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HYPERSPACES OF DITOPOLOGICAL TEXTURE SPACES AND HYPERTEXTURES

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I want to dedicate this manuscript to memory of my advisor who is actually my second father, Dr. L. Michael Brown, sleep in peace.

Abstract. The author consider hyperspaces in the setting of textures and ditopological texture spaces. According to that, the definitions of hypertexture, plain hypertexture and hyperspace of ditopological texture space are presented. Then the author obtained some properties of hypertextures in the categorical respect and give some examples of hypertextures.

1. Introduction

Hyperspace theory has been beginning in the early of XX century with the work of Felix Hausdorff (1868-1942) and Leopold Vietoris (1891-2002). Given a topological space X, the hyperspace $CL(X)$ of all nonempty closed subset of X is equipped with the Vietoris topology [11, Chapter 12, p.750] that is the smallest topology $\mathcal{T}_v$ on $CL(X)$ for which $\{A \in CL(X) \mid A \subseteq U\} \in \mathcal{T}_v$ for $U \in \mathcal{T}$ and $\{A \in CL(X) \mid A \subseteq B\}$ is $\mathcal{T}_v$-closed for each $\mathcal{T}$-closed set B [12]. This definition leads us to involve lower sets with respect to set containment, so it will play a crucial role to obtain *Hypertexture* notion.

Texture spaces have been introduced by L.M. Brown and the primary motivation of ditopological texture spaces is to offer a new extension of classical fuzzy sets [1, 2] and to study the relationship between ditopological texture spaces and fuzzy topologies. Nowadays, the theory is being developed independently of this motivation.

As pointed out in [16], if $(N, \leq)$ is a poset then the set $\mathcal{L} = \{L \subseteq N \mid n \in L, m \leq n \implies m \in L\}$ of *lower subsets* of N is a plain texturing of N. In this paper the author use the same technique to obtain plain and standard texture using

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hyperspace notion. We called these textures plain hypertexture and hypertexture respectively.

The main goal of this article is to introduce *Hyperspaces of Ditopological Texture Spaces* and *Hypertextures*. Basic concepts used in the paper are collected in the section of Preliminaries. In the Third section, Vietoris topology is used in our new setting and the definition of *hyperspace* of a ditopological texture space is given. The fourth section is devoted to Hypertexture notion and in this section we give two types of hypertexture which is called standard hypertexture and plain hypertexture with several examples, and also we investigate some categorical aspects of them. Besides all of these, we obtain a functor from **dfTex** to **dfPTex** which is not exists in classical case where **dfTex** is a category whose objects are texture spaces and whose morphisms are difunctions. If the objects are restricted to be plain textures we obtain the full subcategory **dfPTex** [6, Definition 3.3]. This section is ended by the notion of complementation on *Hypertexture*. The last section is related to future work, we try to sketch our next step.

2. Preliminaries

We recall some basic notions related to textures, ditopological texture spaces and hyperspaces as well for the benefit of general readers who do not have any clue on these subjects. We also refer to [3, 4, 5, 6, 7, 8, 14, 15, 12] for motivation and background material.

Textures: Let S be a set. We work within a subset $\mathcal{S}$ of the power set $\mathcal{P}(S)$ called a *texturing*. A texturing is a point-separating, complete, completely distributive lattice with respect to inclusion. It contains S and $\emptyset$, arbitrary meets coincide with intersections, and finite joins coincide with unions. If $\mathcal{S}$ is a texturing of S the pair $(S, \mathcal{S})$ is called a *texture* space or a *texture* [5].

Most definitions and results concerning textures are most simply expressed using the *p-sets* and *q-sets*: for $s \in S$

$$P_s = \bigcap\{A \in \mathcal{S} \mid s \in A\}, \quad Q_s = \bigvee\{A \in \mathcal{S} \mid s \notin A\}.$$

Example 2.1. (1) The *discrete texture is* $(X, \mathcal{P}(X))$ on the set X. For $x \in X$, $P_x = \{x\}$, $Q_x = X \setminus \{x\}$.

(2) The texture $(L, \mathcal{L})$ is defined, where $L = (0, 1]$ and $\mathcal{L} = \{(0, r] \mid 0 \leq r \leq 1\}$. Here, for $r \in L$, $P_r = Q_r = (0, r]$.

(3) The *unit interval texture is* $(\mathbb{I}, \mathcal{I})$, where $\mathbb{I} = [0, 1]$, $\mathcal{I} = \{[0, r) \mid r \in \mathbb{I}\} \cup \{[0, r] \mid r \in \mathbb{I}\}$. Here, for $r \in \mathbb{I}$, $P_r = [0, r]$ and $Q_r = [0, r)$.

(4) The *product texture* $(S \times T, \mathcal{S} \otimes \mathcal{T})$ of textures $(S, \mathcal{S})$ and $(T, \mathcal{T})$ is defined in [6]. Here the product texturing $\mathcal{S} \otimes \mathcal{T}$ of $S \times T$ consists of arbitrary intersections of sets of the form

$$(A \times T) \cup (S \times B), \ A \in \mathcal{S} \text{ and } B \in \mathcal{T}.$$

For $(s, t) \in S \times T$, $P_{(s,t)} = P_s \times P_t$ and $Q_{(s,t)} = (Q_s \times T) \cup (S \times Q_t)$.

(5) If X is a set, then the product of $(X, \mathcal{P}(X))$ and $(L, \mathcal{L})$ is the texture corresponding to the Hutton algebra $\mathbb{I}^X$ of classic fuzzy subsets of X [5].

Types of texture:

(i) Complemented: If $(S, \mathcal{S})$ is a texture and $\sigma : \mathcal{S} \to \mathcal{S}$ an inclusion reversing involution then $(S, \mathcal{S}, \sigma)$ is referred to as a *complemented texture*. For a discrete texture $\pi_X(A) = X \setminus A$ (set complement) is a common complementation. But not every texture possesses a complementation.

(ii) Simple: If $(S, \mathcal{S})$ is a texture then $M \in \mathcal{S}$ is called a molecule if $M \neq \emptyset$ and $M \subseteq A \cup B, A, B \in \mathcal{S}$ implies $M \subseteq A$ or $M \subseteq B$. For each $s \in S$, P_s is a molecule. The texture $(S, \mathcal{S})$ is called *simple* if p-sets P_s are the only molecules.

(iii) (Nearly, Almost) Plain: If $(S, \mathcal{S})$ is a texture then the point $s \in S$ is called a *plain point* if $P_s \not\subseteq Q_s$.

(a) $(S, \mathcal{S})$ is *plain* if every point $s \in S$ is plain. Equivalently, if $\mathcal{S}$ is closed under arbitrary unions.

(b) $(S, \mathcal{S})$ is *nearly plain* if given $s \in S$ there exists a plain point $w \in S$ with $Q_s = Q_w$ [17].

(c) $(S, \mathcal{S})$ is *almost plain* if given $s, t \in S$ with $P_t \not\subseteq Q_s$ there exists a plain point $u \in S$ with $P_t \not\subseteq Q_u$ and $P_u \not\subseteq Q_s$ [19].

The p-sets and q-sets establish a form of duality with respect to the set complementation to be encoded in general textures. The following auxiliary notion of core set of a set A in $\mathcal{S}$ will be useful to expose the nature of this duality. For a set $A \in \mathcal{S}$, the core of A (denoted by $A^\flat$) is defined by [6, Theorem 1.2].

$$A^\flat = \bigcap \Big\{ \bigcup \{A_i \mid i \in I\} \; | \{A_i \mid i \in I\} \subseteq \mathcal{S}, \; A = \bigvee \{A_i \mid i \in I\} \Big\}.$$

The relation between this concept and the other textural concepts in any texture space is given below, and also we clearly have $A^\flat = A$ for a plain textures.

Theorem 2.2. *In any texture $(S, \mathcal{S})$, the following statements hold:*

(1) $s \notin A \Rightarrow A \subseteq Q_s \Rightarrow s \notin A^\flat$ *for all* $s \in S$, $A \in \mathcal{S}$.
(2) $A^\flat = \{s \mid A \not\subseteq Q_s\}$ *for all* $A \in \mathcal{S}$.
(3) *For* $A_j \in \mathcal{S}$, $j \in J$ *we have* $(\bigvee_{j \in J} A_j)^\flat = \bigcup_{j \in J} A_j^\flat$.
(4) A *is the smallest element of* $\mathcal{S}$ *containing* $A^\flat$ *for all* $A \in \mathcal{S}$.
(5) *For* $A, B \in \mathcal{S}$, *if* $A \not\subseteq B$ *then there exists* $s \in S$ *with* $A \not\subseteq Q_s$ *and* $P_s \not\subseteq B$.
(6) $A = \bigcap \{Q_s \mid P_s \not\subseteq A\}$ *for all* $A \in \mathcal{S}$.
(7) $A = \bigvee \{P_s \mid A \not\subseteq Q_s\}$ *for all* $A \in \mathcal{S}$.

Direlations and difunctions

We denote the p–sets and q–sets for $(S \times T, \mathcal{P}(S) \otimes \mathcal{T})$ by $\overline{P}_{(s,t)}$, $\overline{Q}_{(s,t)}$. Then $r \in \mathcal{P}(S) \otimes \mathcal{T}$ is called a *relation from* $(S, \mathcal{S})$ *to* $(T, \mathcal{T})$ if it satisfies

R1 $r \not\subseteq \overline{Q}_{(s,t)}, P_{s'} \not\subseteq Q_s \implies r \not\subseteq \overline{Q}_{(s',t)}$.

R2 $r \not\subseteq \overline{Q}_{(s,t)} \implies \exists s' \in S$ such that $P_s \not\subseteq Q_{s'}$ and $r \not\subseteq \overline{Q}_{(s',t)}$.

$R \in \mathcal{P}(S) \otimes \mathcal{T}$ is called a *corelation from* $(S, \mathcal{S})$ *to* $(T, \mathcal{T})$ if it satisfies

CR1 $\overline{P}_{(s,t)} \not\subseteq R, P_s \not\subseteq Q_{s'} \implies \overline{P}_{(s',t)} \not\subseteq R$.

CR2 $\overline{P}_{(s,t)} \not\subseteq R \implies \exists s' \in S$ such that $P_{s'} \not\subseteq Q_s$ and $\overline{P}_{(s',t)} \not\subseteq R$.

A pair (r, R) consisting of a relation r and correlation R is now called a *direlation*.

Example 2.3. For any texture $(S, \mathcal{S})$ the *identity direlation* (i, I) on $(S, \mathcal{S})$ is given by

$$i = \bigvee \{\overline{P}_{(s,s)} \mid s \in S\} \text{ and } I = \bigcap \{\overline{Q}_{(s,s)} \mid s \in S\}.$$

Given a direlation $(r, R) : (S, \mathcal{S}) \to (T, \mathcal{T})$ and $B \in \mathcal{T}$ we define $r^{\leftarrow}B,\ R^{\leftarrow}B \in \mathcal{S}$ by

$$\begin{aligned} r^{\leftarrow}B &= \bigvee \{P_s \mid \forall t, r \not\subseteq \overline{Q}_{(s,t)} \implies P_t \subseteq B\}, \\ R^{\leftarrow}B &= \bigcap \{\overline{Q}_s \mid \forall t, \overline{P}_{(s,t)} \not\subseteq R \implies B \subseteq Q_t\}. \end{aligned}$$

A *difunction* $(f, F) : (S, \mathcal{S}) \to (T, \mathcal{T})$ is a direlation that is characterized by the equality $f^{\leftarrow}B = F^{\leftarrow}B$ for all $B \in \mathcal{T}$.

If $(f, F) : (S, \mathcal{S}) \to (T, \mathcal{T})$ is a difunction then by [6, Corollary 2.12] the map $\theta : \mathcal{T} \to \mathcal{S}$ defined by $\theta(B) = f^{\leftarrow}B = F^{\leftarrow}B$ preserves arbitrary joins and intersections.

Conversely, by [7, Proposition 4.1] if $\theta : \mathcal{T} \to \mathcal{S}$ is a mapping that preserves arbitrary joins and intersections then there exists a unique difunction $(f, F) : (S, \mathcal{S}) \to (T, \mathcal{T})$ that satisfies $f^{\leftarrow}B = \theta(B) = F^{\leftarrow}B$ for all $B \in \mathcal{T}$.

Textures and difunctions form a category denoted by **dfTex**, and also plain textures and difunctions between them form a category denoted by **dfPTex**.

Ditopology:

For a texture $(S, \mathcal{S})$, the texturing $\mathcal{S}$ is usually not closed under the operation of taking the set complement. Hence we must forgo the usual relation between open and closed sets and consider a *dichotomous topology* (*ditopology* for short) consisting of a topology (family of open sets) $\tau \subseteq \mathcal{S}$ and a generally unrelated cotopology (family of closed sets) $\kappa \subseteq \mathcal{S}$. We then call $(S, \mathcal{S}, \tau, \kappa)$ a *ditopological texture space* [4].

The notion of ditopology can also be used in other settings. For example it has recently been carried over to completely distributive lattices, producing "Hutton dispaces" [20].

It should be stressed that a ditopology is considered as a single structure, with the open and closed sets playing an equal role. This is in contrast to a bitopology consisting of two distinct topologies, complement with their open and closed sets.

Let $(S, \mathcal{S}, \sigma)$ be complemented texture and (τ, κ) be a ditopology on $(S, \mathcal{S})$, if $\kappa = \sigma(\tau)$, then the ditopology (τ, κ) is said to be *complemented*.

Hyperspace:

If $(X, \mathcal{T})$ is a topological space then the notion of a *hyperspace* of $(X, \mathcal{T})$ is meant a specified family of subsets of X with a topology depending on $\mathcal{T}$ and referred to here as the Vietoris topology. For convenience, a hyperspace is generally assumed not to contain the empty set $\emptyset$, while to avoid pathology all its members are taken to be closed sets under the topology $\mathcal{T}$. Hence the largest hyperspace of $(X, \mathcal{T})$ is the set

$$CL(X) = \{A \subseteq X \mid A \text{ is a non-empty } \mathcal{T}\text{-closed subset of } X\}$$

with the *Vietoris topology*, that is the smallest topology $\mathcal{T}_v$ on $CL(X)$ for which $\{A \in CL(X) \mid A \subseteq U\} \in \mathcal{T}_v$ for $U \in \mathcal{T}$ and $\{A \in CL(X) \mid A \subseteq B\}$ is $\mathcal{T}_v$-closed for each $\mathcal{T}$-closed set B. As here we will generally follow the notation of [12] for basic concepts relating to hyperspaces. As seen in [12], for example, stronger conditions on the elements of the hyperspace may need to be imposed to ensure better properties of the hyperspace or a closer relation between the properties of the topologies $\mathcal{T}$ and $\mathcal{T}_v$.

3. Basic Definitions and the Discrete Case

To study hyperspaces in our new setting, we will need to replace the topological space $(X, \mathcal{T})$ with a ditopological texture space $(S, \mathcal{S}, \tau, \kappa)$. This introduces with a new element, namely the texturing $\mathcal{S}$, as well as replacing the topology $\mathcal{T}$ by the ditopology (τ, κ). It is natural to restrict our attention to the sets in $\mathcal{S}$ when defining required notion of hyperspace, and bearing in mind that we may wish to impose additional conditions as in the classical case. Now, we will base it on a set $H \subseteq \mathcal{S}$. Letting $\mathcal{H}$ be a texturing of H, this leads to the texture $(H, \mathcal{H})$, and the notion of Vietoris topology $\mathcal{T}_v$ generalizes naturally to the *Vietoris ditopology* (τ_v, κ_v), where τ_v is the smallest topology on H for which $\{A \in H \mid A \subseteq G\} \in \tau_v$ for $G \in \tau$ and κ_v the smallest cotopology on H for which $\{A \in H \mid A \subseteq K\} \in \kappa_v$ for all $K \in \kappa$. Hence we make the following general definition:

Definition 3.1. With the notation as above a *hyperspace* of a ditopological texture space $(S, \mathcal{S}, \tau, \kappa)$ is defined as the ditopological texture space of the form $(H, \mathcal{H}, \tau_v, \kappa_v)$.

The following example shows that Definition 3.1 includes the classical case. Here, as usual, we represent a topological space $(X, \mathcal{T})$ by the complemented ditopological texture space $(X, \mathcal{P}(X), \pi_X, \mathcal{T}, \mathcal{T}^c)$, where $\pi_X(A) = X \setminus A$ for $A \in \mathcal{P}(X)$ is the usual set complement and $\mathcal{T}^c = \{X \setminus A \mid A \in \mathcal{T}\}$. We will have more to say regarding complementation in a more general setting later on.

Example 3.2. Let $(X, \mathcal{T})$ be a topological space and $(CL(X), \mathcal{T}_v)$ a hyperspace. The corresponding (complemented) ditopological spaces are $(X, \mathcal{P}(X), \pi_X, \mathcal{T}, \mathcal{T}^c)$ and $(CL(X), \mathcal{P}(CL(X)), \pi_{CL(X)}, \mathcal{T}_v, \mathcal{T}_v^c)$, respectively. Then by setting $H = CL(X)$, the families $\mathcal{H} = \mathcal{P}(H)$, $\tau_v = \mathcal{T}$ and $\kappa_v = \mathcal{T}^c$ give us a natural representation of $(CL(X), \mathcal{T}_v)$ as the (complemented) hyperspace of $(X, \mathcal{P}(X), \pi_X, \mathcal{T}, \mathcal{T}^c)$ is $(H, \mathcal{H}, \pi_H, \tau_v, \kappa_v)$.

For the remainder of this section we continue to consider discrete textures but generalize the classical case by permitting general ditopologies (τ, κ) on $(X, \mathcal{P}(X))$. Hence, in what follows we consider the complemented texture $(X, \mathcal{P}(X), \pi_X)$ and a ditopology (τ, κ) which is not necessarily complemented, that is for which $\kappa \neq \tau^c$. Our ditopological hyperspace is now $(CL(X), \mathcal{P}(CL(X)), \pi_{CL(X)}, \tau_v, \kappa_v)$. We can expect a close relationship here with the bitopological [13] case and the reader is referred in particular to the work of Bruce S. Burdick [9, 10] in this respect.

4. Hypertextures

Rather than restricting the elements of the hyperspace as above we show in this section and the next that by taking $H = \mathcal{S}$ and choosing the texturing $\mathcal{H}$ carefully we can in fact obtain closer links between the original ditopologies and Vietoris ditopologies than in the classical situation. This can be regarded as an important bonus for working in a textural setting. We concentrate in this section on defining two suitable texturings of $\mathcal{S}$, referred to here as *Hypertextures*. The fact that the definition of the Vietoris ditopology involves lower sets with respect to the relation set inclusion will play an important role, here.

Definition 4.1. Let $(S, \mathcal{S})$ be a texture. For $A \in \mathcal{S}$ we set

$$\widehat{A} = \{B \in \mathcal{S} \mid B \subseteq A\} \text{ and } \widehat{\mathcal{S}} = \{\widehat{A} \mid A \in \mathcal{S}\}.$$

Also,

$$\mathcal{L}_{\mathcal{S}} = \{\mathcal{B} \subseteq \mathcal{S} \mid B \in \mathcal{B}, A \subseteq B \implies A \in \mathcal{B}\}.$$

It is immediate from the definitions that $\widehat{\mathcal{S}} \subseteq \mathcal{L}_{\mathcal{S}}$. Both $\widehat{\mathcal{S}}$ and $\mathcal{L}_{\mathcal{S}}$ are texturings of $\mathcal{S}$. Indeed $(\mathcal{S}, \subseteq)$ and $(\widehat{\mathcal{S}}, \subseteq)$ are clearly isomorphic as complete lattices under the mapping $\theta : \mathcal{S} \to \widehat{\mathcal{S}}, A \mapsto \widehat{A}$, while $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$ is the plain texture associated with the partially ordered set $(\mathcal{S}, \subseteq)$ as in [16]. We will refer to $(\mathcal{S}, \widehat{\mathcal{S}})$ as the *standard hypertexture* of $(S, \mathcal{S})$ (or just as the *hypertexture* if there is no fear of confusion), while $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$ will be called the *plain hypertexture*.

Proposition 1. *In $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$, we have the following equalities*

$$P_A = \{B \in \mathcal{S} \mid B \subseteq A\} \text{ and } Q_A = \{B \in \mathcal{S} \mid A \not\subseteq B\}$$

for $A \in \mathcal{S}$.

Proof. We show the first equality, the other one can be easily shown by using definition.

We begin by proving $\{B \in \mathcal{S} : B \subseteq A\} \subseteq \bigcap\{\mathcal{B} \in \mathcal{L}_{\mathcal{S}} : A \in \mathcal{B}\}$, take $C \in \{B \in \mathcal{S} : B \subseteq A\}$ and for any $A \in \mathcal{B}$, we obtain $C \subseteq A \implies C \in \mathcal{B}$.

On the other hand, suppose that $\bigcap\{\mathcal{B} \in \mathcal{L}_{\mathcal{S}} : A \in \mathcal{B}\} \not\subseteq \{B \in \mathcal{S} : B \subseteq A\}$, so there exists $C \in \bigcap\{\mathcal{B} \in \mathcal{L}_{\mathcal{S}} : A \in \mathcal{B}\}$, but $C \notin \{B \in \mathcal{S} : B \subseteq A\}$. If we choose $A \in \mathcal{B} =\downarrow A \in \mathcal{L}_{\mathcal{S}} \implies C \in\downarrow A$ and hence we obtain $C \subseteq A$, this contradicts with $C \not\subseteq A$. □

Using the similar idea, if we choose the texture $(\mathcal{S}, \widehat{\mathcal{S}})$, then we have the followings:

$$P_A = \bigcap\{\widehat{B} \mid A \in \widehat{B}\} = \widehat{A} \text{ and } Q_A = \bigvee\{\widehat{B} \mid A \notin \widehat{B}\} = \left(\bigvee\{B \mid A \not\subseteq B\}\right)^{\frown}.$$

The following lemma gives a necessary and sufficient condition for the equality $\widehat{\mathcal{S}} = \mathcal{L}_{\mathcal{S}}$.

Lemma 4.2. *For a given texture $(S, \mathcal{S})$ we have $\widehat{\mathcal{S}} = \mathcal{L}_{\mathcal{S}}$ if and only if every lower set in $\mathcal{S}$ contains the union of the members of $\mathcal{S}$.*

Proof. Necessity is clear since for each A in $\mathcal{S}$, $\widehat{A}$ is a lower set in which A is the largest, hence the union of the members of $\mathcal{S}$. For sufficiency take $\mathcal{B} \in \mathcal{L}_{\mathcal{S}}$ and let $A = \bigcup\{B \mid B \in \mathcal{B}\}$. Then by hypothesis $A \in \mathcal{B}$ so for $B \in \mathcal{B}$ we have $B \subseteq A$ since $\mathcal{B}$ is a lower set. Hence $\mathcal{B} \subseteq \widehat{A}$. Likewise $\widehat{A} \subseteq \mathcal{B}$, which completes the proof. □

Example 4.3. (1) We consider the discrete texture $(X, \mathcal{P}(X))$ in the case that $X = \{a, b\}$ is a two-point set. We have $\widehat{\mathcal{P}(X)} \neq \mathcal{L}_{\mathcal{P}(X)}$, showing that these texturings are different even in this simple case. Indeed,

$$\widehat{\mathcal{P}(X)} = \{\widehat{\{a,b\}}, \widehat{\{a\}}, \widehat{\{b\}}, \widehat{\emptyset}\} = \{\mathcal{P}(X), \{\{a\}, \emptyset\}, \{\{b\}, \emptyset\}, \{\emptyset\}\},$$

while $\{\{a\}, \{b\}, \emptyset\}$ is the one and unique lower set in $\mathcal{P}(X)$ not belonging to $\widehat{\mathcal{P}(X)}$ so

$$\mathcal{L}_{\mathcal{P}(X)} = \widehat{\mathcal{P}(X)} \cup \{\{a\}, \{b\}, \emptyset\}.$$

Let us note that in $(\mathcal{P}(X), \widehat{\mathcal{P}(X)})$ we have

$$\begin{aligned} P_{\{a,b\}} &= \widehat{\{a,b\}} = \mathcal{P}(X) & Q_{\{a,b\}} &= \widehat{\{a,b\}} = \mathcal{P}(X), \\ P_{\{a\}} &= \widehat{\{a\}} = \{\{a\}, \emptyset\} & Q_{\{a\}} &= \widehat{\{b\}} = \{\{b\}, \emptyset\}, \\ P_{\{b\}} &= \widehat{\{b\}} = \{\{b\}, \emptyset\} & Q_{\{b\}} &= \widehat{\{a\}} = \{\{a\}, \emptyset\}, \\ P_{\emptyset} &= \widehat{\{\emptyset\}} = \{\emptyset\} & Q_{\emptyset} &= \widehat{\{\emptyset\}} = \{\emptyset\}, \end{aligned}$$

while in $(\mathcal{P}(X), \mathcal{L}_{\mathcal{P}(X)})$ we have

$$\begin{aligned} P_{\{a,b\}} &= \{\{a,b\},\{a\},\{b\},\emptyset\} & Q_{\{a,b\}} &= \{\{a\},\{b\},\emptyset\}, \\ P_{\{a\}} &= \{\{a\},\emptyset\} & Q_{\{a\}} &= \{\{b\},\emptyset\}, \\ P_{\{b\}} &= \{\{b\},\emptyset\} & Q_{\{b\}} &= \{\{a\},\emptyset\}, \\ P_{\emptyset} &= \{\emptyset\} & Q_{\emptyset} &= \emptyset. \end{aligned}$$

Clearly in $(\mathcal{P}(X), \mathcal{L}_{\mathcal{P}(X)})$, we have $P_A \not\subseteq Q_A$ for all $A \in \mathcal{P}(X)$ which confirms that this texture is plain. On the other hand, in $(\mathcal{P}(X), \widehat{\mathcal{P}(X)})$ we have $P_{\{a,b\}} = Q_{\{a,b\}}$ and $P_\emptyset = Q_\emptyset$, so this texture is not plain. The q-sets of the points $\{a,b\}$ and $\emptyset$ in $(\mathcal{P}(X), \widehat{\mathcal{P}(X)})$ are not equal to the q-sets of either of the plain points $\{a\}$ or $\{b\}$ so this texture is not nearly plain either (see [17] for a discussion of nearly plain textures).

(2) Now let us consider the texture $(L, \mathcal{L})$ where $L = (0,1]$ and $\mathcal{L} = \{(0,r] \mid 0 \le r \le 1\}$, $(0,0]$ being interpreted as the empty set. This is the Hutton texture of the unit interval. It is well known to be a simple but non-plain texture, and indeed for $0 \le r \le 1$ we have $P_r = Q_r = (0,r]$. Again we consider briefly the textures $(\mathcal{L}, \widehat{\mathcal{L}})$ and $(\mathcal{L}, \mathcal{L}_{\mathcal{L}})$. Clearly we have lower sets in $\mathcal{L}$ of the form $\{(0,s] \mid 0 \le s < k\}$, $0 < k \le 1$, which do not belong to $\widehat{\mathcal{L}}$ so again $\widehat{\mathcal{L}} \subset \mathcal{L}_{\mathcal{L}}$. In $\widehat{\mathcal{L}}$ we have $P_{(0,r]} = Q_{(0,r]} = \widehat{(0,r]}$ for $0 \le r \le 1$ so the texture $(\mathcal{L}, \widehat{\mathcal{L}})$ is not plain and likewise it is not nearly plain. In $(\mathcal{L}, \mathcal{L}_{\mathcal{L}})$, however, we have $P_{(0,r]} = \widehat{(0,r]}$, $Q_{(0,r]} = \{(0,k] \mid 0 \le k < r\}$ for $0 \le r \le 1$. In particular $P_\emptyset = P_{(0,0]} = \{\emptyset\}$, $Q_\emptyset = Q_{(0,0]} = \{\emptyset\}$, so $P_{(0,r]} \not\subseteq Q_{(0,r]}$ for all r which confirms that $(\mathcal{L}, \mathcal{L}_{\mathcal{L}})$ is plain.

(3) An important texture is the unit interval texture $(I, \mathcal{I})$ where $I = [0,1]$ and $\mathcal{I} = \{[0,r], [0,r) \mid 0 \le r \le 1\}$. It is well known that this is a plain texture with its canonical ditopology which plays the same role in ditopological texture spaces as the unit interval in general topology. Again we consider briefly the textures $(\mathcal{I}, \widehat{\mathcal{I}})$ and $(\mathcal{I}, \mathcal{L}_{\mathcal{I}})$. In $(\mathcal{I}, \widehat{\mathcal{I}})$ we have $\widehat{[0,r]} = \{[0,s],[0,s) \mid 0 \le s \le r \le 1\}$ and $\widehat{[0,r)} = \{[0,s) \mid 0 \le s \le r \le 1\} \cup \{[0,s]\} \mid 0 \le s < r \le 1\}$.

The lower sets in $\mathcal{L}_{\mathcal{I}}$ not belonging to $\widehat{\mathcal{I}}$ have the form $\{[0,s),[0,s] \mid 0 \le s < r\}$ for $0 < r \le 1$, so $\widehat{\mathcal{I}} \subset \mathcal{L}_{\mathcal{I}}$ and $(\mathcal{I}, \widehat{\mathcal{I}})$ is not plain. The p,q-sets in $(\mathcal{I}, \widehat{\mathcal{I}})$ are $P_{[0,r]} = \widehat{[0,r]}$, $P_{[0,r)} = \widehat{[0,r)}$; $Q_{[0,r]} = \widehat{[0,r)} = Q_{[0,r)}$ so $[0,r], 0 \le r \le 1$ are plain points and $[0,r), 0 \le r \le 1$ are not. Since the q-set of $[0,r)$ is equal to the q-set of the plain point $[0,r]$ for all r, it follows that $(\mathcal{I}, \widehat{\mathcal{I}})$ is a nearly plain texture. In $(\mathcal{I}, \mathcal{L}_{\mathcal{I}})$ the p,q-sets are easily seen to be the same with the p,q-sets in $(\mathcal{I}, \widehat{\mathcal{I}})$, except for $Q_{[0,r)} = \{[0,s),[0,s] \mid 0 \le s < r\}$, so we now have $P_{[0,r)} \not\subseteq Q_{[0,r)}$ confirming that $(\mathcal{I}, \mathcal{L}_{\mathcal{I}})$ is plain.

We begin by investigating the relationship between the textures $(S, \mathcal{S})$ and $(\mathcal{S}, \widehat{\mathcal{S}})$ in more detail. We have already mentioned the isomorphism $\theta : \mathcal{S} \to \widehat{\mathcal{S}}$, $A \mapsto \widehat{A}$ and we denote its inverse by $\eta : \widehat{\mathcal{S}} \to \mathcal{S}$. By [7, Proposition 4.1] we have a difunction $(h, H) : (S, \mathcal{S}) \to (\mathcal{S}, \widehat{\mathcal{S}})$ characterized by $h^{\leftarrow}\widehat{B} = \eta(\widehat{B}) = H^{\leftarrow}\widehat{B}\ \forall \widehat{B} \in \widehat{\mathcal{S}}$ and a difunction $(k, K) : (\mathcal{S}, \widehat{\mathcal{S}}) \to (S, \mathcal{S})$ characterized by $k^{\leftarrow}A = \theta(A) = K^{\leftarrow}A\ \forall A \in \mathcal{S}$.

Proposition 2. *The textures $(S, \mathcal{S})$ and $(\mathcal{S}, \widehat{\mathcal{S}})$ are* **dfTex** *isomorphic.*

Proof. With the notation above we prove $k \circ h = i_S$, where (i_S, I_S) is the identity difunction on $(S, \mathcal{S})$. By [6, Lemma 2.7 and Definition 2.8] it is sufficient to prove $(k \circ h)^{\leftarrow}A = i_S^{\leftarrow}A = A$ for all $A \in \mathcal{S}$. By [6, Lemma 2.16] we have $(k \circ h)^{\leftarrow} = h^{\leftarrow}(k^{\leftarrow}A)) = h^{\leftarrow}(\theta(A)) = h^{\leftarrow}(\widehat{A}) = \eta(\widehat{A}) = A$ by the characteristic properties of h and k. The equality $K \circ H = I_S$ follows likewise, giving $(k, K) \circ (h, H) = (i_S, I_S)$. Finally $(h, H) \circ (k, K) = (i_{\mathcal{S}}, I_{\mathcal{S}})$ follows by a similar argument, so (h, H) (and (k, K)) set up a **dfTex** isomorphism between $(S, \mathcal{S})$ and $(\mathcal{S}, \widehat{\mathcal{S}})$. □

Since almost plainness [17] is preserved under **dfTex** isomorphisms we have:

Corollary 1. *The texture $(\mathcal{S}, \widehat{\mathcal{S}})$ is almost plain if (and only if) $(S, \mathcal{S})$ is almost plain.* □

In view of the complete lattice isomorphism $\theta : \mathcal{S} \to \widehat{\mathcal{S}}, A \mapsto \widehat{A}$ and as a result of the Theorem 2.2, we have the following corollary. In addition, we have also similar corollary for $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$, but here we give the following statements only for the texture $(\mathcal{S}, \widehat{\mathcal{S}})$, briefly.

Corollary 2. *In hypertexture $(\mathcal{S}, \widehat{\mathcal{S}})$, we have*

(1) $A \notin \widehat{B} \implies \widehat{B} \subseteq Q_A \implies A \notin \widehat{A}^{\flat}$ *for all* $A \in \mathcal{S}$ *and* $\widehat{B} \in \widehat{\mathcal{S}}$,
(2) $\widehat{A}^{\flat} = \{B | \widehat{A} \not\subseteq Q_B\}$ *for all* $\widehat{A} \in \widehat{\mathcal{S}}$,
(3) $\widehat{A_j} \in \widehat{\mathcal{S}},\ j \in J$ *we have* $(\widehat{\bigvee_{j\in J} A_j})^{\flat} = \bigcup_{j\in J} \widehat{A_j}^{\flat}$,
(4) $\widehat{A}$ *is the smallest element of* $\widehat{\mathcal{S}}$ *containing* $\widehat{A}^{\flat}$ *for all* $A \in \mathcal{S}$,
(5) *For* $A, B \in \widehat{\mathcal{S}}$, *if* $\widehat{A} \not\subseteq \widehat{B}$ *then there exists* $C \in \widehat{\mathcal{S}}$ *with* $\widehat{A} \not\subseteq Q_C$ *and* $P_C \not\subseteq \widehat{B}$,
(6) $\widehat{A} = \bigcap\{Q_B | P_B \not\subseteq \widehat{A}\}$ *for all* $\widehat{A} \in \widehat{\mathcal{S}}$,
(7) $\widehat{A} = \bigvee\{P_B | \widehat{A} \not\subseteq Q_B\}$ *for all* $\widehat{A} \in \widehat{\mathcal{S}}$.

Let us now consider the relation between the textures $(S, \mathcal{S})$ and $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$.

Theorem 4.4. *The function*

$$\mathcal{L}_{\mathcal{S}} \xrightarrow{\gamma} \mathcal{S},\ \mathcal{B} \mapsto \bigvee \mathcal{B} = \bigvee_{B\in\mathcal{B}} B$$

defines a difunction $(l, L) : (S, \mathcal{S}) \to (\mathcal{S}, \mathcal{L}_{\mathcal{S}})$ characterized by $l^{\leftarrow}\mathcal{B} = \gamma(\mathcal{B}) = L^{\leftarrow}\mathcal{B}$ for all $\mathcal{B} \in \mathcal{L}_{\mathcal{S}}$.

Proof. Clearly γ maps $\mathcal{L}_{\mathcal{S}}$ into $\mathcal{S}$ so in order to apply [7, Proposition 4.1] we must verify that it preserves arbitrary joins and meets.

To show γ preserves joins we take $\mathcal{B}_i \in \mathcal{L}_\mathcal{S}$ for $i \in I$ and note from the definition that $\bigcup_{i\in I} \mathcal{B}_i \mapsto \bigvee \left(\bigcup_{i\in I} \mathcal{B}_i\right)$ under γ. Hence we must show that $\bigvee \left(\bigcup_{i\in I} \mathcal{B}_i\right) = \bigcup_{i\in I} \left(\bigvee \mathcal{B}_i\right)$. Let $A \in \bigcup_{i\in I} \mathcal{B}_i$. Then there exists $i \in I$ with $A \in \mathcal{B}_i$ so $A \subseteq \bigvee \mathcal{B}_i \subseteq \bigcup_{i\in I} \left(\bigvee \mathcal{B}_i\right)$ and we deduce $\bigvee \left(\bigcup_{i\in I} \mathcal{B}_i\right) \subseteq \bigcup_{i\in I} \left(\bigvee \mathcal{B}_i\right)$. Now suppose that $\bigcup_{i\in I} \left(\bigvee \mathcal{B}_i\right) \not\subseteq \bigvee \left(\bigcup_{i\in I} \mathcal{B}_i\right)$. Then there exists $s \in S$ with $\bigcup_{i\in I} \left(\bigvee \mathcal{B}_i\right) \not\subseteq Q_s$ and $P_s \not\subseteq \bigvee \left(\bigcup_{i\in I} \mathcal{B}_i\right)$ so there exists $i \in I$ with $\bigvee \mathcal{B}_i \not\subseteq Q_s$ and now we have $A \in \mathcal{B}_i$ with $A \not\subseteq Q_s$ which gives the contradiction $P_s \subseteq A \subseteq \bigvee \mathcal{B}_i \subseteq \bigvee \left(\bigcup \mathcal{B}_i\right)$.

To establish the preservation of meets we again take $\mathcal{B}_i \in \mathcal{L}_\mathcal{S}$ for $i \in I$ and note from the definition that $\bigcap_{i\in I} \mathcal{B}_i \mapsto \bigvee \left(\bigcap_{i\in I} \mathcal{B}_i\right)$ under γ. Hence we must show that $\bigvee \left(\bigcap_{i\in I} \mathcal{B}_i\right) = \bigcap_{i\in I} \left(\bigvee \mathcal{B}_i\right)$. Now $A \in \bigcap_{i\in I} \mathcal{B}_i \implies A \subseteq \bigvee \mathcal{B}_i$ for all i so $A \subseteq \bigcap_{i\in I} \left(\bigvee \mathcal{B}_i\right)$ and we have $\bigvee \left(\bigcap_{i\in I} \mathcal{B}_i\right) \subseteq \bigcap_{i\in I} \left(\bigvee \mathcal{B}_i\right)$. Now suppose that $\bigcap_{i\in I} \left(\bigvee \mathcal{B}_i\right) \not\subseteq \bigvee \left(\bigcap_{i\in I} \mathcal{B}_i\right)$. Then there exists $s \in S$ with $\bigcap_{i\in I} \left(\bigvee \mathcal{B}_i\right) \not\subseteq Q_s$ and $P_s \not\subseteq \bigvee \left(\bigcap_{i\in I} \mathcal{B}_i\right)$. Now take $t \in S$ with $P_s \not\subseteq Q_t$ and $P_t \not\subseteq \bigvee \left(\bigcap_{i\in I} \mathcal{B}_i\right)$. We deduce $P_s \subseteq \bigvee \mathcal{B}_i \not\subseteq Q_t$ for all $i \in I$, so there exists $A_i \in \mathcal{B}_i, i \in I$ with $A_i \not\subseteq Q_t$. Now $\mathcal{B}_i$ is a lower set, so $P_t \subseteq \bigcap_{i\in I} A_i \in \mathcal{B}_i$ for all $i \in I$, whence $P_t \subseteq \bigvee \left(\bigcap_{i\in I} \mathcal{B}_i\right)$ which is a contradiction. Finally, it preserves arbitrary joins and meets, that is, we can apply [7, Proposition 4.1], thereby γ maps $\mathcal{L}_\mathcal{S}$ into $\mathcal{S}$ and defines a difunction characterized by the conditions given in the statement of the Theorem. □

In the light of the above discussion, we have the following clear corollary.

Corollary 3. *For the difunctions (k, K) and (l, L) defined on $(\mathcal{S}, \widehat{\mathcal{S}})$ and $(S, \mathcal{S})$, respectively, the composition of these two difunctions is also difunction and it can be easily characterized by $(l \circ k)^{\leftarrow}\mathcal{B} = \widehat{\bigvee \mathcal{B}} = (L \circ K)^{\leftarrow}\mathcal{B}$ for $\mathcal{B} \in \mathcal{L}_\mathcal{S}$.*

We know from [6, p.190] that the category whose objects are textures and whose morphisms are difunctions is denoted by **dfTex**, and if the objects restricted to plain textures we obtain full subcategory **dfPTex** and we have inclusion functor $\mathfrak{P}$: **dfPTex** $\to$ **dfTex**. Since $(\mathcal{S}, \mathcal{L}_\mathcal{S})$ is a plain texture for any texture $(S, \mathcal{S})$ it is natural to ask whether it can be used as a basis for a functor from **dfTex** to **dfPTex** which is does not exist in classical case. The following proposition is an affirmative answers for this question.

Proposition 3. *Let $\mathfrak{B}$ be defined by $\mathfrak{B}(S, \mathcal{S}) = (\mathcal{S}, \mathcal{L}_\mathcal{S})$ and for a **dfTex** morphism $(f, F) : (S, \mathcal{S}) \to (T, \mathcal{T})$ let $\mathfrak{B}(f, F) = (g, G) : (\mathcal{S}, \mathcal{L}_\mathcal{S}) \to (\mathcal{T}, \mathcal{L}_\mathcal{T})$ be characterized by $g^{\leftarrow}\mathcal{B} = \{A \in \mathcal{S} \mid \exists B \in \mathcal{B}, A \subseteq f^{\leftarrow}B\} = G^{\leftarrow}\mathcal{B}$ for $\mathcal{B} \in \mathcal{L}_\mathcal{T}$. Then $\mathfrak{B}$:* **dfTex** $\to$ **dfPTex** *is a functor.*

Proof. It is clear that for $\mathcal{B} \in \mathcal{L}_\mathcal{T}$ the set $\beta(\mathcal{B}) = \{A \in \mathcal{S} \mid \exists B \in \mathcal{B}, A \subseteq f^{\leftarrow}B\}$ is a lower set in $\mathcal{S}$ so β certainly maps into $\mathcal{L}_\mathcal{S}$ and it is trivial that it preserves arbitrary intersections and unions. Hence the difunction $(g, G) : (\mathcal{S}, \mathcal{L}_\mathcal{S}) \to (\mathcal{T}, \mathcal{L}_\mathcal{T})$ is well defined by $g^{\leftarrow}\mathcal{B} = \beta(\mathcal{B}) = G^{\leftarrow}\mathcal{B}$.

We begin by showing $\mathfrak{B}$ preserves composition of morphisms. Let

$$(S,\mathcal{S}) \xrightarrow{\quad (f,F) \quad} (T,\mathcal{T}) \xrightarrow{\quad (m,M) \quad} (U,\mathcal{U})$$

be morphisms and $\mathfrak{B}(f,F) = (g,G), \mathfrak{B}(m,M) = (n,N), \mathfrak{B}((m,M)\circ(f,F)) = (r,R)$. For $\mathcal{C} \in \mathcal{L}_{\mathcal{U}}$ we have

$$(n\circ g)^{\leftarrow}\mathcal{C} = g^{\leftarrow}(n^{\leftarrow}\mathcal{C}) = g^{\leftarrow}\mathcal{B}$$

where $\mathcal{B} = \{B \in \mathcal{T} \mid \exists\, C \in \mathcal{C}, B \subseteq m^{\leftarrow}C\} \in \mathcal{L}_{\mathcal{T}}$. Now

$$\begin{aligned} g^{\leftarrow}\mathcal{B} &= \{A \in \mathcal{S} \mid \exists\, B \in \mathcal{B}, A \subseteq f^{\leftarrow}B\} \\ &= \{A \in \mathcal{S} \mid \exists\, B \in \mathcal{T}, \exists C \in \mathcal{C}, B \subseteq m^{\leftarrow}C, A \subseteq f^{\leftarrow}B\} \\ &= \{A \in \mathcal{S} \mid \exists C \in \mathcal{C}, A \subseteq f^{\leftarrow}(m^{\leftarrow}C)\} \\ &= \{A \in \mathcal{S} \mid \exists C \in \mathcal{C}, A \subseteq (m\circ f)^{\leftarrow}C)\} \\ &= r^{\leftarrow}\mathcal{C} \end{aligned}$$

which gives $\mathfrak{B}((m,M)\circ(f,F)) = \mathfrak{B}(m,M)\circ\mathfrak{B}(f,F)$. Finally we establish that $\mathfrak{B}$ preserves identity morphisms. Let $\mathfrak{B}(i_S, I_S) = (j,J)$. Then for $\mathcal{B} \in \mathcal{L}_{\mathcal{S}}$ we have $j^{\leftarrow}\mathcal{B} = \{A \in \mathcal{S} \mid \exists B \in \mathcal{B}, A \subseteq i_S^{\leftarrow}B\} = \mathcal{B}$ since $i_S^{\leftarrow}B = B$. Hence $\mathfrak{B}(i_S, I_S)$ is the identity on $\mathfrak{B}(S,\mathcal{S}) = (\mathcal{S},\mathcal{L}_{\mathcal{S}})$, as required. □

We end this section by considering complementation. Hence throughout σ will denote an order-reversing involution $\sigma : \mathcal{S} \to \mathcal{S}$. It is natural to ask if σ can be suitably extended to the textures $(\mathcal{S},\widehat{\mathcal{S}})$ and $(\mathcal{S},\mathcal{L}_{\mathcal{S}})$. Now, we have:

Proposition 4. *If the texture $(S,\mathcal{S})$ is complemented with $\sigma : \mathcal{S} \to \mathcal{S}$, then the mapping $\widehat{\sigma} : \widehat{\mathcal{S}} \to \widehat{\mathcal{S}}$ defined by $\widehat{\sigma}(\widehat{A}) = \widehat{\sigma(A)}$, $A \in \mathcal{S}$ describes a complementation on $(\mathcal{S},\widehat{\mathcal{S}})$.*

Proof. In view of the complete lattice isomorphism $\theta : \mathcal{S} \to \widehat{\mathcal{S}}$, $A \mapsto \widehat{A}$ mentioned earlier, it can be shown easily the with help of following information. In general, a texturing need not be closed under set complementation, but it may be that there exists a map $\sigma : \mathcal{S} \to \mathcal{S}$ satisfying some suitable conditions [6, p. 172]. Thus the map σ satisfies $A \subseteq B \implies \sigma(B) \subseteq \sigma(A)$ for all $A, B \in \mathcal{S}$ by using the complete lattice isomorphism and we have $\theta(\sigma(B)) \subseteq \theta(\sigma(A)) \implies \widehat{\sigma(B)} \subseteq \widehat{\sigma(A)}$. That is, $\widehat{\sigma}(\widehat{B}) \subseteq \widehat{\sigma}(\widehat{A})$ for all $\widehat{A}, \widehat{B} \in \widehat{\mathcal{S}}$, and for the second condition satisfied by σ, we have $\widehat{\sigma}(\widehat{\sigma}(\widehat{A})) = \widehat{\sigma}(\widehat{\sigma(A)}) = \widehat{\sigma(\sigma(A))} = \widehat{A}$ for all $\widehat{A} \in \widehat{\mathcal{S}}$. Finally, the map $\widehat{\sigma}$ defines a complementation on $(\mathcal{S},\widehat{\mathcal{S}})$. □

For $(\mathcal{S},\mathcal{L}_{\mathcal{S}})$ we begin by recalling from [16, Theorem 2.10] that every complementation σ_L on a plain texture $(N,\mathcal{L}_N)$ is grounded, that is generated by an order reversing involution $n \mapsto n'$ on the partially ordered set $(N,\leq)$ by the equality $\sigma_L(P_n) = Q_{n'}$. If we use the same idea by taking order reversing involution σ on

$(\mathcal{S},\subseteq)$, then we can obtain a complementation σ_L on $(\mathcal{S},\mathcal{L}_\mathcal{S})$. The following gives an explicit formulae for this complementation.

Lemma 4.5. *By the above notation, the complementation σ_L on $(\mathcal{S},\mathcal{L}_\mathcal{S})$ defined by $\sigma_L(P_A)=Q_{\sigma(A)}$ for all $A\in\mathcal{S}$ is given explicitly by*

$$\sigma_L(\mathcal{B})=\{A\in\mathcal{S}\mid A\notin\sigma(\mathcal{B})\},\ \forall\,\mathcal{B}\in\mathcal{L}_\mathcal{S},$$

where $\sigma(\mathcal{B})=\{\sigma(B)\mid B\in\mathcal{B}\}$.

Proof. We note first that for $\mathcal{B}\in\mathcal{L}_\mathcal{S}$ we have $\mathcal{B}=\bigcup\limits_{A\in\mathcal{B}}\downarrow A=\bigcup\{P_A\mid A\in\mathcal{B}\}$ where $\downarrow A$ denotes the lower set of A, so

$$\sigma_L(\mathcal{B})=\sigma_L\big(\bigcup\{P_B\mid B\in\mathcal{B}\}\big)=\bigcap\big\{\sigma_L(P_B)\mid B\in\mathcal{B}\big\}=\bigcap\big\{Q_{\sigma(B)}\mid B\in\mathcal{B}\big\}.$$

Suppose first that $\bigcap\big\{Q_{\sigma(B)}\mid B\in\mathcal{B}\big\}\not\subseteq\{A\in\mathcal{S}\mid A\notin\sigma(\mathcal{B})\}$. Then we have $A\in\bigcap_{B\in\mathcal{B}}Q_{\sigma(B)}$ with $A\in\sigma(\mathcal{B})$. Hence we have $B\in\mathcal{B}$ with $A=\sigma(B)$ and so we have $A\in Q_{\sigma(B)}=\{A\in\mathcal{S}\mid\sigma(B)\not\subseteq A\}$. This contradicts with $A=\sigma(B)$.

Secondly, suppose that $\{A\in\mathcal{S}\mid A\notin\sigma(\mathcal{B})\}\not\subseteq\bigcap\big\{Q_{\sigma(B)}\mid B\in\mathcal{B}\big\}$. Now we have $B\in\mathcal{B}$ with $\{A\in\mathcal{S}\mid A\notin\sigma(\mathcal{B})\}\not\subseteq Q_{\sigma(B)}$ and so $A\in\mathcal{S}$ with $A\notin\sigma(\mathcal{B})$ and $A\notin Q_{\sigma(B)}=\{A\in\mathcal{S}\mid\sigma(B)\not\subseteq A\}$. This gives us $\sigma(B)\subseteq A$, whence $\sigma(A)\subseteq B$ and so $\sigma(A)\in\mathcal{B}$ as $\mathcal{B}$ is a lower set. But now $A\in\sigma(\mathcal{B})$, which is a contradiction. □

In view of [16, Proposition 2.8], we have the following useful characterization.

Proposition 5. *Let σ_L be a complementation on $(\mathcal{S},\mathcal{L}_\mathcal{S})$ and define $\underline{\sigma}:\mathcal{S}\to\mathcal{L}_\mathcal{S}$ by $\underline{\sigma}(A)=\sigma_L(P_A)$ for all $A\in\mathcal{S}$. Then we have the following properties:*

(i) *$\forall A,B\in\mathcal{S},\ A\subseteq B\Leftrightarrow\underline{\sigma}(B)\subseteq\underline{\sigma}(A)$,*
(ii) *$\forall A,B\in\mathcal{S},\ A\in\underline{\sigma}(B)\implies B\in\underline{\sigma}(A)$,*
(iii) *$\forall B\in\mathcal{S},\ \mathcal{B}\in\mathcal{L}_\mathcal{S},\ B\notin\mathcal{B}\implies\exists\,A\in\mathcal{S}$ with $\mathcal{B}\subseteq\underline{\sigma}(A)$ and $B\notin\underline{\sigma}(A)$.*

Conversely, if $\underline{\sigma}:\mathcal{S}\to\mathcal{L}_\mathcal{S}$ is a mapping satisfying the conditions above (i)-(iii), then $\sigma_L:\mathcal{L}_\mathcal{S}\to\mathcal{L}_\mathcal{S}$ defined by

$$\sigma_L(\mathcal{B})=\bigcap\{\underline{\sigma}(B)|B\in\mathcal{B}\}\tag{4.1}$$

is a complementation on $\mathcal{L}_\mathcal{S}$ satisfying $\underline{\sigma}(A)=\sigma_L(P_A)$ for each $A\in\mathcal{S}$.

Proof. With the given hypothesis, we have:

(i) $A\subseteq B\Leftrightarrow P_A\subseteq P_B\Leftrightarrow\sigma_L(P_B)\subseteq\sigma_L(P_A)\Leftrightarrow\underline{\sigma}(B)\subseteq\underline{\sigma}(A)$,
(ii) $A\in\underline{\sigma}(B)\implies P_A\subseteq\sigma_L(P_B)\implies P_B=\sigma_L(\sigma_L(P_B))\subseteq\sigma_L(P_A)\implies B\in\underline{\sigma}(A)$,
(iii) if $B\notin\mathcal{B}$ then $P_B\not\subseteq\mathcal{B}$ so $\sigma_L(\mathcal{B})\not\subseteq\sigma_L(P_B)=\underline{\sigma}(B)$, so there exists $A\in\mathcal{S}$ with $A\in\sigma_L(\mathcal{B})$ and $A\notin\sigma_L(P_B)=\underline{\sigma}(B)$, this gives us $P_A\subseteq\sigma_L(\mathcal{B})$ thus we get $\sigma_L(\sigma_L(\mathcal{B}))\subseteq\sigma_L(P_A)=\underline{\sigma}(A)\implies\mathcal{B}\subseteq\underline{\sigma}(A)$. Also, $B\notin\underline{\sigma}(A)$ by (ii), since $A\notin\underline{\sigma}(B)$.

Conversely, let $\underline{\sigma} : \mathcal{S} \to \mathcal{L}_{\mathcal{S}}$ be a map satisfying (i)-(iii) and for $\mathcal{B} \in \mathcal{L}_{\mathcal{S}}$ define $\sigma_L(\mathcal{B})$ by (4.1). To show $\sigma_L(\mathcal{S}) \in \mathcal{L}_{\mathcal{S}}$, let $A \in \sigma_L(\mathcal{B})$ and take $C \subseteq A$. In this case, $A \in \sigma_L(\mathcal{B}) = \bigcap\{\underline{\sigma}(B)|B \in \mathcal{B}\}$, so we have $A \in \underline{\sigma}(B)$ for all $B \in \mathcal{B}$. By (ii) we obtain $B \in \underline{\sigma}(A)$ since $C \subseteq A$, then we have $\underline{\sigma}(A) \subseteq \underline{\sigma}(C)$ by (i). Hence we get $B \in \underline{\sigma}(C)$ for all $B \in \mathcal{B}$ and again using (ii) we have $C \in \underline{\sigma}(B)$ for all $B \in \mathcal{B}$. Thus, $C \in \sigma_L(\mathcal{B})$ holds and thereby $\sigma_L(B) \in \mathcal{L}_{\mathcal{S}}$, so we deduce that $\sigma_L : \mathcal{L}_{\mathcal{S}} \to \mathcal{L}_{\mathcal{S}}$ is a mapping.

For $\mathcal{B} \subseteq \mathcal{C}$ in $\mathcal{L}_{\mathcal{S}}$, by (4.1), we obtain $\sigma_L(\mathcal{C}) \subseteq \sigma_L(\mathcal{B})$. In order to show that σ_L is a complementation, we should prove $\sigma_L(\sigma_L(\mathcal{B})) = \mathcal{B}$. To show that equality, we begin by proving the following;

$$P_A = \sigma_L(\underline{\sigma}(A)), \ \forall A \in \mathcal{S}. \tag{4.2}$$

By (4.1), we have $\sigma_L(\underline{\sigma}(A)) = \cap\{\underline{\sigma}(K)|K \in \underline{\sigma}(A)\}$, and $K \in \underline{\sigma}(A) \implies A \in \underline{\sigma}(K) \implies P_A \subseteq \underline{\sigma}(K)$ by (ii), so clearly $P_A \subseteq \sigma_L(\underline{\sigma}(A))$. To show $\sigma_L(\underline{\sigma}(A)) \subseteq P_A$, let take $B \notin P_A$. Then we have $B \not\subseteq A \implies \underline{\sigma}(A) \not\subseteq \underline{\sigma}(B)$ by (i), so we may take $K \in \underline{\sigma}(A)$ satisfying $K \notin \underline{\sigma}(B)$. By (ii) we have $B \notin \underline{\sigma}(K)$, and so $B \notin \sigma_L(\underline{\sigma}(A))$ which gives $\sigma_L(\underline{\sigma}(A)) \subseteq P_A$ and hence (4.2) is satisfied.

Now we ready to show $\sigma_L(\sigma_L(\mathcal{B})) = \mathcal{B}$, first suppose that $\sigma_L(\sigma_L(\mathcal{B})) \not\subseteq \mathcal{B}$ for some $\mathcal{B} \in \mathcal{L}_{\mathcal{S}}$ and take $B \in \sigma_L(\sigma_L(\mathcal{B}))$ with $B \notin \mathcal{B}$. By (iii) we have $A \in \mathcal{S}$ satisfying $\mathcal{B} \subseteq \underline{\sigma}(A)$ and $B \notin \underline{\sigma}(A)$. From the first inclusion, we obtain $P_A = \sigma_L(\underline{\sigma}(A)) \subseteq \sigma_L(\mathcal{B})$ by (4.2) so $A \in \sigma_L(\mathcal{B})$. Now by using (4.1) for $\sigma_L(\mathcal{B})$ replaced with $\mathcal{B}$ we get $\sigma_L(\sigma_L(\mathcal{B})) \subseteq \underline{\sigma}(A)$, which gives a contradiction. Hence $\sigma_L(\sigma_L(\mathcal{B})) \subseteq \mathcal{B}$.

To prove the opposite inclusion, suppose that $\mathcal{B} \not\subseteq \sigma_L(\sigma_L(\mathcal{B}))$, so there exits $B \in \mathcal{B}$ such that $B \notin \sigma_L(\sigma_L(\mathcal{B})) = \bigcap\{\underline{\sigma}(A)|A \in \sigma_L(\mathcal{B})\}$. Thus, for all $A \in \sigma_L(\mathcal{B})$ we have $B \notin \underline{\sigma}(A)$ and this implies $A \notin \underline{\sigma}(B)$ by (ii), but this contradicts with $A \in \sigma_L(\mathcal{B})$.

This completes the proof that σ_L is a complementation, and by using (4.2) we obtain $\sigma_L(P_A) = \sigma_L(\sigma_L(\underline{\sigma}(A))) = \underline{\sigma}(A)$, as required. □

Now, as we indicated earlier, by using the idea in [16, Theorem 2.10], we give the following theorem

Theorem 4.6. *Any complementation σ_L on the plain hypertexture $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$ is grounded, and the corresponding involution $A \to A' = \sigma(A)$ is order reversing. Conversely, if $A \to A' = \sigma(A)$ is an order reversing involution on $(\mathcal{S}, \subseteq)$ then $\underline{\sigma}(A) = Q_{\sigma(A)}$ defines a grounded complementation σ_L on $\mathcal{L}_{\mathcal{S}}$ for which $\underline{\sigma}(A) = \sigma_L(P_A)$ for all $A \in \mathcal{S}$.*

Proof. It can be easily proved by using Lemma 4.5 and Proposition 5. □

The following example illustrates the above construction.

Example 4.7. Consider the texture $(L, \mathcal{L})$ of Examples 4.3(2). The standard complementation for this texture is λ defined by $\lambda((0, r]) = (0, 1 - r], \ 0 \leq r \leq 1$.

As noted earlier there are two types of lower set in $\mathcal{L}$ defining the p-sets and q-sets in $(\mathcal{L}, \mathcal{L}_{\mathcal{L}})$, respectively:

$$P_{(0,r]} = \{(0,k] \mid 0 \le k \le r\} \text{ and } Q_{(0,r]} = \{(0,k] \mid 0 \le k < r\}.$$

By using the equalities $\lambda_L(P_{(0,r]}) = Q_{\lambda((0,r])} = Q_{(0,1-r]}$, $\lambda_L(Q_{(0,r]}) = P_{\lambda((0,r])} = P_{(0,1-r]}$ or the formula given in Lemma 4.5 we clearly have:

$$\lambda_L(\{(0,k] \mid 0 \le k \le r\}) = \{(0,s] \mid 0 \le s < 1-r\}$$

and

$$\lambda_L(\{(0,k] \mid 0 \le k < r\}) = \{(0,s] \mid 0 \le s \le 1-r\}.$$

We note that λ is not restriction of λ_L on $(\mathcal{L}, \widehat{\mathcal{L}})$. However, we do have the following commutativity diagram, which represents a form of compatibility:

$$\begin{array}{ccc} \mathcal{L}_{\mathcal{L}} & \xrightarrow{\ \gamma\ } & \mathcal{L} \\ \downarrow \lambda_L & & \downarrow \lambda \\ \mathcal{L}_{\mathcal{L}} & \xrightarrow{\ \gamma\ } & \mathcal{L} \end{array}$$

We must establish $\lambda(\gamma(\mathcal{B})) = \gamma(\lambda_L(\mathcal{B}))$ for all $\mathcal{B} \in \mathcal{L}_{\mathcal{L}}$. There are two cases to consider:

1) If $\mathcal{B}$ has the form $\{(0,k] \mid 0 \le k \le r\}$, $0 \le r \le 1$, then $\gamma(\mathcal{B}) = (0,r]$, $\lambda((0,r]) = (0,1-r]$ and $\lambda_L(\mathcal{B}) = \{(0,s] \mid 0 \le s < 1-r\}$. Thus, $\gamma(\lambda_L(\mathcal{B})) = (0,1-r]$ establishes the required equality.

2) If $\mathcal{B}$ has the form $\{(0,k] \mid 0 \le k < r\}$, $0 \le r \le 1$, then the proof is similar and is omitted.

Note 1. *It will probably strike the reader that the complemented texture* $(\mathcal{L}, \mathcal{L}_{\mathcal{L}}, \lambda_L)$ *bears a close resemblance to the unit interval texture (Examples 4.3(3)) with its standard complementation. Indeed it is not difficult to prove that these two textures are actually isomorphic in the sense of* [4]*, and the details are left to the interested reader. This shows that the plain hypertexture of a texture with very poor mathematical properties (for example* $(L, \mathcal{L}, \lambda)$ *has no plain points at all) can, in certain cases, be a texture with excellent properties.*

We now present an example which shows that the complementation σ_L does not always have the compatibility property mentioned above.

Example 4.8. Consider again the texture $(X, \mathcal{P}(X))$, $X = \{a, b\}$, of Examples 4.3(1). The standard complementation π, $\pi(A) = X \setminus A$ on $(X, \mathcal{P}(X))$ gives $\pi(\{a,b\}) = \emptyset$, $\pi(\{a\}) = \{b\}$, $\pi(\{b\}) = \{a\}$ and $\pi(\emptyset) = \{a, b\}$. Consider X which have the discrete ordering. In this case, it is generated by the (necessarily order reversing) involution $n \mapsto n$ on X, see [16]. For the complementation π_L on $(\mathcal{P}(X), \mathcal{L}_{\mathcal{P}(X)})$ we obtain the following results from Lemma 4.5 :

$\mathcal{B}$	$\pi(\mathcal{B})$	$\pi_L(\mathcal{B})$
$\{\{a,b\},\{a\},\{b\},\emptyset\}$	$\{\emptyset,\{b\},\{a\},\{a,b\}\}$	$\emptyset$
$\{\{a\},\{b\},\emptyset\}$	$\{\{b\},\{a\},\{a,b\}\}$	$\{\emptyset\}$
$\{\{a\},\emptyset\}$	$\{\{b\},\{a,b\}\}$	$\{\{a\},\emptyset\}$
$\{\{b\},\emptyset\}$	$\{\{a\},\{a,b\}\}$	$\{\{b\},\emptyset\}$
$\{\emptyset\}$	$\{\{a,b\}\}$	$\{\{a\},\{b\},\emptyset\}$
$\emptyset$	$\emptyset$	$\{\{a,b\},\{a\},\{b\},\emptyset\}$

We note that while π interchanges $\{a\}$ and $\{b\}$, π_L does not interchange $\{\{a\},\emptyset\}$ and $\{\{b\},\emptyset\}$, so we do not have compatibility in the sense of Example 4.7.

In place of the involution $n \mapsto n$ let us consider the (necessarily order reversing) involution $a \mapsto b$, $b \mapsto a$. This generates a complementation ϖ on $(X, \mathcal{P}(X))$, which leads to the complementation ϖ_L on $(\mathcal{P}(X), \mathcal{L}_{\mathcal{P}(X)})$. It is trivial to verify that ϖ_L is the same as π_L except that $\varpi_L(\{\{a\},\emptyset\}) = \{\{b\},\emptyset\}$ and $\varpi_L(\{\{b\},\emptyset\}) = \{\{a\},\emptyset\}$. It follows easily that the following diagram is commutative so this time ϖ_L has the required compatibility property.

$$\begin{array}{ccc} \mathcal{L}_{\mathcal{P}(X)} & \xrightarrow{\gamma} & \mathcal{P}(X) \\ \downarrow \varpi_L & & \downarrow \pi \\ \mathcal{L}_{\mathcal{P}(X)} & \xrightarrow{\gamma} & \mathcal{P}(X) \end{array}$$

Comment. It is not known if we can always find a compatible complementation on the plain hypertexture of a given texture.

5. Conclusion and Future Work

In this paper, we define hypertexture notion which is inspired by the hyperspace notion, and we investigate its properties, there is a naturally question arises: What will we do for the next step? Let us consider a ditopological texture space $(S, \mathcal{S}, \tau, \kappa)$ and the Vietoris ditopology on the corresponding standard and plain hypertextures $(\mathcal{S}, \widehat{\mathcal{S}})$, $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$, respectively. Therefore, we already begin with the following definition.

Definition 5.1. Let $(S, \mathcal{S}, \tau, \kappa)$ be a ditopological texture space.

(1) The *Vietoris ditopology* for $(\mathcal{S}, \widehat{\mathcal{S}})$ is $(\widehat{\tau}, \widehat{\kappa})$ where $\widehat{\tau}$ is the smallest topology on $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$ satisfying $\widehat{G} \in \widehat{\tau}$ whenever $G \in \tau$ and $\widehat{\kappa}$ is the smallest cotopology on $(\mathcal{S}, \widehat{\mathcal{S}})$ satisfying $\widehat{K} \in \widehat{\kappa}$ whenever $K \in \kappa$.
(2) The *Vietoris ditopology* for $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$ is (τ_v, κ_v) where τ_v is the smallest topology on $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$ satisfying $\widehat{G} \in \tau_v$ whenever $G \in \tau$ and κ_v is the smallest cotopology on $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$ satisfying $\widehat{K} \in \kappa_v$ whenever $K \in \kappa$.

In view of the isomorphism $A \mapsto \widehat{A}$ we have at once:

Lemma 5.2. *With the above notation, the equalities* $\widehat{\tau} = \{\widehat{G} \mid G \in \tau\}$ *and* $\widehat{\kappa} = \{\widehat{K} \mid K \in \kappa\}$ *are trivial.*

Corollary 4. *The difunction* (h, H) *corresponding to the isomorphism* $A \mapsto \widehat{A}$ *as above is a dihomeomorphism between* $(S, \mathcal{S}, \tau, \kappa)$ *and* $(\mathcal{S}, \widehat{\mathcal{S}}, \widehat{\tau}, \widehat{\kappa})$.

It follows that (h, H) preserves the “point free" properties of ditopological texture spaces, including the compactness properties and the separation properties of [8]. Also, the notion of extended real dicompactness [20] is preserved under dihomeomorphisms and so (h, H) preserves this property too. However, it cannot be expected that properties depending on the point structure will preserve in general. For a counterexample we need only consider the texture $(\{a, b\}, \mathcal{P}(\{a, b\}))$ of Examples 4.3(1) with the discrete ditopology $\tau = \kappa = \mathcal{P}(\{a, b\})$. This is trivially a bi-T_2 plain dicompact, hence real dicompact space. However, by the discussion in Examples 4.3(1) the image of $(\{a, b\}, \mathcal{P}(\{a, b\}))$ under (h, H) is not nearly plain and so cannot support a real dicompact ditopology by [18, Proposition 2.9].

Let us now consider the Vietoris ditopology (τ_v, κ_v) on $(\mathcal{S}, \mathcal{L}_{\mathcal{S}})$ and the difunction $(l, L) : (S, \mathcal{S}) \to (\mathcal{S}, \mathcal{L}_{\mathcal{S}})$ defined in Theorem 4.4. In this case, the following lemma is obvious.

Lemma 5.3. *The difunction* $(l, L) : (S, \mathcal{S}, \tau, \kappa) \to (\mathcal{S}, \mathcal{L}_{\mathcal{S}}, \tau_v, \kappa_v)$ *is bicontinuous.*

Now we can state that the functor $\mathfrak{B}$ described in Proposition 3 can be regarded as mapping from the category **dfDitop** of ditopological texture spaces to the category **dfPDitop** of plain ditopological texture spaces.

Proposition 6. *Let* $\mathfrak{B}$ *be defined by* $\mathfrak{B}(S, \mathcal{S}, \tau, \kappa) = (\mathcal{S}, \mathcal{L}_{\mathcal{S}}, \tau_v, \kappa_v)$ *and for a* ***dfDitop*** *morphism* $(f, F) : (S, \mathcal{S}, \tau, \kappa) \to (T, \mathcal{T}, \mu, \nu)$ *let* $\mathfrak{B}(f, F) = (g, G) : (\mathcal{S}, \mathcal{L}_{\mathcal{S}}, \tau_v, \kappa_v) \to (\mathcal{T}, \mathcal{L}_{\mathcal{T}}, \mu_v, \nu_v)$ *be characterized by* $g^{\leftarrow}\mathcal{B} = \{A \in \mathcal{S} \mid \exists B \in \mathcal{B}, A \subseteq f^{\leftarrow}B\} = G^{\leftarrow}\mathcal{B}$ *for* $\mathcal{B} \in \mathcal{L}_{\mathcal{T}}$. *Then* $\mathfrak{B}$: **dfDitop** $\to$ **dfPDitop** *is a functor.*

In addition, we continue to investigate the other categorical structure of this new notion which we call it Hyperdispace and also we will work on some separation axioms, dicompactness together with difilters for this new structure.

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