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A QUARTER SYMMETRIC NON-METRIC CONNECTION IN A KENMOTSU MANIFOLD

MOHIT KUMAR DWIVEDI

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Abstract. In a Riemannian manifold, the existence of a quarter symmetric non-metric connection is proved. We find formula for curvature tensor of this new connection. We also study this connection in Kenmotsu manifold and find the first and second Bianchi identities for the curvature tensor. Finally we get some identities for projective curvature tensor.

1. Introduction

Let M be an n-dimensional differentiable manifold equipped with a linear connection $\tilde{\nabla}$. The torsion tensor $\tilde{T}$ of $\tilde{\nabla}$ is given by

$$\tilde{T}(X,Y) \equiv \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X,Y],$$

and the curvature tensor $\tilde{R}$ of $\tilde{\nabla}$ is

$$\tilde{R}(X,Y)Z \equiv \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X,Y]} Z.$$

The connection $\tilde{\nabla}$ is symmetric if its torsion tensor $\tilde{T}$ vanishes, otherwise it is non-symmetric. If there is a Riemannian metric g in M such that $\tilde{\nabla} g = 0$, the connection $\tilde{\nabla}$ is a metric connection , otherwise it is non-metric. It is well known that a linear connection is symmetric and metric if and only if it is the Levi-Civita connection.

Hayden [7] introduced a metric connection $\tilde{\nabla}$ with a non-zero torsion on a Riemannian manifold. Such a connection is called a Hayden connection. On the other hand, in a Riemannian manifold given a 1-form ω, the *Weyl connection* $\tilde{\nabla}$ constructed with ω and its associated vector B (Folland 1970, [5]) is a symmetric non-metric connection. In fact, the Riemannian metric of the manifold is recurrent with respect to the Weyl connection with the recurrence 1-form ω, that is, $\tilde{\nabla} g = \omega \otimes g$. Another symmetric non-metric connection is projectively related to the Levi-Civita connection (cf. Yano [19], Smaranda [16]).

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Friedmann and Schouten ([4], [14]) introduced the idea of a semi-symmetric linear connection in a differentiable manifold. A linear connection is said to be a semi-symmetric connection if its torsion tensor $\tilde{T}$ is of the form

$$\tilde{T}(X,Y) = u(Y)X - u(X)Y, \tag{1.1}$$

where u is a 1-form. A Hayden connection with the torsion tensor of the form (1.1) is a semi-symmetric metric connection. In 1970, Yano [20] considered a semi-symmetric metric connection and studied some of its properties. Some different kind of semi-symmetric connections are studied in [1], [2], [9] and [15].

In 1975, S. Golab [6] defined and studied quarter-symmetric linear connections in differentiable manifolds. A linear connection is said to be a quarter-symmetric connection if its torsion tensor $\tilde{T}$ is of the form

$$\tilde{T}(X,Y) = u(Y)\varphi X - u(X)\varphi Y, \qquad X, Y \in TM, \tag{1.2}$$

where u is a 1-form and φ is a tensor of type $(1,1)$. Note that a quarter-symmetric metric connection is a Hayden connection with the torsion tensor of the form (1.2). Studies of various types of quarter-symmetric metric connections and their properties include [11], [12], [13] and [21] among others.

On the other hand, there is well known class of almost contact metric manifolds introduced by K. Kenmotsu, which is now known as Kenmotsu manifolds [8].

In this paper we study a quarter-symmetric non-metric connection. The paper is organized as follows. In section 2, we get a quarter symmetric non-metric connection. In this section the curvature tensor of the Riemannian manifold with respect to the defined quarter symmetric non-metric connection is also find. In last of this section first Bianchi identity for the curvature tensor of the Riemannian manifold with respect given quarter symmetric non-metric connection is find. In section 3 we study this quarter symmetric non-metric connection in Kenmotsu manifold. We have given the covariant derivative of a 1-form and the torsion tensor we also get the curvature tensor of the Kenmotsu manifold with respect to the defined quarter symmetric non-metric connection and find first and second Bianchi identities. In last of this section we have given the Ricci-tensor, scalar curvature and the projective curvature of the Kenmotsu manifold with respect to the defined quarter symmetric non-metric connection.

2. A quarter-symmetric connection

In this section we find the existance of a quarter-symmetric non-metric connection

Theorem 2.1. *Let M be an n-dimensional Riemannian manifold equipped with the Levi-Civita connection ∇ of its Riemannian metric g. Let η be a 1-form and φ a $(1,1)$ tensor field in M such that*

$$\eta(X) \equiv g(\xi, X), \tag{2.1}$$

$$g(\varphi X, Y) = -g(X, \varphi Y) \tag{2.2}$$

for all $X, Y \in TM$. Then there exists a unique quarter symmetric non-metric connection $\tilde{\nabla}$ in M given by

$$\tilde{\nabla}_X Y = \nabla_X Y - \eta(X)\varphi Y - g(X,Y)\xi, \tag{2.3}$$

which satisfies

$$\tilde{T}(X,Y) = \eta(Y)\varphi X - \eta(X)\varphi Y, \tag{2.4}$$

and

$$(\tilde{\nabla}_X g)(Y,Z) = \eta(Y)g(X,Z) + \eta(Z)g(X,Y), \tag{2.5}$$

where $\tilde{T}$ *is the torsion tensor of* $\tilde{\nabla}$.

Proof. The equation (2.4) of [18] is

$$\begin{aligned}\tilde{\nabla}_X Y &= \nabla_X Y + u(Y)\varphi_1 X - u(X)\varphi_2 Y - g(\varphi_1 X, Y)U \\ &\quad - f_1\{u_1(X)Y + u_1(Y)X - g(X,Y)U_1\} \\ &\quad - f_2 g(X,Y)U_2.\end{aligned}$$

Taking

$$\varphi_1 = 0,\ \varphi_2 = \varphi,\ u = u_1 = u_2 = \eta,\ f_1 = 0,\ f_2 = 1 \ \textit{ and } U_2 = \xi, \tag{2.6}$$

in above equation we get (2.3). The equations (2.5) and (2.6) of [18] are

$$\tilde{T}(X,Y) = u(Y)\varphi X - u(X)\varphi Y,$$

$$\begin{aligned}(\tilde{\nabla}_X g)(Y,Z) &= 2f_1 u_1(X)g(Y,Z) \\ &\quad + f_2\{u_2(Y)g(X,Z) + u_2(Z)g(X,Y)\}.\end{aligned}$$

Using (2.6) in above equations we get respectively (2.4) and (2.5).

Conversely, a connection defined by (2.3) satisfies the conditions (2.4) and (2.5).

Proposition 2.1. *Let* M *be an* n*-dimensional Riemannian manifold. For the quarter symmetric connection defined by* (2.3) *the covariant derivatives of the torsion tensor* $\tilde{T}$ *and any* 1*-form* π *are given respectively by*

$$\begin{aligned}(\tilde{\nabla}_X \tilde{T})(Y,Z) &= ((\tilde{\nabla}_X \eta)Z)\varphi Y - ((\tilde{\nabla}_X \eta)Y)\varphi Z \\ &\quad + \eta(Z)(\tilde{\nabla}_X \varphi)Y - \eta(Y)(\tilde{\nabla}_X \varphi)Z,\end{aligned} \tag{2.7}$$

and

$$(\tilde{\nabla}_X \pi)Y = (\nabla_X \pi)Y + \eta(X)\pi(\varphi Y) + g(X,Y)\pi(\xi) \tag{2.8}$$

for all $X, Y, Z \in TM$.

Proof. Using (2.8) and (2.3) in

$$(\tilde{\nabla}_X \tilde{T})(Y,Z) = \tilde{\nabla}_X \tilde{T}(Y,Z) - \tilde{T}(\tilde{\nabla}_X Y, Z) - \tilde{T}(Y, \tilde{\nabla}_X Z)$$

we obtain (2.7). Similarly, using (2.3) in

$$(\tilde{\nabla}_X \pi)Y = \tilde{\nabla}_X \pi Y - \pi(\tilde{\nabla}_X Y)$$

we get (2.8).

In an n-dimensional Riemannian manifold M, for the quarter symmetric connection defined by (2.3), let us write

$$\tilde{T}(X,Y,Z) = g(\tilde{T}(X,Y), Z), \quad X, Y, Z \in TM. \tag{2.9}$$

Proposition 2.2. *Let* M *be an* n*-dimensional Riemannian manifold. Then*

$$\begin{aligned}&\tilde{T}(X,Y,Z) + \tilde{T}(Y,Z,X) + \tilde{T}(Z,X,Y) \\ &= 2\eta(X)g(Y,\varphi Z) + 2\eta(Y)g(Z,\varphi X) + 2\eta(Z)g(X,\varphi Y).\end{aligned} \tag{2.10}$$

Proof. In view of (2.7) and (2.9) we get (2.10).

Theorem 2.2. *Let M be an n-dimensional Riemannian manifold equipped with the Levi-Civita connection ∇ of its Riemannian metric g. Then the curvature tensor $\tilde{R}$ of the quarter symmetric connection defined by* (2.3) *is given by*

$$\begin{aligned}\tilde{R}(X,Y)Z &= R(X,Y)Z - \tilde{T}(X,Y,Z)\xi - 2d\eta(X,Y)\varphi Z \\ &+ \eta(X)(\nabla_Y\varphi)Z - \eta(Y)(\nabla_X\varphi)Z \\ &+ g(Y,Z)\{\eta(X)\xi - \nabla_X\xi + \eta(X)\varphi\xi\} \\ &- g(X,Z)\{\eta(Y)\xi - \nabla_Y\xi + \eta(Y)\varphi\xi\} \end{aligned} \tag{2.11}$$

for all $X,Y,Z \in TM$, where R is the curvature of Levi-Civita connection.

Proof. In view of (2.3), (2.2), (2.4) and (2.9) we get (2.11).

Theorem 2.3. *In an n-dimensional Riemannian manifold the first Bianchi identity for the curvature tensor of the Riemannian manifold with respect to the quarter symmetric connection defined by* (2.3) *is*

$$\begin{aligned} & \tilde{R}(X,Y)Z + \tilde{R}(Y,Z)X + \tilde{R}(Z,X)Y \\ = & -\left\{\tilde{T}(X,Y,Z)\xi + \tilde{T}(Y,Z,X)\xi + \tilde{T}(Z,X,Y)\xi\right\} \\ & + \eta(X)B(Y,Z) + \eta(Y)B(Z,X) + \eta(Z)B(X,Y) \\ & - 2d\eta(X,Y)\varphi Z - 2d\eta(Y,Z)\varphi X - 2d\eta(Z,X)\varphi Y \end{aligned} \tag{2.12}$$

for all $X,Y,Z \in TM$, where

$$B(X,Y) = (\nabla_X\varphi)Y - (\nabla_Y\varphi)X. \tag{2.13}$$

Proof. From (2.11) we get

$$\begin{aligned} & \tilde{R}(X,Y)Z + \tilde{R}(Y,Z)X + \tilde{R}(Z,X)Y \\ = & \; 2\eta(X)g(\varphi Y,Z)\xi + 2\eta(Y)g(\varphi Z,X)\xi + 2\eta(Z)g(\varphi X,Y)\xi \\ & + \eta(X)(\nabla_Y\varphi)Z - \eta(X)(\nabla_Z\varphi)Y + \eta(Y)(\nabla_Z\varphi)X \\ & - \eta(Y)(\nabla_X\varphi)Z + \eta(Z)(\nabla_X\varphi)Y - \eta(Z)(\nabla_Y\varphi)X \\ & - ((\nabla_X\eta)Y)\varphi Z + ((\nabla_Y\eta)X)\varphi Z - ((\nabla_Y\eta)Z)\varphi X \\ & + ((\nabla_Z\eta)Y)\varphi X - ((\nabla_Z\eta)X)\varphi Y + ((\nabla_X\eta)Z)\varphi Y. \end{aligned}$$

Using (2.13) and (2.10) in the previous equation we get (2.12).

Let us write the curvature tensor $\tilde{R}$ as a $(0,4)$ -tensor by

$$\tilde{R}(X,Y,Z,W) = g\left(\tilde{R}(X,Y)Z,W\right), \qquad X,Y,Z,W \in TM. \tag{2.14}$$

Then we have the following:

Theorem 2.4. *Let M be a Riemannian manifold. Then*

$$\tilde{R}(X,Y,Z,W) + \tilde{R}(Y,X,Z,W) = 0, \tag{2.15}$$

for all $X,Y,Z,W \in TM$.

Proof. Using (3.25) in (2.14) we get

$$\begin{aligned}\tilde{R}(X,Y,Z,W) &= R(X,Y,Z,W) - \eta(Z)g(\tilde{T}(X,Y),W) \\ &+ g(Y,Z)g(X,W) - g(X,Z)g(Y,W). \end{aligned} \tag{2.16}$$

Interchanging X and Y in the previous equation and adding the resultant equation in (3.28) and using (2.4) we get (2.15).

3. Quarter symmetric non-metric connection in a Kenmotsu manifold

Let M be a $(2n+1)$-dimensional almost contact metric manifold [3] equipped with an almost contact metric structure (φ,ξ,η,g) consisting of a $(1,1)$ tensor field φ, a vector field ξ, a 1-form η and a compatible Riemannian metric g satisfying

$$\varphi^2 = -\,I + \eta\otimes\xi, \quad \eta(\xi)=1, \quad \varphi\xi = 0, \quad \eta\circ\varphi = 0, \tag{3.1}$$

$$g(X,Y) = g(\varphi X,\varphi Y) + \eta(X)\,\eta(Y), \tag{3.2}$$

$$g(\varphi X,Y) = -g(X,\varphi Y), \quad g(\xi,X) = \eta(X) \tag{3.3}$$

for all $X,Y\in TM$. An almost contact metric manifold is called a *Kenmotsu manifold* if it satisfies [8]

$$(\nabla_X\varphi)\,Y = \eta(Y)\,\varphi X - g(\varphi X,Y)\,\xi, \qquad X,Y\in TM, \tag{3.4}$$

where ∇ is Levi-Civita connection of the Riemannian metric. From the above equation it follows that

$$\nabla_X\xi = -X + \eta(X)\,\xi, \tag{3.5}$$

$$(\nabla_X\eta)\,Y = -\,g(X,Y) + \eta(X)\,\eta(Y) \tag{3.6}$$

and consequently

$$d\eta = 0. \tag{3.7}$$

where

$$d\eta(X,Y) = \frac{1}{2}\left((\nabla_X\eta)Y - (\nabla_Y\eta)X\right), \quad X,Y\in TM. \tag{3.8}$$

Moreover, the curvature tensor R, the Ricci tensor S, and the Ricci operator Q satisfy [8]

$$R(X,Y)\,\xi = \eta(X)\,Y - \eta(Y)\,X, \tag{3.9}$$

$$S(X,\xi) = -2n\eta(X), \tag{3.10}$$

$$Q\xi = -2n\xi. \tag{3.11}$$

The equation (3.9) is equivalent to

$$R(\xi,X)\,Y = \eta(Y)\,X - g(X,Y)\,\xi, \tag{3.12}$$

which implies that

$$R(\xi,X)\,\xi = X - \eta(X)\,\xi. \tag{3.13}$$

From (3.9) and (3.12), we have

$$\eta(R(X,Y)\,\xi) = 0, \tag{3.14}$$

$$\eta(R(\xi,X)\,Y) = \eta(X)\,\eta(Y) - g(X,Y). \tag{3.15}$$

Theorem 3.1. *Let M be a Kenmotsu manifold. Then for the quarter symmetric connection defined by* (2.3) *it follows that*

$$(\tilde{\nabla}_X \pi)Y = (\nabla_X \pi)Y + \eta(X)\,\pi(\varphi Y) + g(X,Y)\,\pi(\xi)\,. \tag{3.16}$$

In particular,

$$(\tilde{\nabla}_X \eta)Y = \eta(X)\,\eta(Y)\,, \tag{3.17}$$

and

$$\tilde{d}\eta = 0, \tag{3.18}$$

where

$$\tilde{d}\eta(X,Y) = \frac{1}{2}((\tilde{\nabla}_X \eta)Y - (\tilde{\nabla}_Y \eta)X), \quad X,Y \in TM. \tag{3.19}$$

Proof. In view of (2.3) we get (3.16). Replacing π by η in (3.16) and using (3.1) and (3.6) we get (3.17). The equation (3.18) follows from (3.17).

Theorem 3.2. *Let M be a Kenmotsu manifold. Then*

$$(\tilde{\nabla}_X \varphi)Y = \eta(Y)\,\varphi X. \tag{3.20}$$

Consequently,

$$\tilde{T}(X,Y) = (\tilde{\nabla}_X \varphi)Y - (\tilde{\nabla}_Y \varphi)X, \tag{3.21}$$

$$\tilde{\nabla}_X \xi = -X \tag{3.22}$$

for all $X,Y \in TM$.

Proof. In view of (2.3) and (3.1) we get

$$(\tilde{\nabla}_X \varphi)Y = (\nabla_X \varphi)Y - g(X,\varphi Y)\,\xi,$$

which in view of (3.4) gives (3.20). From (3.20) and (2.4) we get (3.21). In view of (2.3), (3.5), (3.1) and (3.3) we get (3.22).

Theorem 3.3. *Let M be a Kenmotsu manifold. Then*

$$(\tilde{\nabla}_X \tilde{T})(Y,Z) = \eta(X)\,\tilde{T}(Y,Z)\,. \tag{3.23}$$

Consequently,

$$(\tilde{\nabla}_X \tilde{T})(Y,Z) + (\tilde{\nabla}_Y \tilde{T})(Z,X) + (\tilde{\nabla}_Z \tilde{T})(X,Y) = 0 \tag{3.24}$$

for all $X,Y,Z \in TM$.

Proof. Using (3.17), (3.20) and (2.4) in (2.7) we obtain (3.23). The equation (3.24) follows from (3.23) and (2.4).

Theorem 3.4. *The curvature tensor $\tilde{R}$ of the quarter symmetric connection in a Kenmotsu manifold is given by*

$$\tilde{R}(X,Y)Z = R(X,Y)Z - \eta(X)\,\tilde{T}(X,Y) + g(Y,Z)\,X - g(X,Z)\,Y \tag{3.25}$$

for all $X,Y,Z \in TM$.

Proof. Using (3.4), (3.1), (3.3), (2.9) and (2.4) in (2.11) we get

$$\begin{aligned}\tilde{R}(X,Y)Z &= R(X,Y)Z - \eta(Y)(\nabla_X\varphi)Z + \eta(X)(\nabla_Y\varphi)Z \\ &+ ((\nabla_Y\eta)X)\varphi Z - ((\nabla_X\eta)Y)\varphi Z \\ &+ g(Y,Z)(-\nabla_X\xi + \eta(X)\xi) \\ &- g(X,Z)(-\nabla_Y\xi + \eta(Y)\xi) \\ &- \eta(Y)g(\varphi X,Z)\xi + \eta(X)g(\varphi Y,Z)\xi.\end{aligned}$$

Using (3.4), (3.7), (3.5) and (3.1) in the above equation we get (3.25).

Now, we have the following theorems.

Theorem 3.5. *Let M be a Kenmotsu manifold. Then*

$$\begin{aligned}&\tilde{R}(X,Y,Z,W) + \tilde{R}(X,Y,W,Z) \\ = &-\eta(Z)(g\tilde{T}(X,Y),W) - \eta(W)(g\tilde{T}(X,Y),Z),\end{aligned} \tag{3.26}$$

$$\begin{aligned}&\tilde{R}(X,Y,Z,W) - \tilde{R}(Z,W,X,Y) \\ = &\ \eta(X)(g\tilde{T}(Z,W),Y) - \eta(Z)(g\tilde{T}(X,Y),W)\end{aligned} \tag{3.27}$$

for all $X,Y,Z,W \in TM$.

Proof. Using (3.25) in (2.14) we get

$$\begin{aligned}\tilde{R}(X,Y,Z,W) &= R(X,Y,Z,W) - \eta(Z)g(\tilde{T}(X,Y),W) \\ &+ g(Y,Z)g(X,W) - g(X,Z)g(Y,W).\end{aligned} \tag{3.28}$$

Interchanging X and Y in the previous equation and adding the resultant equation in (3.28) and using (2.4) we get (2.15). The equation (3.26) can be obtained by interchanging Z and W in (3.28) and adding the resultant equation to (3.28). In the last, interchanging X and Z, and Y and W in (3.28) and subtracting the resultant equation from (3.28) and using (2.4) we get (3.27).

Theorem 3.6. *The first Bianchi identity for curvature tensor of the Kenmotsu manifold with respect to the connection (2.3) is given as*

$$\tilde{R}(X,Y)Z + \tilde{R}(Y,Z)X + \tilde{R}(Z,X)Y = 0 \tag{3.29}$$

for all $X,Y,Z \in TM$.

Proof. Using (3.25) and (2.4) we get (3.29).

Let us define

$$R_0(X,Y)Z = g(Y,Z)X - g(X,Z)Y \tag{3.30}$$

Theorem 3.7. *The second Bianchi identity for curvature tensor of the Kenmotsu manifold with respect to the connection (2.3) is given as*

$$
\begin{aligned}
& (\tilde{\nabla}_X \tilde{R})(Y,Z)W + (\tilde{\nabla}_Y \tilde{R})(Z,X)W + (\tilde{\nabla}_Z \tilde{R})(X,Y)W \\
= \; & \eta(X)\{R(\varphi Y,Z)W + R(Y,\varphi Z)W + R(Y,Z)\varphi W \\
& - \varphi R(Y,Z)W + R_0(Z,Y)W\} \\
& + \eta(Y)\{R(\varphi Z,X)W + R(Z,\varphi X)W \\
& + R(Z,X)\varphi W - \varphi R(Z,X)W + R_0(X,Z)W\} \\
& + \eta(Z)\{R(\varphi X,Y)W + R(X,\varphi Y)W \\
& + R(X,Y)\varphi W - \varphi R(X,Y)W + R_0(Y,X)W\} \\
& + g(X,W)R(Y,Z)\xi + g(Y,W)R(X,Z)\xi \\
& + g(Z,W)R(X,Y)\xi
\end{aligned} \tag{3.31}
$$

for all $X, Y, Z, W \in TM$.

Proof. Using (3.25), (2.4), (3.1), and (3.3) in the following equation

$$
\begin{aligned}
(\tilde{\nabla}_X \tilde{R})(Y,Z)W \;=\; & \tilde{\nabla}_X(\tilde{R}(Y,Z)W) - \tilde{R}(\tilde{\nabla}_X Y,Z)W \\
& - \tilde{R}(Y,\tilde{\nabla}_X Z)W - \tilde{R}(Y,Z)\tilde{\nabla}_X W
\end{aligned} \tag{3.32}
$$

we get

$$
\begin{aligned}
(\tilde{\nabla}_X \tilde{R})(Y,Z)W = \; & \nabla_X(R(Y,Z)W - (X\eta(Z))\eta(W)\varphi Y \\
& - \eta(Z)(X\eta(W))\varphi Y - \eta(Z)\eta(W)\nabla_X \varphi Y \\
& + (X\eta(Y))\eta(W)\varphi Z + \eta(Y)(X\eta(W))\varphi Z \\
& + \eta(Y)\eta(W)\nabla_X \varphi Z + (Xg(Z,W))Y + g(Z,W)\nabla_X Y \\
& - g(Y,W)\nabla_X Z - \eta(X)\varphi R(Y,Z)W - \eta(X)\varphi g(Z,W)Y \\
& + \eta(X)\varphi g(Y,W)Z - g(X,R(Y,Z)W)\xi \\
& + \eta(Z)\eta(W)g(X,\varphi Y)\xi - \eta(Y)\eta(W)g(X,\varphi Z)\xi \\
& - R(\nabla_X Y,Z)W + \eta(Z)\eta(W)\varphi\nabla_X Y - \eta(\nabla_X Y)\eta(W)\varphi Z \\
& - g(Z,W)\nabla_X Y + g(\nabla_X Y,W)Z + \eta(X)R(\varphi Y,Z)W \\
& + \eta(X)g(Z,W)\varphi Y + g(X,Y)R(\xi,Z)W \\
& + g(X,Y)\eta(W)\varphi Z - g(X,Y)\eta(W)Z \\
& - R(Y,\nabla_X Z)W + \eta(\nabla_X Z)\eta(W)\varphi Y \\
& - \eta(Y)\eta(W)\varphi\nabla_X Z + g(\nabla_X Z,W)Y \\
& + g(Y,W)\nabla_X Z + \eta(X)R(Y,\varphi Z)W \\
& - \eta(X)g(Y,W)\varphi Z + g(X,Z)R(Y,\xi)W \\
& - g(X,Z)\eta(W)\varphi Y + g(X,Z)\eta(W)Y \\
& - R(Y,Z)\nabla_X W + \eta(Z)\eta(\nabla_X W)\varphi Y \\
& - \eta(Y)\eta(\nabla_X W)\varphi Z - g(Z,\nabla_X W)Y \\
& + g(Y,\nabla_X W)Z + \eta(X)R(Y,Z)\varphi W \\
& + g(X,W)R(Y,Z)\xi - g(X,W)\eta(Z)\varphi Y \\
& + g(X,W)\eta(Y)\varphi Z + g(X,W)\eta(Z)Y - g(X,W)\eta(Y)
\end{aligned} \tag{3.33}
$$

In view of (3.7), (3.5), (3.3), (3.6) and (3.33) we get (3.31).

Theorem 3.8. *In an n-dimensional Kenmotsu manifold M, the Ricci tensor and the scalar curvature with respect to the connection defined by* (2.3) *are given by*

$$\tilde{S}(Y,Z) = S(Y,Z) + (n-1)\,g(Y,Z), \quad Y,Z \in TM \tag{3.34}$$

and

$$\tilde{r} = r + n(n-1) \tag{3.35}$$

respectively, where S is the Ricci tensor and r is the scalar curvature of M.

Proof. Let $\{e_1, \ldots, e_n\}$ be a basis of M the

$$\tilde{S}(Y,Z) = \sum g\left(\tilde{R}(e_i, Y)\,Z, e_i\right)$$

Using (3.25) and trace$(\varphi) = 0$ in previous equation we get (3.34) and (3.34) gives (3.35).

Theorem 3.9. *The torsion tensor $\tilde{T}$ satisfies the following equation*

$$\tilde{T}\left(\tilde{T}(X,Y),Z\right) + \tilde{T}\left(\tilde{T}(Y,Z),X\right) + \tilde{T}\left(\tilde{T}(Z,X),Y\right) = 0 \tag{3.36}$$

for all $X,Y,Z \in TM$.

Proof. Using (2.4) and (3.1) we get (3.36).

Theorem 3.10. *The projective curvature tensor of Kenmotsu manifold M with respect to the connection defined by* (2.3) *is given as*

$$\tilde{P}(X,Y)\,Z = P(X,Y)\,Z - \eta(Z)\,\tilde{T}(X,Y), \qquad X,Y,Z \in TM, \tag{3.37}$$

where P is the projective curvature tensor of the Kenmotsu manifold. We also have the following identites

$$\tilde{P}(X,Y)\,Z + \tilde{P}(Y,X)\,Z = 0 \tag{3.38}$$

$$\tilde{P}(X,Y)\,Z + \tilde{P}(Y,Z)\,X + \tilde{P}(Z,X)\,Y = 0. \tag{3.39}$$

Proof. The projective curvature tensor is given as [10]

$$\begin{aligned} \tilde{P}(X,Y)\,Z \;\; = \;\; & \tilde{R}(X,Y)\,Z + \frac{1}{n+1}\left(\tilde{S}(X,Y) - \tilde{S}(Y,X)\right)Z \\ & - \frac{n}{n^2-1}\left(\tilde{S}(Y,Z)\,X - \tilde{S}(X,Z)\,Y\right) \\ & - \frac{1}{n^2-1}\left(\tilde{S}(Z,Y)\,X - \tilde{S}(Z,X)\,Y\right) \end{aligned} \tag{3.40}$$

Using (3.34) in (3.40) we get (3.37). In view of (3.37) we get (3.38) and (3.39).

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Department of Mathematics and Astronomy, University of Lucknow-Lucknow
E-mail address: dwivedimohit@yahoo.com