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# ON LIGHTLIKE SUBMANIFOLDS IN SEMI-RIEMANNIAN MANIFOLD WITH A RICCI QUARTER SYMMETRIC METRIC CONNECTION

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Abstract. We study lightlike submanifolds of a semi-Riemannian manifold with a Ricci quarter-symmetric metric connection. We obtain integrability conditions for screen distribution. We also identify the conditions for which the Ricci tensor of a lightlike submanifold with Ricci quarter-symmetric metric connection is symmetric. Then, we conclude our study by showing that the conformal curvature tensor of a lightlike submanifold $M$ of semi-Riemannian space form $\widetilde{M}(c)$ is equal to curvature tensor of $M$ with respect to Ricci quarter-symmetric metric connection.

## 1. Introduction

The idea of a Ricci quarter symmetric metric connection on a Riemannian manifold was introduced and presented by Kamilya and De U.C. [6]. Before this work, a few papers had been written about the studies of various types of a quarter symmetric metric connection and their properties in [3], [8] and [10]. Then in [3] Golap studied some properties of the curvature tensor of a differentiable manifold with respect to the quarter symmetric metric connection. In [6] Kamilya and De U.C. found necessary and sufficient conditions for the symmetry of the Ricci tensor of a Ricci quarter symmetric metric connection, and showed that conformal curvature tensor of induced connection $\nabla$ and linear connection $\widetilde{\nabla}$ are equal. Quarter symmetric metric connection on Riemannian manifold and affinely connected manifold were studied by Rostogi S.C. [10]. In an earlier paper [12], we studied lightlike hypersurfaces of a semi-Riemannian manifold admitting a semi-symmetric metric connection.

In this paper, we study lightlike submanifolds of a semi-Riemannian manifold admitting a Ricci quarter symmetric metric connection since the growing importance of lightlike submanifold in semi-Riemannian geometry, and their use in general relativity. Due to the degeneracy of the metric, basic differences occur between the study of lightlike submanifolds and the classical theory of Riemannian as well

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as semi-Riemannianmanifold.Several papers have been written on lightlike submanifolds in semi-Riemannian manifold recent years (see [1], [2], for instance) but lightlike submanifolds in semi-Riemannian manifold with Ricci quarter-symmetric metric connection has not been studied yet.

In this paper, we have proved that on lightlike submanifold the connection induced from Ricci quarter-symmetric metric connection is Ricci quarter-symmetric non metric, nevertheless, on screen distribution the connection induced from that connection is Ricci quarter-symmetric metric connection. We define the induced geometrical objects with respect to the Ricci quarter-symmetric metric connection on the triplet $(S(TM), S(TM^{\perp}), tr(TM))$. Then we investigate the integrability condition of the screen distribution with respect to the Ricci quarter-symmetric metric connection. Moreover, we give the conditions under which the Ricci tensor of a lightlike submanifold with respect to the Ricci quarter-symmetric metric connection is symmetric. Then, we show that the conformal curvature tensor of a lightlike submanifold $M$ of semi-Riemannian space form $\widetilde{M}(c)$ is equal to curvature tensor of $M$ with respect to Ricci quarter-symmetric metric connection.

## 2. Preliminaries

Let $(\widetilde{M}, \widetilde{g})$ be a real $(m+n)-$dimensional semi-Riemannian manifold of constant index such that $1 \leq \nu \leq m+n-1$ and $(M, g)$ be an $m-$dimensional submanifold of $\widetilde{M}$. In case $\widetilde{g}$ is degenerate on the tangent bundle $TM$ of $M$, $M$ is called a lightlike submanifold of $\widetilde{M}$. Denote by $g$ the induced tensor field of $\widetilde{g}$ on $M$ and suppose $g$ is degenerate. Then, for each tangent space $T_xM$ we consider

$$T_xM^{\perp} = \left\{ Y_x \in T_x\widetilde{M} \mid \widetilde{g}_x\left(Y_x, X_x\right) = 0, \, \forall X_x \in T_xM \right\}$$

which is a degenerate $n-$dimensional subspace of $T_x\widetilde{M}$. Thus, both $T_xM$ and $T_xM^{\perp}$ are degenerate orthogonal subspaces but no longer complementary subspaces. For this case, there exists a subspace $RadT_xM = T_xM \cap T_xM^{\perp}$ called *radical (null) subspace.* If the mapping

$$RadTM : x \in M \longrightarrow RadT_xM$$

defines a smooth distribution on $M$ of *rank* $r > 0$, the submanifold $M$ of $\widetilde{M}$ is called $r-$*lightlike* ($r-$*degenerate*) *submanifold* and $RadTM$ is called the *radical* (*lightlike*) *distribution* on $M$. In the following , there are four possible cases:

*Case 1*. $M$ is called a $r-$lightlike submanifold if $1 \leq r < \min\{m, n\}$.

*Case 2*. $M$ is called a coisotropic submanifold if $1 < r = n < m$.

*Case 3*. $M$ is called an isotropic submanifold if $1 < r = m < n$.

*Case 4*. $M$ is called a totally lightlike submanifold if $1 < r = m = n$ [7].

Throughout to this work, we consider case 1 where there exists a non-degenerate screen distribution $S(TM)$ which is a complementary vector subbundle to $RadTM$ in $TM$. Therefore,

$$TM = RadTM \perp S\left(TM\right), \tag{2.1}$$

in which $\perp$ denotes orthogonal direct sum. Although $S(TM)$ is not unique, it is canonically isomorphic to the factor vector bundle $TM/RadTM$. Denote an $r-$lightlike submanifold by $(M, g, S(TM), S(TM^{\perp}))$, where $S(TM^{\perp})$ is a complementary vector bundle of $RadTM$ in $TM^{\perp}$ and $S(TM^{\perp})$ is non-degenerate with respect to $\widetilde{g}$. Let us define that $tr(TM)$ is a complementary (but never orthogonal) vectors bundle to $TM$ in $T\widetilde{M}_{|_M}$ and

$$tr(TM) = ltr(TM) \perp S(TM^{\perp}), \tag{2.2}$$

where $ltr(TM)$ is an arbitrary lightlike transversal vector bundle of $M$. Then we have

$$\begin{aligned} T\widetilde{M}_{|_M} &= TM \oplus tr(TM) \\ &= (RadTM \oplus ltr(TM)) \perp S(TM) \perp S(TM^{\perp}) \end{aligned} \tag{2.3}$$

where $\oplus$ denotes direct sum, but it is not orthogonal [7].

Now we assume that $\mathcal{U}$ is a local coordinate neighborhood of $M$. We consider the following local quasi-orthonormal field of frames on $\widetilde{M}$ along $M$:

$$\{\xi_1, ...., \xi_r, X_{r+1}, ..., X_m, N_1, ...., N_r, W_{r+1}, ..., W_n\} \tag{2.4}$$

where $\{\xi_1, ...., \xi_r\}$ and $\{N_1, ...., N_r\}$ are lightlike basis of $\Gamma(Rad(TM)_{|\mathcal{U}})$ and $\Gamma(ltr(TM)_{|\mathcal{U}})$, respectively and $\{X_{r+1}, ..., X_m\}$ and $\{W_{r+1}, ..., W_n\}$ are orthonormal basis of $\Gamma(S(TM)_{|\mathcal{U}})$ and $\Gamma(S(TM^{\perp})_{|\mathcal{U}})$, respectively, where the following conditions are satisfied

$$\widetilde{g}(N_i, \xi_j) = \delta_{ij}, \quad 1 \leq i, j \leq r,$$

$\widetilde{g}(N_i, N_j) = \widetilde{g}(N_i, X_k) = 0$, $r + 1 \leq k \leq m$, $X_k \in \Gamma(S(TM)_{|\mathcal{U}})$, $N_i \in \Gamma(ltr(TM)_{|\mathcal{U}})$ [5].

**Example 2.1.** *[7]. Consider in $\mathbb{R}^4_2$ the* $1-$lightlike submanifold $M$ *given by the equations:*

$$x^3 = \frac{1}{\sqrt{2}}(x^1 + x^2), \ x^4 = \frac{1}{2}\log(1 + (x^1 - x^2)^2).$$

*Then we have $TM = Span\{U_1, U_2\}$ and $TM^{\perp} = Span\{H_1, H_2\}$ where we set*

$$U_1 = \sqrt{2}(1 + (x^1 - x^2)^2)\frac{\partial}{\partial x^1} + (1 + (x^1 - x^2)^2)\frac{\partial}{\partial x^3} + \sqrt{2}(x^1 - x^2)\frac{\partial}{\partial x^4},$$

$$U_2 = \sqrt{2}(1 + (x^1 - x^2)^2)\frac{\partial}{\partial x^2} + (1 + (x^1 - x^2)^2)\frac{\partial}{\partial x^3} - \sqrt{2}(x^1 - x^2)\frac{\partial}{\partial x^4},$$

*and*

$$H_1 = \frac{\partial}{\partial x^1} + \frac{\partial}{\partial x^2} + \sqrt{2}\frac{\partial}{\partial x^3},$$

$$H_2 = 2(x^2 - x^1)\frac{\partial}{\partial x^2} + \sqrt{2}(x^2 - x^1)\frac{\partial}{\partial x^3} + (1 + (x^1 - x^2)^2)\frac{\partial}{\partial x^4}.$$

It follows that *$Rad(TM)$ is a distribution on $M$ of rank* 1 *spanned by $\xi = H_1$. Choose $S(TM)$ and $S(TM^{\perp})$ spanned by $U_2$ and $H_2$ which are timelike and spacelike respectively. Finally, the lightlike transversal vector bundle*

$$ltr(TM) = Span\{N = -\frac{1}{2}\frac{\partial}{\partial x^1} + \frac{1}{2}\frac{\partial}{\partial x^2} + \frac{1}{\sqrt{2}}\frac{\partial}{\partial x^3}\},$$

*and the transversal vector bundle*

$$tr(TM) = Span\{N, H_2\}.$$

are obtained.

## 3. Ricci Quarter Symmetric Metric Connection

Let $\widetilde{M}$ denotes a $(m+n)-$dimensional semi-Riemannian manifold with a semi-Riemannian metric $\widetilde{g}$ of index $1 \leq \nu \leq m+n-1$, and $\widetilde{\nabla}$ denotes linear connection in $\widetilde{M}$. The torsion tensor $\widetilde{T}$ of $\widetilde{\nabla}$ is given by

$$\widetilde{T}(\widetilde{X},\widetilde{Y}) = \widetilde{\nabla}_{\widetilde{X}}\widetilde{Y} - \widetilde{\nabla}_{\widetilde{Y}}\widetilde{X} - [\widetilde{X},\widetilde{Y}], \quad \forall \widetilde{X},\widetilde{Y} \in \Gamma(T\widetilde{M})$$

and have type $(1,2)$. A linear connection $\widetilde{\nabla}$ on $\widetilde{M}$ is said to be Ricci quarter symmetric conection if its torsion tensor $\widetilde{T}$ satisfies

$$\widetilde{T}(\widetilde{X},\widetilde{Y}) = \widetilde{\pi}(\widetilde{Y})L\widetilde{X} - \widetilde{\pi}(\widetilde{X})L\widetilde{Y}$$

for any $\widetilde{X},\widetilde{Y} \in \Gamma(T\widetilde{M})$, where $\widetilde{\pi}$ is a $1-$form and $L$ is the Ricci operator of type $(1,1)$ defined by

$$\widetilde{g}(L\widetilde{X},\widetilde{Y}) = S(\widetilde{X},\widetilde{Y}),$$

$S$ is the Ricci tensor of $\widetilde{M}$ .

A linear connection $\widetilde{\nabla}$ is called metric connection if

$$(\widetilde{\nabla}_{\widetilde{X}}\widetilde{g})(\widetilde{Y},\widetilde{Z}) = 0.$$

In the sequel, $\widetilde{M}$ will always denote an $(m+n)$-dimensional semi-Riemannian manifold endowed with a Ricci quarter symmetric metric connection $\widetilde{\nabla}$ given by

$$\widetilde{\nabla}_{\widetilde{X}}\widetilde{Y} = \overset{\circ}{\widetilde{\nabla}}_{\widetilde{X}}\widetilde{Y} + \widetilde{\pi}(\widetilde{Y})L\widetilde{X} - S(\widetilde{X},\widetilde{Y})\widetilde{Q} \tag{3.1}$$

where $\overset{\circ}{\widetilde{\nabla}}$ is a Levi-civita connection with respect to $\widetilde{g}$ [6].

From (2.3), the vector field $\widetilde{Q}$ on $\widetilde{M}$ is decomposed as

$$\widetilde{Q} = Q + \sum_{i=1}^{r}\lambda_i N_i + \sum_{\alpha=r+1}^{n}\lambda_\alpha W_\alpha, \tag{3.2}$$

where $Q$ is a vector field and $\lambda_a$, $1 \leq a \leq n$ are real valued function on $M$.

We show the symmetric linear connection induced on $M$ from $\overset{\circ}{\widetilde{\nabla}}$ on $\widetilde{M}$ by $\overset{\circ}{\nabla}$ , then the Gauss formula with respect to $\overset{\circ}{\nabla}$ is given by

$$\overset{\circ}{\widetilde{\nabla}}_X Y = \overset{\circ}{\nabla}_X Y + \sum_{i=1}^{r}\overset{\circ}{h}{}_i^{\ell}(X,Y)N_i + \sum_{\alpha=r+1}^{n}\overset{\circ}{h}{}_\alpha^{s}(X,Y)W_\alpha \tag{3.3}$$

for $X,Y \in \Gamma(TM)$, $N_i \in \Gamma(ltr(TM))$, and $W_\alpha \in \Gamma(S(TM^\perp))$, where $\{\overset{\circ}{h}{}_i^{\ell}\}$ and $\{\overset{\circ}{h}{}_\alpha^{s}\}$ are called the local lightlike second fundamental forms and the local screen second fundamental forms of $M$ which are symmetric bilinear forms [2]. Let us define the connection $\nabla$ on $M$ that is induced from the Ricci quarter-symmetric metric connection $\widetilde{\nabla}$ on $\widetilde{M}$ given by the equation below is called the Gauss formula with respect to $\widetilde{\nabla}$ for any $X,Y \in \Gamma(TM)$

$$\widetilde{\nabla}_X Y = \nabla_X Y + \sum_{i=1}^{r}h_i^{\ell}(X,Y)N_i + \sum_{\alpha=r+1}^{n}h_\alpha^{s}(X,Y)W_\alpha, \tag{3.4}$$

where $\{h_i^\ell\}$ and $\{h_\alpha^s\}$ are called the local lightlike second fundamental forms and the local screen second fundamental forms of $M$ which are tensors of type $(0,2)$ on $M$.

By virtue of (3.1), we get

$$\widetilde{\nabla}_X Y = \overset{\circ}{\widetilde{\nabla}}_X Y + \widetilde{\pi}(Y)LX - S(X,Y)\widetilde{Q}. \tag{3.5}$$

Thus, on substituting (3.3) and (3.4) into (3.5) we see that

$$\begin{aligned}\nabla_X Y + \sum_{i=1}^{r} h_i^\ell(X,Y)N_i + \sum_{\alpha=r+1}^{n} h_\alpha^s(X,Y)W_\alpha &= \overset{\circ}{\nabla}_X Y + \sum_{i=1}^{r} \overset{\circ}{h}{}_i^\ell(X,Y)N_i - S(X,Y)\widetilde{Q} \\ &\quad + \sum_{\alpha=r+1}^{n} \overset{\circ}{h}{}_\alpha^s(X,Y)W_\alpha + \widetilde{\pi}(Y)LX \end{aligned} \tag{3.6}$$

Since (3.2) and (3.6), we obtain

$$\nabla_X Y = \overset{\circ}{\nabla}_X Y + \pi(Y)LX - g(X,Y)Q \tag{3.7}$$

and

$$\overset{\circ}{h}{}_i^\ell = h_i^\ell + \lambda_i S \text{ and } \overset{\circ}{h}{}_\alpha^s = h_\alpha^s + \lambda_\alpha S,\ 1 \le i \le r,\ r+1 \le \alpha \le n \tag{3.8}$$

for any $X, Y \in \Gamma(TM)$, where $\widetilde{\pi}(X) = \pi(X)$.

According to (3.7) and the connection induced on lightlike submanifold from Levi-Civita connection is not metric, we have

$$\begin{aligned}(\nabla_X g)(Y,Z) &= \sum_{i=1}^{r}\{h_i^\ell(X,Y) + \lambda_i S(X,Y)\}\eta_i(Z) \\ &\quad + \{h_i^\ell(X,Z) + \lambda_i S(X,Z)\}\eta_i(Y)\end{aligned} \tag{3.9}$$

where

$$\eta_i(Z) = g(N_i, Z),\ 1 \le i \le r \tag{3.10}$$

for any $X, Y, Z \in \Gamma(TM)$ and $N_i \in \Gamma(ltr(TM))$. Also from (3.7), the torsion tensor of the connection $\nabla$ is found as :

$$T(X,Y) = \pi(Y)LX - \pi(X)LY. \tag{3.11}$$

Then from (3.9) and (3.11), we have the following proposition :

**Proposition 3.1.** *The induced connection on a lightlike submanifold of a semi-Riemannian manifold admitting a Ricci quarter-symmetric metric connection is Ricci quarter-symmetric, but not metric connection.*

The Weingarten formulae with respect to $\overset{\circ}{\widetilde{\nabla}}$ is given by

$$\overset{\circ}{\widetilde{\nabla}}_X N_i = -\overset{\circ}{A}_{N_i} X + \overset{\circ}{\nabla}{}^\ell_X N_i + \overset{\circ}{D}{}^s(X, N_i),\ 1 \le i \le r, \tag{3.12}$$

and

$$\overset{\circ}{\widetilde{\nabla}}_X W_\alpha = -\overset{\circ}{A}_{W_\alpha} X + \overset{\circ}{D}{}^\ell(X, W_\alpha) + \overset{\circ}{\nabla}{}^s_X W_\alpha,\ r+1 \le \alpha \le n, \tag{3.13}$$

for any $X \in \Gamma(TM)$, $N_i \in \Gamma(ltr(TM)$ and $W_\alpha \in \Gamma(S(TM^\perp))$, where

$$\overset{\circ}{\nabla}{}^\ell_X : \Gamma(ltr(TM) \longrightarrow \Gamma(ltr(TM);\ \overset{\circ}{\nabla}{}^\ell_X(LV) = \overset{\circ}{D}{}^\ell_X(LV),$$

$$\overset{\circ}{\nabla}{}^{s}_{X} : \Gamma(S(TM^{\perp})) \longrightarrow \Gamma(S(TM^{\perp}));\ \overset{\circ}{\nabla}{}^{s}_{X}(SV) = \overset{\circ}{D}{}^{s}_{X}(SV),$$

$$\overset{\circ}{D}{}^{\ell} : \Gamma(TM) \times \Gamma(S(TM^{\perp})) \longrightarrow \Gamma(ltr(TM));\ \overset{\circ}{D}{}^{\ell}(X, SV) = \overset{\circ}{D}{}^{\ell}_{X}(SV),$$

$$\overset{\circ}{D}{}^{s} : \Gamma(TM) \times \Gamma(ltr(TM)) \longrightarrow \Gamma(S(TM^{\perp}));\ \overset{\circ}{D}{}^{s}(X, LV) = \overset{\circ}{D}{}^{s}_{X}(LV)$$

for any $V \in \Gamma(tr(TM))$ such that $L$ and $S$ are the projection morphisms of $tr(TM)$ on $ltr(TM)$ and $S(TM^{\perp})$ respectively. Also $\overset{\circ}{\nabla}{}^{\ell}$ and $\overset{\circ}{\nabla}{}^{s}$ are linear connections on $ltr(TM)$ and $S(TM^{\perp})$, respectively, $\overset{\circ}{A}_{N_i}$ and $\overset{\circ}{A}_{W_\alpha}$ are called the shape operators of $M$ with respect to $N_i$ and $W_\alpha$, respectively [2]. We shall define

$$\rho_{ij}(X) = \widetilde{g}(\overset{\circ}{\nabla}{}^{\ell}_{X} N_i, \xi_j),\ 1 \leq i, j \leq r,$$

$$\sigma_{i\alpha}(X) = \varepsilon_\alpha \widetilde{g}(\overset{\circ}{D}{}^{s}(X, N_i), W_\alpha),\ r+1 \leq \alpha \leq n,\ 1 \leq i \leq r,$$

$$\gamma_{\alpha j}(X) = \widetilde{g}(\overset{\circ}{D}{}^{\ell}(X, W_\alpha), \xi_j),\ r+1 \leq \alpha \leq n,\ 1 \leq j \leq r,$$

$$\mu_{\alpha\beta}(X) = \varepsilon_\beta \widetilde{g}(\overset{\circ}{\nabla}{}^{s}_{X} W_\alpha, W_\beta),\ r+1 \leq \alpha, \beta \leq n,$$

for any $X \in \Gamma(TM)$, $N_i \in \Gamma(ltr(TM)$ and $W_\alpha \in \Gamma(S(TM^{\perp}))$. Thus, (3.12) and (3.13) can be rewritten as :

$$\overset{\circ}{\widetilde{\nabla}}_X N_i = -\overset{\circ}{A}_{N_i} X + \sum_{j=1}^{r} \rho_{ij}(X) N_j + \sum_{\alpha=r+1}^{n} \sigma_{i\alpha}(X) W_\alpha, 1 \leq i \leq r, \tag{3.14}$$

and

$$\overset{\circ}{\widetilde{\nabla}}_X W_\alpha = -\overset{\circ}{A}_{W_\alpha} X + \sum_{j=1}^{r} \gamma_{\alpha j}(X) N_j + \sum_{\beta=r+1}^{n} \mu_{\alpha\beta}(X) W_\beta,\ r+1 \leq \alpha \leq n. \tag{3.15}$$

Let us denote by $P$ the projection morphism of $TM$ on $S(TM)$ with respect to the decomposition (2.1), then we can write $X = PX + \sum_{i=1}^{r} \eta_i(X)\xi_i$ for any $X \in \Gamma(TM)$. Thereafter, by using (3.1), (3.2), (3.14) and (3.15) we obtain the Weingarten formulae with respect to $\widetilde{\nabla}$ given by

$$\begin{aligned} \widetilde{\nabla}_X N_i &= -\overset{\circ}{A}_{N_i} X + \eta_i(Q) LX - S(X, N_i) Q + \sum_{j=1}^{r} \{-S(X, Y)\lambda_i N_i + \rho_{ij}(X) N_j\} \\ &\quad + \sum_{\alpha=r+1}^{n} (\sigma_{i\alpha}(X) - S(X, N_i)\lambda_\alpha) W_\alpha,\ 1 \leq i \leq r, \end{aligned} \tag{3.16}$$

and

$$\begin{aligned} \widetilde{\nabla}_X W_\alpha &= -\overset{\circ}{A}_{W_\alpha} X + \lambda_\alpha \varepsilon_\alpha LX - S(X, W_\alpha) Q + \sum_{j=1}^{r} \{\gamma_{\alpha j}(X) N_j \\ &\quad - S(X, W_\alpha)\lambda_i N_i\} + \sum_{\beta=r+1}^{n} \{\mu_{\alpha\beta}(X) W_\beta - S(X, W_\alpha) W_\alpha\}, \end{aligned} \tag{3.17}$$

for any $X \in \Gamma(TM)$, $N_i \in \Gamma(ltr(TM))$ and $W_\alpha \in \Gamma(S(TM^{\perp}))$.

By considering (3.1), (3.4), (3.16), (3.17) and taking into account that $\overset{\circ}{\widetilde{\nabla}}$ is not metric connection, we derive that

$$\widetilde{g}(N_i, \overset{\circ}{A}_{W_\alpha}X) = S(X, W_\alpha)\pi(N_i) - \sigma_{i\alpha}(X),\ 1 \leq i \leq r, \tag{3.18}$$

$$\begin{aligned} \widetilde{g}(\overset{\circ}{A}_{W_\alpha}X, Y) &= \varepsilon_\alpha\lambda_\alpha S(X, Y) + \varepsilon_\alpha h^S_\alpha(X, Y) + \sum_{j=1}^{r}\gamma_{\alpha j}(X)\eta_\iota(Y) + S(X, W_\alpha)\pi(Y) \\ &\quad + \lambda_i S(X, W_\alpha)\eta_i(Y) + \{h^L_i(X, W_\alpha) + \lambda_i S(X, W_\alpha)\}\eta_i(Y) \end{aligned} \tag{3.19}$$

$$\begin{aligned} g(Y, \overset{\circ}{A}_{N_i}X) &= \sum_{j=1}^{r}\rho_{ij}(X)\eta_j(Y) + \eta_i(\nabla_X Y) - X(\eta_i(Y)) \\ &\quad + \{h^L_i(X, N_i) + \lambda_i S(X, N_i)\}\eta_i(Y) - \eta_i(Q)S(X, Y) \\ &\quad + \lambda_i\eta_i(Y)S(X, N_i) - S(X, N_i)\pi(Y) + \varepsilon_i h^L_i(X, Y) \end{aligned} \tag{3.20}$$

$$h^\ell_i(X, Y) + \widetilde{g}(\nabla_X\xi_i, Y) = \lambda_i g(X, Y) \tag{3.21}$$

for any $X, Y \in \Gamma(TM)$, $\xi_i \in \Gamma(RadTM)$, $W_\alpha \in \Gamma(S(TM^\perp))$, $N_i \in \Gamma(ltr(TM))$.

Presently, from the decomposition (2.1), geometrical objects of screen distribution with respect to $\overset{\circ}{\nabla}$ is given by

$$\overset{\circ}{\nabla}_X PY = \overset{\circ}{\nabla}{}^*_X PY + \sum_{i=1}^{r}\overset{\circ}{h}{}^*_i(X, PY)\xi_i, \tag{3.22}$$

$$\overset{\circ}{\nabla}_X\xi_i = -\overset{\circ}{A}{}^*_{\xi_i}X + \overset{\circ}{\nabla}{}^{*t}_X\xi_i,\ 1 \leq i \leq r \tag{3.23}$$

for any $X, Y \in \Gamma(TM)$ and $\xi_i \in \Gamma(RadTM)$, $1 \leq i \leq r$, where $\{\overset{\circ}{\nabla}{}^*_X PY, \overset{\circ}{A}{}^*_{\xi_i}X\}$ and $\{\sum_{i=1}^{r}\overset{\circ}{h}{}^*_i(X, PY)\xi_i, \overset{\circ}{\nabla}{}^{*t}_X\xi_i\}$ belong to $\Gamma(S(TM))$ and $\Gamma(RadTM)$, respectively. From the above it follows that $\overset{\circ}{\nabla}{}^*$ and $\overset{\circ}{\nabla}{}^{*t}$ are metric linear connections on complementary distributions $S(TM)$ and $Rad(TM)$ respectively, $\overset{\circ}{A}{}^*_{\xi_i}$ are shape operator of $S(TM)$ with respect to $\xi_i$, $\overset{\circ}{h}{}^*_i$ are bilinear forms on $\Gamma(TM) \times \Gamma(S(TM))$. In addition, by using (3.3) and (3.22) we obtain

$$\overset{\circ}{h}{}^\ell_i(X, PY) = \widetilde{g}(\overset{\circ}{A}{}^*_{\xi_i}X, PY),\ 1 \leq i \leq r \tag{3.24}$$

for any $X, Y \in \Gamma(TM)$ [2]. We define

$$u_{ij}(X) = g(\overset{\circ}{\nabla}{}^{*t}_X\xi_i, N_j),\ 1 \leq i, j \leq r$$

for any $X \in \Gamma(TM)$ and $\xi_i \in \Gamma(RadTM)$, $1 \leq i \leq r$. Thus, it follows that

$$\overset{\circ}{\nabla}_X\xi_i = -\overset{\circ}{A}{}^*_{\xi_i}X + \sum_{j=1}^{r}u_{ij}(X)\xi_j,\ 1 \leq i \leq r. \tag{3.25}$$

Analogous to the equation (3.22) we have

$$\nabla_X PY = \nabla^*_X PY + \sum_{i=1}^{r}h^*_i(X, PY)\xi_i, \tag{3.26}$$

where $h_i^*$ is the second fundamental form of distribution $S(TM)$.

From (3.7), we get

$$\nabla_X PY = \overset{\circ}{\nabla}_X PY + \pi(PY)LX - S(X, PY)Q. \tag{3.27}$$

Thus, applying (3.22) and (3.26) in (3.27), we deduce that

$$\nabla_X^* PY + \sum_{i=1}^{r} h_i^*(X, PY)\xi_i = \overset{\circ}{\nabla}{}_X^* PY + \sum_{i=1}^{r} \overset{\circ}{h}{}_i^*(X, PY)\xi_i + \pi(PY)LX - S(X, PY)Q$$

from the above it follows that

$$\nabla_X^* PY = \overset{\circ}{\nabla}{}_X^* PY + \pi(PY)L(PX) - S(X, PY)PQ \tag{3.28}$$

and

$$h_i^*(X, PY) = \overset{\circ}{h}{}_i^*(X, PY) + \eta_i(X)\pi(PY) - S(X, PY)\eta_i(Q), \ 1 \leq i \leq r \tag{3.29}$$

for any $X, Y \in \Gamma(TM)$. Also, taking (3.25) in (3.7) we have

$$\begin{aligned} \nabla_X \xi_i &= -A_{\xi_i}^* X + -S(X, \xi_i)PQ + \sum_{j=1}^{r} \{u_{ij}(X) - S(X, \xi_i)\eta_i(Q) \\ &\quad +\varepsilon_i \lambda_i \eta_i(X)\}\xi_i, \end{aligned} \tag{3.30}$$

where $A_{\xi_i}^* = \overset{\circ}{A}{}_{\xi_i}^* - \lambda_i I$, $1 \leq i \leq r$. We conclude from (3.28) that

$$(\nabla_X^* g)(PY, PZ) = 0 \tag{3.31}$$

and

$$T^*(PX, PY) = \pi(PY)L(PX) - \pi(PX)L(PY). \tag{3.32}$$

From (3.31) and (3.32), the following proposition can be stated as :

**Proposition 3.2.** *The induced connection $\nabla^*$ on a screen distribution of lightlike submanifold is a Ricci quarter-symmetric metric connection.*

**Proposition 3.3.** *Let $(M, g, S(TM), S(TM^{\perp}))$ be a lightlike submanifold of semi-Riemannian manifold $(\widetilde{M}, \widetilde{g})$ admitting a Ricci quarter-symmetric metric connection. The screen distribution $S(TM)$ is integrable if and only if second fundemental form of screen disribution $h_i^*$ and Ricci tensor of $\widetilde{M}$ are symmetric.*

*Proof.* Since the torsion tensor $T$ of $\nabla$ does not vanish, and also by using (3.11), (3.28), (3.30) and equality given by

$$X = PX + \sum_{i=1}^{r} \eta_i(X)\xi_i$$

we can get

$$\begin{aligned}
[X,Y] &= \nabla^*_X PY - \nabla^*_Y PX + \sum_{i=1}^{r}\eta_i(X)A^*_{\xi_i}Y - \eta_i(Y)A^*_{\xi_i}X \\
&\quad + \sum_{i=1}^{r}\{h^*_i(X,PY) - h^*_i(Y,PX) + X(\eta_i(Y)) - Y(\eta_i(X))\}\xi_i \\
&\quad + \sum_{i,j=1}^{r}\{\eta_i(Y)u_{ij}(X) - \eta_i(X)u_{ij}(Y)\}\xi_j \\
&\quad + \{\pi(PY) + \sum_{i=1}^{r}\eta_i(X)\lambda_i\}PY - \{\pi(PX) + \sum_{i=1}^{r}\eta_i(Y)\lambda_i\}PX \\
&\quad + \sum_{i=1}^{r}\{\pi(PX)\eta_i(Y) - \pi(PY)\eta_i(X)\}\xi_i.
\end{aligned} \tag{3.33}$$

Taking the scalar product of the above equation with $N_i$, $1 \leq i \leq r$, we have

$$\begin{aligned}
\widetilde{g}([X,Y],N_i) &= h^*_i(X,PY) - h^*_i(Y,PX) + X(\eta_i(Y)) - Y(\eta_i(X)) \\
&\quad + \sum_{j=1}^{r}\eta_i(Y)u_{ij}(X) - \eta_i(X)u_{ij}(Y) \\
&\quad + \pi(PX)\eta_i(Y) - \pi(PY)\eta_i(X).
\end{aligned} \tag{3.34}$$

From (3.10) , (3.28) and (3.34), we obtain

$$\begin{aligned}
2d\eta_i(X,Y) &= \overset{\circ}{h}{}^*_i(Y,PX) - \overset{\circ}{h}{}^*_i(X,PY) \\
&\quad + \sum_{j=1}^{r}\{\eta_i(X)\{u_{ij}(Y) + \pi(PY)\} \\
&\quad - \eta_i(Y)\{u_{ij}(X) + \pi(PX)\}\} + \{S(Y,PX) - S(X,PY)\}\eta_i(Q),
\end{aligned} \tag{3.35}$$

or

$$2d\eta_i(PX,PY) = h^*_i(PY,PX) - h^*_i(PX,PY) + \{S(PY,PX) - S(PX,PY)\}\eta_i(Q),$$

which proves the theorem. □

## 4. The Gauss and Codazzi Equations

We denote the curvature tensor of $\widetilde{M}$ with respect to Ricci quarter-symmetric metric connection $\widetilde{\widetilde{\nabla}}$ by

$$\widetilde{R}(\widetilde{X},\widetilde{Y})\widetilde{Z} = \widetilde{\widetilde{\nabla}}_{\widetilde{X}}\widetilde{\widetilde{\nabla}}_{\widetilde{Y}}\widetilde{Z} - \widetilde{\widetilde{\nabla}}_{\widetilde{Y}}\widetilde{\widetilde{\nabla}}_{\widetilde{X}}\widetilde{Z} - \widetilde{\widetilde{\nabla}}_{[\widetilde{X},\widetilde{Y}]}\widetilde{Z}$$

and that of $M$ with respect to induced connection $\nabla$ by

$$R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z,$$

where $\widetilde{X},\ \widetilde{Y},\ \widetilde{Z} \in \Gamma(T\widetilde{M})$ and $X,\ Y,\ Z \in \Gamma(TM)$. Then by using (3.4), (3.16), (3.17), we get

$$
\begin{aligned}
\widetilde{R}(X,Y)Z &= R(X,Y)Z + \sum_{i=1}^{r}\{(\nabla_X h_i^\ell)(Y,Z) - (\nabla_Y h_i^\ell)(X,Z) \\
&\quad + h_i^\ell(\pi(Y)X - \pi(X)Y, Z) - \lambda_i h_i^\ell(Y,Z)S(X,N_i) + \lambda_i h_i^\ell(X,Z)S(Y,N_i) \\
&\quad - \lambda_i h_\alpha^S(Y,Z)S(X,W_\alpha)\}N_i + + \sum_{\alpha=r+1}^{n}\{(\nabla_X h_\alpha^s)(Y,Z) - (\nabla_Y h_\alpha^s)(X,Z) \\
&\quad + h_\alpha^s(\pi(Y)X - \pi(X)Y, Z) + h_i^\ell(Y,Z)(\sigma_{i\alpha}(X) - \lambda_\alpha S(X,N_i)) \\
&\quad - h_i^\ell(X,Z)(\sigma_{i\alpha}(Y) - \lambda_\alpha S(Y,N_i)) - h_\alpha^s(Y,Z)S(X,W_\alpha) \\
&\quad + h_\alpha^s(X,Z)S(Y,W_\alpha)\}W_\alpha + \sum_{i=1}^{r} h_i^\ell(X,Z)\{-\overset{\circ}{A}_{N_i}X + \eta(Q)LX \\
&\quad - S(X,N_i)Q + p_{ij}(X)N_j\} - \sum_{i=1}^{r} h_i^\ell(Y,Z)\{-\overset{\circ}{A}_{N_i}Y + \eta(Q)LY \\
&\quad - S(Y,N_i)Q + p_{ij}(Y)N_j\} + \sum_{\alpha=r+1}^{n} h_\alpha^s(Y,Z)\{-A_{W_\alpha}X + \lambda_\alpha\varepsilon_\alpha LX \\
&\quad - S(X,W_\alpha)Q + \gamma_{\alpha j}(X)N_j + \mu_{\alpha\beta}(X)W_\beta\} - \sum_{\alpha=r+1}^{n} h_\alpha^s(X,Z)\{-A_{W_\alpha}Y \\
&\quad + \lambda_\alpha\varepsilon_\alpha LY - S(Y,W_\alpha)Q + \gamma_{\alpha j}(Y)N_j + \mu_{\alpha\beta}(Y)W_\beta\}
\end{aligned} \tag{4.1}
$$

for any $X, Y, Z \in \Gamma(TM)$, $N_i \in \Gamma(ltr(TM))$ and $W_\alpha \in \Gamma(S(TM^\perp))$. From (3.26), (3.30) and (4.1), we have the Gauss and Codazzi equations of the lightlike submanifold with a Ricci quarter symmetric metric connection:

$$
\begin{aligned}
\widetilde{g}(\widetilde{R}(X,Y)PZ, PU) &= g(R(X,Y)PZ, PU) \\
&\quad - \sum_{i=1}^{r} h_i^\ell(Y,PZ)\{\overset{*}{h}_i(X,PU) - \pi(PU)\eta_i(X) + S(X,PU)\eta(Q)\} \\
&\quad + h_i^\ell(X,PZ)\{\overset{*}{h}_i(Y,PU) - \pi(PU)\eta_i(Y) + S(Y,PU)\eta(Q)\} \\
&\quad + h_i^\ell(Y,PZ)\eta(Q)S(X,PU) - S(X,N_i)g(Q,PU) \\
&\quad + S(Y,N_i)g(Q,PU) - h_i^\ell(X,PZ)\eta(Q)S(Y,PU) \\
&\quad + \sum_{\alpha=r+1}^{n} h_\alpha^s(Y,PZ)g(\overset{\circ}{A}_{W_\alpha}X, PU) + \lambda_\alpha\varepsilon_\alpha h_\alpha^s(Y,PZ)S(X,PU) \\
&\quad - S(X,W_\alpha)h_\alpha^s(Y,PZ)\pi(PU) + h_\alpha^s(X,PZ)g(\overset{\circ}{A}_{W_\alpha}Y, PU) \\
&\quad + \lambda_\alpha\varepsilon_\alpha h_\alpha^s(X,PZ)S(Y,PU) - S(Y,W_\alpha)h_\alpha^s(X,PZ)\pi(PU)
\end{aligned} \tag{4.2}
$$

$$
\begin{aligned}
\widetilde{g}(\widetilde{R}(X,Y)\xi_i, N_i) &= g(R(X,Y)\xi_i, N_i) \\
&+ \sum_{\alpha=r+1}^{n} \{-h^s_\alpha(Y,\xi_i)\sigma_{i\alpha}(X) + \lambda_\alpha \varepsilon_\alpha h^s_\alpha(Y,\xi_i) S(X,N_i) \\
&+ h^s_\alpha(Y,\xi_i) S(X,W_\alpha)\pi(N_i) + h^s_\alpha(X,\xi_i)\sigma_{i\alpha}(X) \\
&+ h^s_\alpha(X,\xi_i)\sigma_{i\alpha}(Y) - \lambda_\alpha\varepsilon_\alpha h^s_\alpha(X,\xi_i) S(Y,N_i) \\
&+ h^s_\alpha(X,\xi_i) S(Y,W_\alpha)\pi(N_i)
\end{aligned} \tag{4.3}
$$

$$
\begin{aligned}
\widetilde{g}(R(X,Y)\xi_i, N_i) &= h^*_i(Y, A^*_{\xi_i}X) - h^*_i(X, A^*_{\xi_i}Y) + 2du_{ii}(X,Y) \\
&+ \varepsilon_i\lambda_i g((\nabla_X L)(PY) - L(\nabla_X PY), N_i) \\
&+ \eta_i(Q)\{(\nabla_X S)(Y,\xi_i) - S(\nabla_X Y,\xi_i) + S(Y,\nabla_X\xi_i)\} \\
&- X(\eta_i(Q))S(Y,\xi_i) + \varepsilon_i\lambda_i 2d\eta_{ii}(X,Y) \\
&+ \{U_{ij}(Y) - S(Y,\xi_i)\eta_i(Q) + \varepsilon_i\lambda_i\eta_i(Y)\}\eta_i(\nabla_X\xi_i) \\
&- \varepsilon_i\lambda_i g((\nabla_Y L)(PX) - L(\nabla_Y PX), N_i) \\
&+ \eta_i(Q)\{(\nabla_Y S)(X,\xi_i) - S(\nabla_Y X,\xi_i) + S(X,\nabla_Y\xi_i)\} \\
&+ Y(\eta_i(Q))S(X,\xi_i) - \{U_{ij}(X) - S(X,\xi_i)\eta_i(Q) \\
&+ \varepsilon_i\lambda_i\eta_i(X)\}\eta_i(\nabla_Y\xi_i).
\end{aligned} \tag{4.4}
$$

## 5. The Ricci Tensor

Let $(M, g, S(TM), S(TM^\perp))$ be an $m-$dimensional lightlike submanifold of an $(m+n)-$dimensional semi-Riemannian manifold $(\widetilde{M}, \widetilde{g})$ admitting a Ricci quarter-symmetric metric connection. By using (3.7), we have

$$
\begin{aligned}
R(X,Y)Z &= \overset{\circ}{R}(X,Y)Z - M(Y,Z)LX + M(X,Z)LY \\
&- S(Y,Z)QX + S(X,Z)QY + \pi(Z)[(\nabla_X L)(Y) - (\nabla_Y L)(X)] \\
&- [(\nabla_X S)(Y) - (\nabla_Y S)(X)]Q
\end{aligned} \tag{5.1}
$$

,where $M$ is tensor of type $(0,2)$ defined by

$$M(X,Y) = g(QX,Y) = (\nabla_X\pi)(Y) - \pi(Y)\pi(LX) + \frac{1}{2}\pi(Q)S(X,Y)$$

and $Q$ is a tensor field of type (2,1) defined by

$$QX = \nabla_X Q - \pi(LX)Q + \frac{1}{2}\pi(Q)LX.$$

Similar to the definition of the Ricci tensor of $M$ with respect to the symmetric connection, the Ricci tensor of $M$ with respect to Ricci quarter-symmetric metric connection is defined by

$$Ric(X,Y) = trace\{Z \to R(X,Z)Y\} \tag{5.2}$$

for any $X, Y, Z \in \Gamma(TM)$. Then the Ricci tensor of an $m-$dimensional lightlike submanifold $M$ with respect to Ricci quarter-symmetric metric connection is given by

$$Ric(X,Y) = \sum_{i=1}^{r} \widetilde{g}(R(X,\xi_i)Y, N_i) + \sum_{k=r+1}^{m} \varepsilon_k g(R(X,X_k)Y, X_k) \tag{5.3}$$

where $\{X_{r+1}, ..., X_m\}$ is an orthonormal basis of screen distribution $\Gamma(S(TM))$.

Thus, by using (3.7), (5.1) and (5.3) we obtain
(5.4)

$$Ric(X,Y) - Ric(Y,X) = \overset{\circ}{Ric}(X,Y) - \overset{\circ}{Ric}(Y,X) + \frac{r(m-2)}{m}[M(Y,X) - M(X,Y)]$$

for any $X, Y \in \Gamma(TM)$,where $r$ is scalar curvature.

From (5.4) we have

**Proposition 5.1.** *Let $(M, g, S(TM), S(TM^{\perp}))$ be a lightlike submanifold of semi-Riemannian manifold $(\widetilde{M}, \widetilde{g})$ admitting a Ricci quarter-symmetric metric connection. Then Ricci tensor of a lightlike submanifold with respect to the Ricci quarter-symmetric metric connection is symmetric if and only if the Ricci tensor of a lightlike submanifold with respect to the symmetric connection and the tensor of $M$ are symmetric.*

We assume that the $1-$form $\pi$ is closed. In this case we can define the sectional curvature for a section in $\widetilde{M}$ with respect to the Ricci quarter-symmetric metric connection (see [5]).

Now, suppose that the Ricci quarter-symmetric metric connection $\widetilde{\nabla}$ is of constant sectional curvature, then $\widetilde{R}(\widetilde{X}, \widetilde{Y})\widetilde{Z}$ should be in the form of

$$\widetilde{R}(\widetilde{X}, \widetilde{Y})\widetilde{Z} = c\{\widetilde{g}(\widetilde{X}, \widetilde{Z})\widetilde{Y} - \widetilde{g}(\widetilde{Y}, \widetilde{Z})\widetilde{X}\} \tag{5.5}$$

for any $\widetilde{X}, \widetilde{Y}, \widetilde{Z} \in \Gamma(T\widetilde{M})$, where $c$ is a certain scalar. Thus, $\widetilde{M}$ is called a semi-Riemannian space form with respect to the Ricci quarter-symmetric metric connection and is denoted by $\widetilde{M}(c)$.

**Proposition 5.2.** *Let $(M, g, S(TM), S(TM^{\perp}))$ be a lightlike submanifold of $(m+n)-$dimensional semi-Riemannian space form $\widetilde{M}(c)$ with a Ricci quarter-symmetric metric connection. Then we have*

$$\begin{aligned} Ric(X,Y) &= (m-1)kg(X,Y) - \frac{r}{m}[\widetilde{g}(X,Y)\widetilde{M}(Y,N_i) + \varepsilon_\alpha \widetilde{M}(X,Y) \\ &\quad -\widetilde{g}(X_a,Y)\widetilde{M}(X,X_a) + m\widetilde{g}(X,Y)] \end{aligned} \tag{5.6}$$

*for any $X, Y, Z \in \Gamma(TM)$, where $m$ is the trace of the tensor $\widetilde{M}(X,Y)$.*

*Proof.* Appliying (5.1) in (5.3) and considering (5.5), then (5.6) is obtained. □

From (5.6), we have the following corollary :

**Corollary 5.1.** *Let $(M, g, S(TM), S(TM^{\perp}))$ be a lightlike submanifold of semi-Riemannian space form $\widetilde{M}(c)$ with a Ricci quarter-symmetric metric connection. If $\widetilde{M}$ vanishes, then $M$ is an Einstein manifold.*

Denoting the conformal curvature tensors of type $(0,4)$ of the semi-symmetric metric connections $\nabla$ by $C$ , we have

$$\begin{aligned} C(X,Y,Z,U) &= R(X,Y,Z,U) - \frac{1}{m-2}\{Ric(Y,Z)g(X,U) - Ric(X,Z)g(Y,Z) \\ &\quad +g(Y,Z)Ric(X,U) - g(X,Z)Ric(Y,U)\}. \end{aligned} \tag{5.7}$$

**Proposition 5.3.** *Let $(M, g, S(TM), S(TM^{\perp}))$ be a lightlike submanifold of semi-Riemannian space form $\widetilde{M}(c)$ with a Ricci quarter-symmetric metric connection. If $M$ is Einstein and $r = 2k(m-1)$, then conformal curvature tensors of the Ricci Quarter-symmetric metric connections $\nabla$ is equal to curvature tensor $R$ of Lightlike submanifold with respect to the Ricci quarter-symmetric connection.*

*Proof.* By using the definition of Einstein manifold and (5.3) in (5.7), we obtain

$$C(X,Y,Z,U)=R(X,Y,Z,U)+g(Y,Z)g(X,U)\{\frac{r-2k(m-1)}{(m-1)(m-2)}\},$$

which proves assertion of the theorem. □

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