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ON ϕ-SYMMETRIC KENMOTSU MANIFOLDS WITH RESPECT TO QUARTER-SYMMETRIC METRIC CONNECTION

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Abstract. The object of the paper is to study $\phi-$symmetric Kenmotsu manifolds with respect to quarter-symmetric metric connection. We characterize locally $\phi-$symmetric, $\phi-$symmetric and locally concircular $\phi-$symmetric Kenmotsu manifolds with respect to quarter-symmetric metric connection and obtain intersting results.

1. Introduction

In 1924, A. Friedman and J.A. Schouten ([11, 19]) introduced the notion of a semi-symmetric linear connection on a differentiable manifold. H.A. Hayden [13] defined a metric connection with torsion on a Riemannian manifold. In 1970, K. Yano [26] studied some curvature and derivational conditions for semi-symmetric connections in Riemannian manifolds. In 1975, S. Golab [12] initiated the study of quarter-symmetric linear connection on a differentiable manifold. A linear connection $\widetilde{\nabla}$ in an n-dimensional differentiable manifold is said to be a quarter-symmetric connection if its torsion tensor T is of the form

$$T(X,Y) = \widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X - [X,Y] \tag{1.1}$$

$$= \eta(Y)\phi X - \eta(X)\phi Y, \tag{1.2}$$

where η is a 1-form and ϕ is a tensor of type $(1,1)$. In addition, a quarter-symmetric linear connection $\widetilde{\nabla}$ satisfies the condition

$$(\widetilde{\nabla}_X g)(Y,Z) = 0$$

for all $X, Y, Z \in \chi(M)$, where $\chi(M)$ is the Lie algebra of vector fields of the manifold M, then $\widetilde{\nabla}$ is said to be a quarter-symmetric metric connection. If we replace ϕX by X and ϕY by Y in (1.2) then the connection is called a semi-symmetric metric connection [26]. In [22] M.M. Tripathi, [4] C.S. Bagewadi, D.G. Prakasha and Venkatesha, [8] U.C. De and G. Pathak studied semi-symmetric metric connection

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in a Kenmotsu manifold. In [23, 24], M.M. Tripathi studied semi-symmetric non-metric connection in a Kenmotsu manifold. In 1980, R. S. Mishra and S. N. Pandey [15] studied quarter-symmetric metric connection and in particular, Ricci quarter-symmetric symmetric metric connection on Riemannian, Sasakian and Kaehlerian manifolds. Note that a quarter-symmetric metric connection is a Hayden connection with the torsion tensor of the form $(1, 2)$. Studies of various types of quarter-symmetric metric connection and their properties include ([10, 3, 15, 17, 18]) and [27] among others.

The notion of locally symmetry of Riemannian manifolds have been weakened by many authors in several ways to a different extent. As a weaker version of local symmetry, T. Takahashi [21] introduced the notion of locally $\phi-$symmetry on Sasakian manifolds. In the context of contact geometry the notion of $\phi-$symmetry is introduced and studied by E. Boeckx, P. Buecken and L. Vanhecke [7] with several examples. The notion of ϕ-symmetry on Sasakian manifolds with respect to quarter-symmetric metric connection was studied in [16].

On the other hand K. Kenmotsu [14] defined a type of contact metric manifold which is now a days called Kenmotsu manifold. It may be mentioned that a Kenmotsu manifold is not a Sasakian manifold.

In the present paper, we study quarter-symmetric metric connection in a Kenmotsu manifold. The paper is organized as follows: In section 2, we give a brief account of Kenmotsu manifolds. In section 3 we give the relation between the Levi-Civita connection and the quarter-symmetric metric connection on a Kenmotsu manifold. In the next section, we characterize locally $\phi-$symmetric Kenmotsu manifold with respect to quarter-symmetric metric connection. In section 5, we study $\phi-$symmetric Kenmotsu manifolds with respect to quarter-symmetric metric connection. In section 6, we characterize locally concircular $\phi-$symmetric Kenmotsu manifolds with respect to quarter-symmetric metric connection.

2. Kenmotsu manifolds

An $n(= 2m+1)$-dimensional differentiable manifold M is called an almost contact Riemannian manifold if either its structural group can be reduced to $U(n) \times 1$ or equivalently, there is an almost contact structure (ϕ, ξ, η) consisting of a $(1, 1)$ tensor field ϕ, a vector field ξ and a $1-$form η satisfying

$$\phi^2 = -I + \eta \otimes \xi \tag{2.1}$$

$$\eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta \circ \phi = 0. \tag{2.2}$$

Let g be a compatible Riemannian metric with (ϕ, ξ, η), that is,

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$$

or equivalently,

$$g(X, \phi Y) = -g(\phi X, Y) \quad and \quad g(X, \xi) = \eta(X)$$

for any vector fields X, Y on M [6]. If, moreover

$$(\nabla_X \phi)Y = -\eta(Y)\phi X - g(X, \phi Y)\xi, \tag{2.3}$$

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.4}$$

for any $X, Y \in \chi(M)$,then (M, ϕ, ξ, η, g) is called an almost Kenmotsu manifold. Here ∇ denotes the Riemannian connection of g.

An almost Kenmotsu manifold become a Kenmotsu manifold if

$$g(X, \phi Y) = d\eta(X, Y) \quad \text{for all vector fields X, Y.} \tag{2.5}$$

In a Kenmotsu manifold M the following relations hold [14]:

$$(\nabla_X \eta)Y = g(X, Y) - \eta(X)\eta(Y), \tag{2.6}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.7}$$

$$S(X, \xi) = -(n-1)\eta(X), \tag{2.8}$$

for every vector fields X, Y, Z on M where R and S are the Riemannian curvature tensor and the Ricci tensor with respect to Levi-Civita connection, respectively.

Definition 2.1. A Kenmotsu manifold M is said to be locally ϕ-symmetric if

$$\phi^2((\nabla_W R)(X, Y)Z) = 0, \tag{2.9}$$

for all vector fields X, Y, Z, W orthogonal to ξ. This notion was introduced by T. Takahashi for Sasakian manifolds.

Definition 2.2. A Kenmotsu manifold M is said to be ϕ-symmetric if

$$\phi^2((\nabla_W R)(X, Y)Z) = 0, \tag{2.10}$$

for arbitrary vector fields X, Y, Z, W.

Definition 2.3. A Kenmotsu manifold M is said to be locally concircular ϕ-symmetric if

$$\phi^2((\nabla_W \widetilde{C})(X, Y)Z) = 0, \tag{2.11}$$

for all vector fields X, Y, Z, W orthogonal to ξ, where $\widetilde{C}$ is the concircular curvature tensor given by [25]

$$\widetilde{C}(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y]. \tag{2.12}$$

Here R and r are the Riemannian curvature tensor and scalar curvature tensor, respectively.

3. Relation between the Levi-Civita connection and the quarter-symmetric metric connection in a Kenmotsu manifolds

Let $\widetilde{\nabla}$ be a linear connection and ∇ be a Riemannian connection of an almost contact metric manifold M such that

$$\widetilde{\nabla}_X Y = \nabla_X Y + H(X, Y), \tag{3.1}$$

where H is a tensor of type $(1,1)$. For $\widetilde{\nabla}$ to be a quarter-symmetric metric connection in M, we have [12]

$$H(X, Y) = \frac{1}{2}[T(X, Y) + T'(X, Y) + T'(Y, X)] \tag{3.2}$$

and

$$g(T'(X, Y), Z) = g(T(Z, X), Y). \tag{3.3}$$

From (1.1) and (3.3) we get

$$T'(X, Y) = g(\phi Y, X)\xi - \eta(X)\phi Y. \tag{3.4}$$

Using (1.1) and (3.4) in (3.2) we obtain

$$H(X,Y) = -\eta(X)\phi Y.$$

Hence a quarter-symmetric metric connection $\widetilde{\nabla}$ in a Kenmotsu manifold is given by

$$\widetilde{\nabla}_X Y = \nabla_X Y - \eta(X)\phi Y. \tag{3.5}$$

Therefore equation (3.5) is the relation between the Levi-Civita connection and the quarter-symmetric metric connection on a Kenmotsu manifold.

A relation between the curvature tensor of M with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$ and the Levi-Civita connection ∇ is given by [20]

$$\begin{aligned} \widetilde{R}(X,Y)Z &= R(X,Y)Z - 2d\eta(X,Y)\phi Z + [\eta(X)g(\phi Y,Z) - \eta(Y)g(\phi X,Z)]\xi \\ &\quad +[\eta(Y)\phi X - \eta(X)\phi Y]\eta(Z). \end{aligned} \tag{3.6}$$

where $\widetilde{R}$ and R are the Riemannian curvatures of the connection $\widetilde{\nabla}$ and ∇, respectively. From (3.6), it follows that

$$\widetilde{S}(Y,Z) = S(Y,Z) - 2d\eta(\phi Z,Y) + g(\phi Y,Z) + \psi\eta(Y)\eta(Z), \tag{3.7}$$

where $\widetilde{S}$ and S are the Ricci tensors of the connection $\widetilde{\nabla}$ and ∇, respectively and $\psi = \sum_{i=1}^{n} g(\phi e_i, e_i) = Trace of \phi$. From (3.7) it is clear that in a Kenmotsu manifold the Ricci tensor with respect to the quarter-symmetric metric connection is not symmetric. Contracting (3.7), we get

$$\widetilde{r} = r + 2(n-1), \tag{3.8}$$

where $\widetilde{r}$ and r are the scalar curvatures of the connection $\widetilde{\nabla}$ and ∇, respectively.

4. Locally $\phi-$symmetric Kenmotsu manifolds with respect to the quarter-symmetric metric connection

Analogous to the definition of locally $\phi-$symmetric Kenmotsu manifolds with respect to Levi-Civita connection, we define a locally $\phi-$symmetric Kenmotsu manifold with respect to the quarter-symmetric metric connection by

$$\phi^2((\widetilde{\nabla}_W \widetilde{R})(X,Y)Z) = 0 \tag{4.1}$$

for all vector fields X, Y, Z, W orthogonal to ξ.
Using (3.5) we can write

$$(\widetilde{\nabla}_W \widetilde{R})(X,Y)Z = (\nabla_W \widetilde{R})(X,Y)Z - \eta(W)\phi\widetilde{R}(X,Y)Z. \tag{4.2}$$

Now differentiating (3.6) with respect to W, we obtain

$$\begin{aligned} (\nabla_W \widetilde{R})(X,Y)Z &= (\nabla_W R)(X,Y)Z - 2d\eta(X,Y)(\nabla_W \phi)Z + \{(\nabla_W \eta)(X)g(\phi Y,Z) \\ &\quad -(\nabla_W \eta)(Y)g(\phi X,Z)\}\xi + \{g(\phi Y,Z)\eta(X) - g(\phi X,Z)\eta(Y)\}(\nabla_W \xi) \\ &\quad +\{\eta(Y)\phi X - \eta(X)\phi Y\}(\nabla_W \eta)(Z) + \{(\nabla_W \eta)(Y)\phi X \\ &\quad +\eta(Y)(\nabla_W \phi)(X) - (\nabla_W \eta)(X)\phi Y - \eta(X)(\nabla_W \phi)(Y)\}\eta(Z). \end{aligned} \tag{4.3}$$

Using (2.3) and (2.6), in (4.3) we get

$$\begin{aligned}(\nabla_W\widetilde{R})(X,Y)Z &= (\nabla_W R)(X,Y)Z - 2\eta(X,Y)\{g(\phi W,Z)\xi - \eta(Z)\phi W\}\\ &+\{g(X,W)g(\phi Y,Z) - g(Y,W)g(\phi X,Z)\}\xi\\ &-2\{\eta(X)\eta(W)g(\phi Y,Z) - \eta(Y)\eta(W)g(\phi X,Z)\}\xi\\ &+\{g(\phi Y,Z)\eta(X) - g(\phi X,Z)\eta(Y)\}(W) + \{g(W,Z) - \eta(W)\eta(Z)\}\\ &\times\{\eta(Y)\phi X - \eta(X)\phi Y\} + \{g(Y,W)\phi X - g(X,W)\phi Y\\ &+g(\phi W,X)\eta(Y)\xi - g(\phi W,Y)\eta(X)\xi - \eta(Y)\eta(W)\phi X\\ &-\eta(X)\eta(W)\phi Y - 2\eta(X)\eta(Y)\phi W\}\eta(Z). \qquad (4.4)\end{aligned}$$

With the help of (2.2) and (4.4), in (4.2) we obtain

$$\begin{aligned}\phi^2(\widetilde{\nabla}_W\widetilde{R})(X,Y)Z &= \phi^2(\nabla_W R)(X,Y)Z - 2d\eta(X,Y)\eta(Z)\phi W + \{g(\phi Y,Z)\eta(X)\\ &-g(\phi X,Z)\eta(Y)\}(\phi^2 W) + \{g(W,Z) - \eta(W)\eta(Z)\}\{\eta(Y)\phi^2(\phi X)\\ &-\eta(X)\phi^2(\phi Y)\} + \{g(Y,W)\phi^2(\phi X) - g(X,W)\phi^2(\phi Y)\\ &-\eta(Y)\eta(W)\phi^2(\phi X) - \eta(X)\eta(W)\phi^2(\phi Y)\\ &-2\eta(X)\eta(Y)\phi^2(\phi W)\}\eta(Z) - \eta(W)\phi^2(\phi\widetilde{R})(X,Y)Z. \qquad (4.5)\end{aligned}$$

If we consider X, Y, Z, W orthogonal to ξ, (4.5) reduces to

$$\phi^2((\widetilde{\nabla}_W\widetilde{R})(X,Y)Z) = \phi^2((\nabla_W R)(X,Y)Z). \qquad (4.6)$$

Hence we can state the following:

Theorem 4.1. *For a Kenmotsu manifold the quarter-symmetric metric connection $\widetilde{\nabla}$ is locally $\phi-$symmetric if and only if the Levi-Civita connection ∇ is so.*

5. $\phi-$symmetric Kenmotsu manifolds with respect to the quarter-symmetric metric connection

A Kenmotsu manifold M is said to be ϕ-symmetric with respect to quarter-symmetric metric connection if

$$\phi^2((\widetilde{\nabla}_W\widetilde{R})(X,Y)Z) = 0 \qquad (5.1)$$

for arbitrary vector fields X, Y, Z, W.

Let us consider a $\phi-$symmetric Kenmotsu manifolds with respect to quarter-symmetric metric connection. Then by virtue of (2.1) and (5.1) we have

$$-(\widetilde{\nabla}_W\widetilde{R})(X,Y)Z + \eta((\widetilde{\nabla}_W\widetilde{R})(X,Y)Z)\xi = 0, \qquad (5.2)$$

from which it follows

$$-g((\widetilde{\nabla}_W\widetilde{R})(X,Y)Z,U) + \eta((\widetilde{\nabla}_W\widetilde{R})(X,Y)Z)g(\xi,U) = 0. \qquad (5.3)$$

Let $\{e_i\}$ $i = 1, 2, ..., n$, be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X = U = e_i$ in (5.3) and taking summation over i, $1 \le i \le n$, we get

$$-g((\widetilde{\nabla}_W\widetilde{S})(Y,Z) + \sum_{i=1}^{n}\eta((\widetilde{\nabla}_W\widetilde{R})(e_i,Y)Z)\eta(e_i) = 0. \qquad (5.4)$$

The second term of (5.4) by putting $Z = \xi$ takes the form

$$\eta((\widetilde{\nabla}_W\widetilde{R})(e_i,Y)\xi)\eta(e_i) = g((\widetilde{\nabla}_W\widetilde{R})(e_i,Y)\xi,\xi)g(e_i,\xi), \qquad (5.5)$$

which is denoted by E. In this case E vanishes. Since by using (3.5), we can write

$$g((\widetilde{\nabla}_W \widetilde{R})(e_i, Y)\xi, \xi) = g((\nabla_W \widetilde{R})(e_i, Y)\xi, \xi) - \eta(W).\eta(\phi\widetilde{R}(e_i, Y)\xi) \tag{5.6}$$

By (2.2) and (4.4), we obtain from (5.6)

$$g((\widetilde{\nabla}_W \widetilde{R})(e_i, Y)\xi, \xi) = g((\nabla_W R)(e_i, Y)\xi, \xi). \tag{5.7}$$

Also, in a Kenmotsu manifold M we have [9] $g((\nabla_W R)(e_i, Y)\xi, \xi) = 0$ and thus from (5.7) we have

$$g((\widetilde{\nabla}_W \widetilde{R})(e_i, Y)\xi, \xi) = 0. \tag{5.8}$$

By replacing Z by ξ in (5.4) and using (5.8), we get

$$(\widetilde{\nabla}_W \widetilde{S})(Y, \xi) = 0. \tag{5.9}$$

We know that

$$(\widetilde{\nabla}_W \widetilde{S})(Y, \xi) = \widetilde{\nabla}_W \widetilde{S}(X, \xi) - \widetilde{S}(\widetilde{\nabla}_W Y, \xi) - \widetilde{S}(Y, \widetilde{\nabla}_W \xi) \tag{5.10}$$

By making use of (2.4), (2.8), (3.5) and (3.7)

$$\begin{aligned}(\widetilde{\nabla}_W \widetilde{S})(Y, \xi) \;\; = \;\; & -S(Y, W) + 2d\eta(\phi Y, W) - g(\phi Y, W) \\ & +\{\psi - (n-1)\}g(Y, W) - \psi\eta(Y)\eta(W).\end{aligned} \tag{5.11}$$

Applying (5.11) in (5.9), we obtain

(5.12)

$$-S(Y, W) + 2d\eta(\phi Y, W) - g(\phi Y, W) + \{\psi - (n-1)\}g(Y, W) - \psi\eta(Y)\eta(W) = 0.$$

Then contracting the last equation, one can get

$$r = -n(n-1). \tag{5.13}$$

This leads to the following:

Theorem 5.1. *Let M be a ϕ-symmetric Kenmotsu manifolds with respect to quarter-symmetric metric connection $\widetilde{\nabla}$. Then the manifold has a constant negative scalar curvature r with respect to Levi-Civita connection ∇ of M given by (5.13).*

6. Locally concircular $\phi-$symmetric Kenmotsu manifolds with respect to the quarter-symmetric metric connection

A Kenmotsu manifold M is said to be a locally concircular $\phi-$symmetric with respect to quarter-symmetric metric connection if

$$\phi^2((\widetilde{\nabla}_W \widetilde{\widetilde{C}})(X, Y)Z) = 0 \tag{6.1}$$

for all vector fields X, Y, Z, W orthogonal to ξ, where $\widetilde{\widetilde{C}}$ is the concircular curvature tensor with respect to quarter-symmetric metric connection given by

$$\widetilde{\widetilde{C}}(X, Y)Z = \widetilde{R}(X, Y)Z - \frac{\widetilde{r}}{n(n-1)}[g(Y, Z)X - g(X, Z)Y]. \tag{6.2}$$

where $\widetilde{R}$ and $\widetilde{r}$ are the Riemannian curvature tensor and scalar curvature with respect to quarter-symmetric metric connection $\widetilde{\nabla}$, respectively. Using (3.5) we can write

$$(\widetilde{\nabla}_W \widetilde{\widetilde{C}})(X, Y)Z = (\nabla_W \widetilde{\widetilde{C}})(X, Y)Z - \eta(W)\phi\widetilde{\widetilde{C}}(X, Y)Z. \tag{6.3}$$

Now differentiating (6.2) with respect to W, we obtain

$$(\nabla_W \tilde{\tilde{C}})(X,Y)Z = (\nabla_W \tilde{R})(X,Y)Z - \frac{(\nabla_W \tilde{r})}{n(n-1)}[g(Y,Z)X - g(X,Z)Y]. \tag{6.4}$$

By making use of (4.4) and (3.8) in (6.4), we have

$$\begin{aligned}(\nabla_W \tilde{\tilde{C}})(X,Y)Z &= (\nabla_W R)(X,Y)Z - 2d\eta(X,Y)\{g(\phi W,Z)\xi - \eta(Z)\phi W\}\\
&+\{g(X,W)h(\phi Y,Z) - g(Y,W)g(\phi X,Z)\}\xi\\
&-2\{\eta(X)\eta(W)g(\phi Y,Z) - \eta(Y)\eta(W)g(\phi X,Z)\}\xi\\
&+\{g(\phi Y,Z)\eta(X) - g(\phi X,Z)\eta(Y)\}(W) + \{g(W,Z) - \eta(W)\eta(Z)\}\\
&\times\{\eta(Y)\phi X - \eta(X)\phi Y\} + \{g(Y,W)\phi X - g(X,W)\phi Y\\
&+g(\phi W,X)\eta(Y)\xi - g(\phi W,Y)\eta(X)\xi - \eta(Y)\eta(W)\phi X\\
&-\eta(X)\eta(W)\phi Y - 2\eta(X)\eta(Y)\phi W\}\eta(Z)\\
&-\frac{(\nabla_W r)}{n(n-1)}[g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{6.5}$$

Taking account of (2.12), we write (6.5) as

$$\begin{aligned}(\nabla_W \tilde{\tilde{C}})(X,Y)Z &= (\nabla_W \tilde{C})(X,Y)Z - 2d\eta(X,Y)\{g(\phi W,Z)\xi - \eta(Z)\phi W\}\\
&+\{g(X,W)g(\phi Y,Z) - g(Y,W)g(\phi X,Z)\}\xi\\
&-2\{\eta(X)\eta(W)g(\phi Y,Z) - \eta(Y)\eta(W)g(\phi X,Z)\}\xi\\
&+\{g(\phi Y,Z)\eta(X) - g(\phi X,Z)\eta(Y)\}(W) + \{g(W,Z) - \eta(W)\eta(Z)\}\\
&\times\{\eta(Y)\phi X - \eta(X)\phi Y\} + \{g(Y,W)\phi X - g(X,W)\phi Y\\
&+g(\phi W,X)\eta(Y)\xi - g(\phi W,Y)\eta(X)\xi - \eta(Y)\eta(W)\phi X\\
&-\eta(X)\eta(W)\phi Y - 2\eta(X)\eta(Y)\phi W\}\eta(Z). \end{aligned} \tag{6.6}$$

Applying (2.2) and (6.6), in (6.3) we have

$$\begin{aligned}\phi^2(\tilde{\nabla}_W \tilde{\tilde{C}})(X,Y)Z &= \phi^2(\nabla_W \tilde{C})(X,Y)Z + 2d\eta(X,Y)\eta(Z)\phi^2(\phi W)\\
&+\{g(\phi Y,Z)\eta(X) - g(\phi X,Z)\eta(Y)\}\phi^2(W)\\
&+\{g(W,Z) - \eta(W)\eta(Z)\} \times \{\eta(Y)\phi^{(}\phi X) - \eta(X)\phi^2(\phi Y)\}\\
&+\{g(Y,W)\phi^2(\phi X) - g(X,W)\phi^2(\phi Y) - \eta(Y)\eta(W)\phi^2(\phi X)\\
&-\eta(X)\eta(W)\phi^2(\phi Y) - 2\eta(X)\eta(Y)\phi^2(\phi W)\}\eta(Z)\\
&-\eta(W)\phi^2(\phi\tilde{\tilde{C}})(X,Y)Z. \end{aligned} \tag{6.7}$$

If we consider X, Y, Z, W orthogonal to ξ, (6.7) reduces to

$$\phi^2(\tilde{\nabla}_W \tilde{\tilde{C}})(X,Y)Z = \phi^2(\nabla_W \tilde{C})(X,Y)Z.$$

Hence we have the following:

Theorem 6.1. *For a Kenmotsu manifold the quarter-symmetric metric connection $\tilde{\nabla}$ is locally concircular ϕ-symmetric if and only if the Levi-Civita connection ∇ is so.*

Next, from (2.2) and (6.5) in (6.3), we have

$$\begin{aligned}\phi^2(\widetilde{\nabla}_W\widetilde{\widetilde{C}})(X,Y)Z &= \phi^2(\nabla_W R)(X,Y)Z + 2d\eta(X,Y)\eta(Z)\phi^2(\phi W)\\ &+\{g(\phi Y,Z)\eta(X) - g(\phi X,Z)\eta(Y)\}\phi^2(W)\\ &+\{g(W,Z)-\eta(W)\eta(Z)\}\times\{\eta(Y)\phi^(\phi X)-\eta(X)\phi^2(\phi Y)\}\\ &+\{g(Y,W)\phi^2(\phi X)-g(X,W)\phi^2(\phi Y)-\eta(Y)\eta(W)\phi^2(\phi X)\\ &-\eta(X)\eta(W)\phi^2(\phi Y)-2\eta(X)\eta(Y)\phi^2(\phi W)\}\eta(Z)\\ &-\eta(W)\phi^2(\phi\widetilde{\widetilde{C}})(X,Y)Z - \frac{\nabla_W r}{n(n-1)}[g(Y,Z)\phi^2X - g(X,Z)\phi^2Y]\\ &-\eta(W)\phi^2(\phi\widetilde{\widetilde{C}})(X,Y)Z. \qquad (6.8)\end{aligned}$$

If we take X, Y, Z, W orthogonal to ξ, (6.8) reduces to

$$\phi^2(\nabla_W\widetilde{\widetilde{C}})(X,Y)Z = \phi^2(\nabla_W R)(X,Y)Z - \frac{\nabla_W r}{n(n-1)}[g(Y,Z)\phi^2X - g(X,Z)\phi^2Y].$$

If r is constant, then $\nabla_W r$ is zero. Therefore, (6.9) yields

$$\phi^2(\nabla_W\widetilde{\widetilde{C}})(X,Y)Z = \phi^2(\nabla_W R)(X,Y)Z.$$

Thus, we can state the following:

Theorem 6.2. *Let M be an locally concircular ϕ-symmetric Kenmotsu manifold with respect to quarter-symmetric metric connection $\widetilde{\nabla}$. If the scalar curvature r with respect to Levi-Civita connection is constant, then M is locally ϕ-symmetric with respect to Levi-civita connection ∇.*

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