end{aligned} \tag{4.5}$$

This system has various solutions for $a_1 \neq 0$:
Solution 1: When the solution of the system (4.5) is chosen as

$$\begin{aligned} a_0 &= -\frac{\nu\,\ln(A) + \sqrt{\nu^2\,(\ln(A))^2 - 2\,\varepsilon\,K}}{\varepsilon} \\ a_1 &= 2\,\frac{\nu\,\ln(A)}{\varepsilon} \\ c &= -\sqrt{\nu^2\,(\ln(A))^2 - 2\,\varepsilon\,K} \end{aligned} \tag{4.6}$$

the solution of (4.2) is constructed as

$$u(\xi) = -\frac{\nu\,\ln(A) + \sqrt{\nu^2\,(\ln(A))^2 - 2\,\varepsilon\,K}}{\varepsilon} + 2\,\frac{\nu\,\ln(A)}{\varepsilon}\frac{1}{1 + dA^{\xi}}, \tag{4.7}$$

where $\sqrt{\nu^2\,(\ln(A))^2 - 2\,\varepsilon\,K} \geq 0$ and $\varepsilon \neq 0$. Thus, the solution of the ctfBE (1.1) is expressed as

$$u_1(x,t) = -\frac{\nu\,\ln(A) + \sqrt{\nu^2\,(\ln(A))^2 - 2\,\varepsilon\,K}}{\varepsilon} + 2\,\frac{\nu\,\ln(A)}{\varepsilon}\frac{1}{1 + dA^{x + \sqrt{\nu^2(\ln(A))^2 - 2\,\varepsilon\,K}\,\frac{t^{\alpha}}{\alpha}}} \tag{4.8}$$

Solution 2: When the solution of the system (4.5) is chosen as

$$\begin{aligned} a_0 &= -\frac{\nu\,\ln(A) - \sqrt{\nu^2\,(\ln(A))^2 - 2\,\varepsilon\,K}}{\varepsilon} \\ a_1 &= 2\,\frac{\nu\,\ln(A)}{\varepsilon} \\ c &= \sqrt{\nu^2\,(\ln(A))^2 - 2\,\varepsilon\,K} \end{aligned} \tag{4.9}$$

the solution of the ODE (4.2) can be written as

$$u(\xi) = -\frac{\nu\,\ln(A) - \sqrt{\nu^2\,(\ln(A))^2 - 2\,\varepsilon\,K}}{\varepsilon} + 2\,\frac{\nu\,\ln(A)}{\varepsilon}\frac{1}{1 + dA^{\xi}} \tag{4.10}$$

with the conditions $\sqrt{\nu^2\,(\ln(A))^2 - 2\,\varepsilon\,K} \geq 0$ and $\varepsilon \neq 0$. Thus, the exact solution of the ctfBE (1.1) is written in an explicit form as

$$u_2(x,t) = -\frac{\nu\,\ln(A) - \sqrt{\nu^2\,(\ln(A))^2 - 2\,\varepsilon\,K}}{\varepsilon} + 2\,\frac{\nu\,\ln(A)}{\varepsilon}\frac{1}{1 + dA^{x - \sqrt{\nu^2(\ln(A))^2 - 2\,\varepsilon\,K}\,\frac{t^{\alpha}}{\alpha}}} \tag{4.11}$$

Some solutions derived from $u_1(x,t)$ for the parameter values $\varepsilon = 1$, $A = 3$, $d = 1$, $K = 1$, $\nu = 4$ are illustrated in Fig 1(a)-1(d) for various values of the derivative order α. In all choices of α, an initial wave moves to the right along the space axis. When the derivative order α changes from zero to one, the shape of the wave does not change. On the other hand, the speed of the wave is larger at the beginning of the motion but gets slower as time variable increases for smaller values of α. When α is chosen as one, we observe that the initial wave moves with a constant speed.

5. The solution of the ctfKdVE

The transformation (3.2) converts the ctfKdVE to

$$-cu^{'}+\varepsilon uu^{'}+\beta u^{'''}=0 \tag{5.1}$$

Integrating the equation (5.1)once changes it to

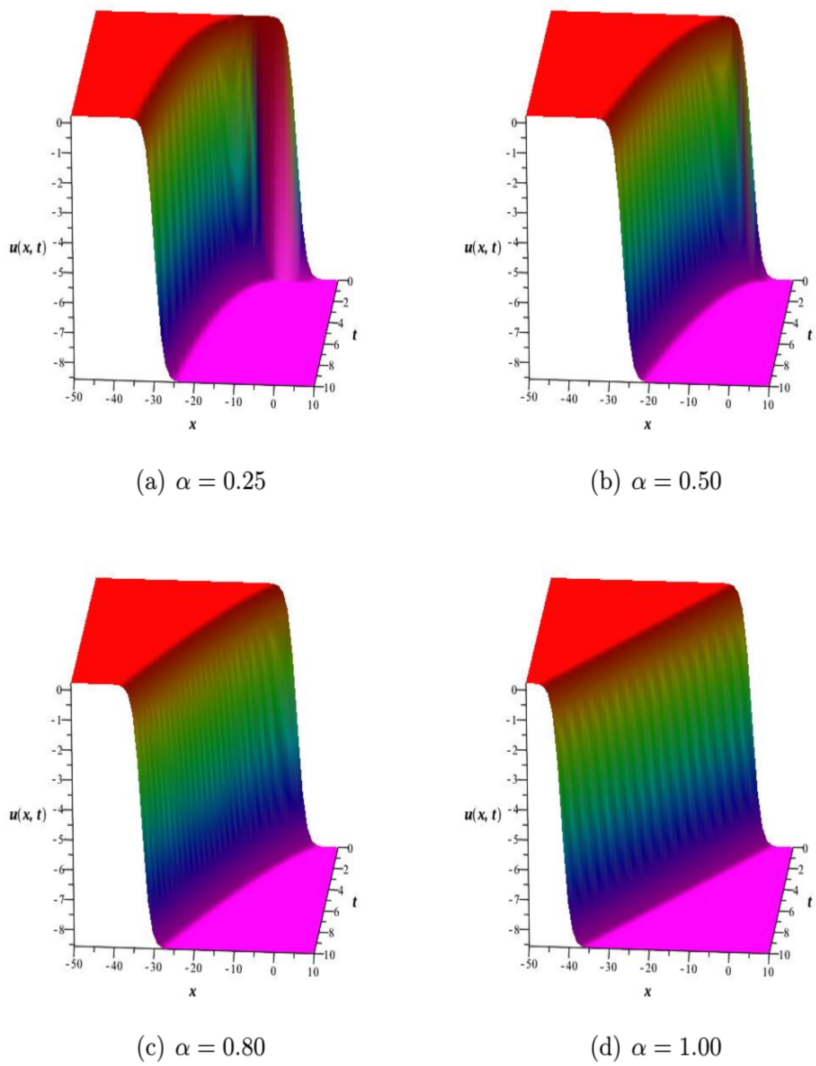
$$-cu+\varepsilon\frac{1}{2}u^2+\beta u^{''}=K \tag{5.2}$$

where K is the integration constant. The balance of u^2 and $u^{''}$ gives $n=2$. Substituting the predicted solution $u(\xi)=a_0+a_1Q(\xi)+a_2Q^2(\xi)$, $a_2\neq 0$ into (5.2) yields

$$\begin{aligned}&\left(\frac{1}{2}\varepsilon\, {a_2}^2+6\,\beta\, a_2\,(\ln(A))^2\right)Q^4(\xi)+\left(2\,\beta\, a_1\,(\ln(A))^2-10\,\beta\, a_2\,(\ln(A))^2+\varepsilon\, a_1a_2\right)Q^3(\xi)\\&+\left(\frac{1}{2}\varepsilon\, {a_1}^2+\varepsilon\, a_0a_2-3\,\beta\, a_1\,(\ln(A))^2+4\,\beta\, a_2\,(\ln(A))^2-ca_2\right)Q^2(\xi)\\&+\left(-ca_1+\varepsilon\, a_0a_1+\beta\, a_1\,(\ln(A))^2\right)Q(\xi)+\frac{1}{2}\varepsilon\, {a_0}^2-K-ca_0=0\end{aligned} \tag{5.3}$$

in the arranged form. Forcing the coefficients of the powers of $Q(\xi)$ and the constant term to be zero gives an algebraic system

$$\begin{aligned}\frac{1}{2}\varepsilon\, {a_0}^2-K-ca_0&=0\\-ca_1+\varepsilon\, a_0a_1+\beta\, a_1\,(\ln(A))^2&=0\\\frac{1}{2}\varepsilon\, {a_1}^2+\varepsilon\, a_0a_2-3\,\beta\, a_1\,(\ln(A))^2+4\,\beta\, a_2\,(\ln(A))^2-ca_2&=0\\2\,\beta\, a_1\,(\ln(A))^2-10\,\beta\, a_2\,(\ln(A))^2+\varepsilon\, a_1a_2&=0\\\frac{1}{2}\varepsilon\, {a_2}^2+6\,\beta\, a_2\,(\ln(A))^2&=0\end{aligned} \tag{5.4}$$



(a) $\alpha = 0.25$ (b) $\alpha = 0.50$

(c) $\alpha = 0.80$ (d) $\alpha = 1.00$

FIGURE 1. Illustrations of $u_1(x,t)$ for $\varepsilon = 1$, $A = 3$, $d = 1$, $K = 1$, $\nu = 4$

Solution 1: The solution

$$
\begin{aligned}
a_0 &= \frac{-\beta\,(\ln(A))^2 + \sqrt{\beta^2\,(\ln(A))^4 - 2\,\varepsilon\,K}}{\varepsilon}\\
a_1 &= 12\,\frac{\beta\,(\ln(A))^2}{\varepsilon}\\
a_2 &= -12\,\frac{\beta\,(\ln(A))^2}{\varepsilon}\\
c &= \sqrt{\beta^2\,(\ln(A))^4 - 2\,\varepsilon\,K}
\end{aligned}
\tag{5.5}
$$

of the system (5.4) gives the solution of the ODE (5.2) as

$$\begin{aligned} u(\xi) \quad = \quad & \frac{-\beta\,(\ln(A))^2 + \sqrt{\beta^2\,(\ln(A))^4 - 2\,\varepsilon\,K}}{\varepsilon} + 12\,\frac{\beta\,(\ln(A))^2}{\varepsilon}\,\frac{1}{1+dA^{\xi}} \\ & -12\,\frac{\beta\,(\ln(A))^2}{\varepsilon}\,\frac{1}{(1+dA^{\xi})^2} \end{aligned} \tag{5.6}$$

where $\sqrt{\beta^2\,(\ln(A))^4 - 2\,\varepsilon\,K} \geq 0$ and $\varepsilon \neq 0$. Thus, the exact solution of the ct-fKdVE (1.2) is written in an explicit form as

$$\begin{aligned} u_3(x,t) \quad = \quad & \frac{-\beta\,(\ln(A))^2 + \sqrt{\beta^2\,(\ln(A))^4 - 2\,\varepsilon\,K}}{\varepsilon} \\ & +12\,\frac{\beta\,(\ln(A))^2}{\varepsilon}\,\frac{1}{1+dA^{x-\sqrt{\beta^2(\ln(A))^4-2\,\varepsilon\,K}\frac{t^{\alpha}}{\alpha}}} \\ & -12\,\frac{\beta\,(\ln(A))^2}{\varepsilon}\,\frac{1}{\left(1+dA^{x-\sqrt{\beta^2(\ln(A))^4-2\,\varepsilon\,K}\frac{t^{\alpha}}{\alpha}}\right)^2} \end{aligned} \tag{5.7}$$

Solution 2: Similarly, the solution

$$\begin{gathered} a_0 = -\frac{\beta\,(\ln(A))^2 + \sqrt{\beta^2\,(\ln(A))^4 - 2\,\varepsilon\,K}}{\varepsilon} \\ a_1 = 12\,\frac{\beta\,(\ln(A))^2}{\varepsilon} \\ a_2 = -12\,\frac{\beta\,(\ln(A))^2}{\varepsilon} \\ c = -\sqrt{\beta^2\,(\ln(A))^4 - 2\,\varepsilon\,K} \end{gathered} \tag{5.8}$$

of the system (5.4) gives the solution of the ODE (5.2) as

$$\begin{gathered} u(\xi) = -\frac{\beta\,(\ln(A))^2 + \sqrt{\beta^2\,(\ln(A))^4 - 2\,\varepsilon\,K}}{\varepsilon} \\ +12\,\frac{\beta\,(\ln(A))^2}{\varepsilon}\,\frac{1}{1+dA^{\xi}} \\ -12\,\frac{\beta\,(\ln(A))^2}{\varepsilon}\,\frac{1}{(1+dA^{\xi})^2} \end{gathered} \tag{5.9}$$

where $\sqrt{\beta^2\,(\ln(A))^4 - 2\,\varepsilon\,K} \geq 0$ and $\varepsilon \neq 0$. Thus, the exact solution of the ctfKdVE (1.2) is written in an explicit form as

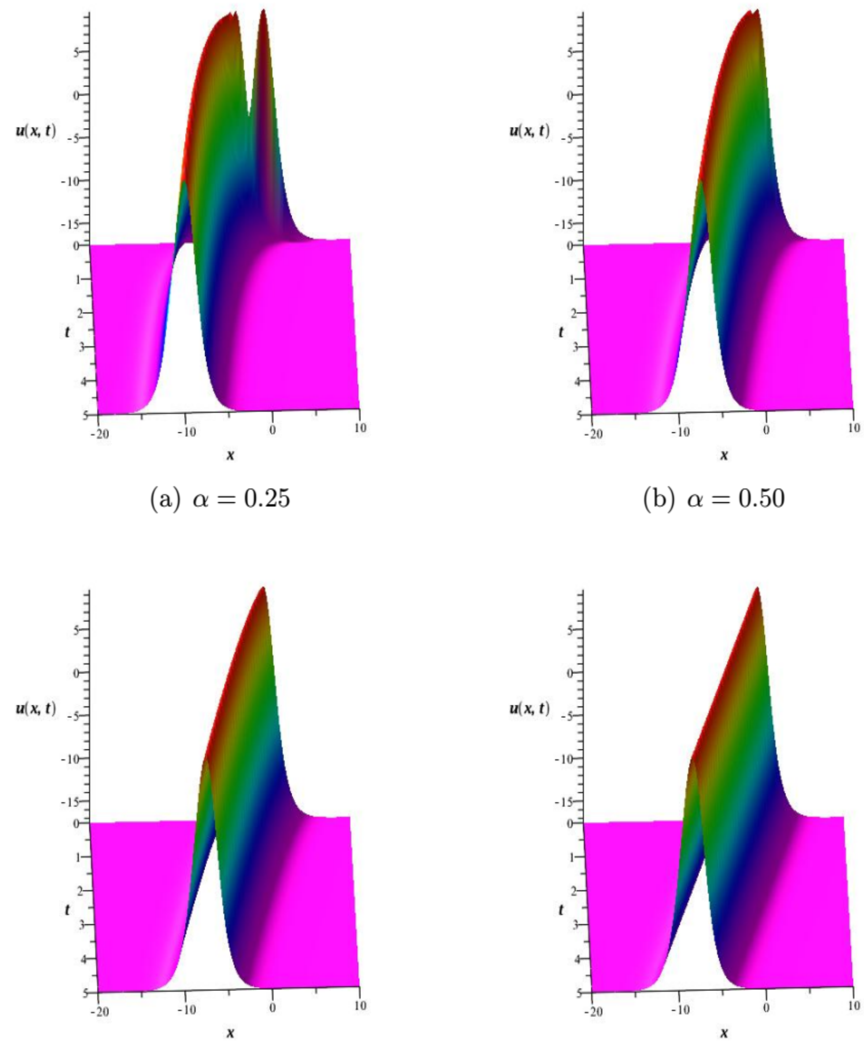
$$
\begin{aligned}
u_4(x,t) \quad = \quad & -\frac{\beta\,(\ln(A))^2 + \sqrt{\beta^2\,(\ln(A))^4 - 2\,\varepsilon\,K}}{\varepsilon} + 12\,\frac{\beta\,(\ln(A))^2}{\varepsilon}\,\frac{1}{1 + dA^{x + \sqrt{\beta^2(\ln(A))^4 - 2\,\varepsilon\,K}\,\frac{t^\alpha}{\alpha}}} \\
& -12\,\frac{\beta\,(\ln(A))^2}{\varepsilon}\,\frac{1}{\left(1 + dA^{x + \sqrt{\beta^2(\ln(A))^4 - 2\,\varepsilon\,K}\,\frac{t^\alpha}{\alpha}}\right)^2}
\end{aligned}
\tag{5.10}
$$

Some solutions are generated from the general form of the solution $u_4(x,t)$ by choosing the parameters $\varepsilon = 1/5$, $A = 3$, $d = 1$, $K = 1$, $\beta = 3/2$, Fig 2(a)-2(d). This solution represents motion of an initial positive pulse along $x-$axis without changing its shape and direction as time proceeds. The speed of the pulse is higher when t and α are smaller but as t increases the speed decreases. The change in the speed of the pulse decreases as α approaches 1.

6. Conclusion

Some conformable time fractional partial differential equations are solved by using the modified Kudryashov method. Both the ctfBE and the ctfKdVE equations are reduced to some nonlinear ODEs of integer order by using compatible wave transformations. The balance between the nonlinear term and the term with the highest order derivative gives the highest power of the series forming the solution. Substituting the solution into the resultant ODEs and some computer algebra give the relations between the parameters of the equations and the coefficients of the finite series solution.

Some explicit solutions are given for the conformable time fractional Burgers' and the conformable time fractional KdV equations. The solutions are illustrated for particular choices of the parameters and various values of α.



(a) $\alpha = 0.25$ (b) $\alpha = 0.50$

(c) $\alpha = 0.80$ (d) $\alpha = 1.00$

FIGURE 2. Illustrations of $u_4(x,t)$ for $\varepsilon = 1/5$, $A = 3$, $d = 1$, $K = 1$, $\beta = 3/2$

REFERENCES

[1] Demiray, S. T., Pandir, Y., & Bulut, H. (2016). All exact travelling wave solutions of Hirota equation and Hirota-Maccari system, *Optik-International Journal for Light and Electron Optics*, 127(4), 1848-1859.

[2] Aksoy, E., Kaplan, M., & Bekir, A. (2016). Exponential rational function method for space-time fractional differential equations *Waves in random and complex media*, 26(2), 142-151.

[3] Korkmaz, A., Exact solutions of space-time fractional EW and modified EW equations, *Chaos, Solitons & Fractals*, 2017, 96, 132-138.

[4] Guner, O., Korkmaz, A. & Bekir, A., Dark Soliton Solutions of Space-Time Fractional Sharma-Tasso-Olver and Potential Kadomtsev-Petviashvili Equations, *Communications in Theoretical Physics*, 2017, 67, 182.

[5] Guner, O., Bekir, A. & Korkmaz, A., Tanh-type and sech-type solitons for some space-time fractional PDE models, *The European Physical Journal Plus*, 2017, 132, 92.

[6] Ekici, M., Mirzazadeh, M., Eslami, M., Zhou, Q., Moshokoa, S. P., Biswas, A., & Belic, M. (2016). Optical soliton perturbation with fractional-temporal evolution by first integral method with conformable fractional derivatives, *Optik-International Journal for Light and Electron Optics*, 127(22), 10659-10669.

[7] Mohyud-Din, S. T., & Bibi, S. (2017). Exact solutions for nonlinear fractional differential equations using exponential rational function method, *Optical and Quantum Electronics*, 49(2), 64.

[8] Gurefe, Y., Misirli, E., Sonmezoglu, A., & Ekici, M. (2013). Extended trial equation method to generalized nonlinear partial differential equations *Applied Mathematics and Computation*, 219(10), 5253-5260.

[9] Mirzazadeh, M. (2015). Soliton solutions of Davey-Stewartson equation by trial equation method and ansatz approach,*Nonlinear Dynamics*, 82(4), 1775-1780.

[10] Atangana, A. (2016). Exact solution of the time-fractional groundwater flow equation within a leaky aquifer equation, *Journal of Vibration and Control*, 22(7), 1749-1756.

[11] Esen, A., Yagmurlu, N. M., & Tasbozan, O. (2016). Double exp-function method for multisoliton solutions of the Tzitzeica-Dodd-Bullough equation,*Acta Mathematicae Applicatae Sinica*, English Series, 32(2), 461-468.

[12] Zhang S., Xia T., A generalized F-expansion method and new exact solutions of Konopelchenko-Dubrovsky equations, *Applied Mathematics and Computation* (2006), 183(2), 1190-1200.

[13] Zayed, E. M. E., & Gepreel, K. A., The (G'/G)-expansion method for finding traveling wave solutions of nonlinear partial differential equations in mathematical physics, *Journal of Mathematical Physics* (2009), 50(1), 013502.

[14] Rezazadeh, H., Ziabary B. P., Sub-equation Method for the Conformable Fractional Generalized Kuramoto-Sivashinsky Equation, *Computational Research Progress in Applied Science & Engineering*, 2(3), 106-109, 2016.

[15] Tasbozan, O., Çenesiz, Y., & Kurt, A. (2016). New solutions for conformable fractional Boussinesq and combined KdV-mKdV equations using Jacobi elliptic function expansion method,*The European Physical Journal Plus*, 131(7), 244.

[16] A. Kurt, O. Tasbozan, Y. Cenesiz, Homotopy Analysis Method for Conformable Burgers-Korteweg-de Vries Equation, *Bulletin of Mathematical Sciences and Applications*, 17, 17-23, 2016

[17] Kudryashov, N. A. One method for finding exact solutions of nonlinear differential equations, *Communications in Nonlinear Science and Numerical Simulation* (2012), 17(6), 2248-2253.

[18] Kabir, M. M., Khajeh, A., Abdi Aghdam, E., & Yousefi Koma, A., Modified Kudryashov method for finding exact solitary wave solutions of higher-order nonlinear equations, *Mathematical methods in the Applied Sciences* (2011), 34(2), 213-219.

[19] Tandogan, Y. A., Pandir, Y., & Gurefe, Y., Solutions of the nonlinear differential equations by use of modified Kudryashov method, *Turkish Journal of Mathematics and Computer Science* (2013), 1, 1-7.

[20] Hosseini, K., Mayeli, P., & Ansari, R. (2017). Modified Kudryashov method for solving the conformable time-fractional Klein-Gordon equations with quadratic and cubic nonlinearities, *Optik-International Journal for Light and Electron Optics*, 130, 737-742.

[21] Hosseini, K., Bekir, A., & Ansari, R. (2016). New exact solutions of the conformable time-fractional Cahn-Allen and Cahn-Hilliard equations using the modified Kudryashov method, *Optik-International Journal for Light and Electron Optics*

[22] Khalil, R., Al Horani, M., Yousef, A., & Sababheh, M., A new definition of fractional derivative, *Journal of Computational and Applied Mathematics* (2014), 264, 65-70.

[23] Atangana, A., Baleanu, D., & Alsaedi, A., New properties of conformable derivative, *Open Mathematics* (2015) , 13(1), 1-10.

[24] Çenesiz, Y., Baleanu, D., Kurt, A., & Tasbozan, O., New exact solutions of Burgers' type equations with conformable derivative,*Waves in Random and Complex Media* (2016), 1-14.

[25] Abdeljawad, T., On conformable fractional calculus, *Journal of computational and Applied Mathematics* (2015), 279, 57-66.

[26] Eslami, M., & Rezazadeh, H., The first integral method for Wu-Zhang system with conformable time-fractional derivative, *Calcolo* (2016), 53, 475-485.

Current address: Alper Korkmaz: Çankırı Karatekin University, Department of Mathematics, Çankırı, Turkey.

E-mail address: akorkmaz@karatekin.edu.tr,alperkorkmaz7@gmail.com