International Electronic Journal of Geometry
Volume 4 No. 1 pp. 129–137 (2011) ©IEJG

ON A QUARTER SYMMETRIC NON-METRIC CONNECTION IN AN LORENTZIAN PARA-SASAKIAN MANIFOLDS

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(Communicated by Cihan ÖZGÜR)

Abstract. In this paper we define and study a quarter-symmetric non-metric connection on an Lorentzian para-Sasakian manifold.

1. Introduction

In 1975, Golab [3] introduced the notion of quarter-symmetric connection in a Riemannian manifold with affine connection. This was further developed by Yano and Imai [12], Rastogi [8],[9], Mishra and Pandey [6], Mukhopadhyay, Roy and Barua [7], Biswas and De [1], Sengupta and Biswas [10], Singh and Pandey [11] and many other geometers.

In this paper we define and study a quarter-symmetric non-metric connection on an Lorentzian para-Sasakian manifold and prove its existence. Some properties of the curvature tensor and the Ricci tensor of the quarter-symmetric non-metric connection is found. A necessary and sufficient condition for the Ricci tensor of $\overline{D}$ to be symmetric and skew-symmetric under certain conditions. A necessary and sufficient condition for the projective Ricci tensor of $\overline{D}$ to be skew-symmetric under certain condition. We also find the necessary and sufficient condition for the Einstein manifold of the connection D is equal to the Einstein manifold of the connection $\overline{D}$ under certain condition.

2. Preliminaries

An $n-$dimensional differential manifold M^n is a Lorentzian para-Sasakian (LP-Sasakian) manifold if it admits a (1, 1)-tensor field ϕ, contravariant vector field ξ, a covariant vector field η, and a Lorentzian metric g, which satisfy

$$\phi^2 X = X + \eta(X)\xi, \tag{2.1}$$

$$\eta(\xi) = -1, \tag{2.2}$$

2000 *Mathematics Subject Classification.* 53C25.

Key words and phrases. Lorentzian para-Sasakian manifold, Projective Ricci tensor, Einstein manifold.

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{2.3}$$

$$g(X, \xi) = \eta(X), \tag{2.4}$$

$$(D_X\phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.5}$$

and

$$D_X\xi = \phi X, \tag{2.6}$$

for any vector fields X and Y, where D denotes covariant differentiation with respect to g (Matsumoto [4] and Matsumoto and Mihai [5]).

In an LP-Sasakian manifold M^n with structure (ϕ, ξ, η, g), it is easily seen that

$$(a) \quad \phi\xi = 0 \quad (b) \quad \eta(\phi X) = 0 \quad (c) \quad \text{rank}(\phi) = (n-1). \tag{2.7}$$

Let us put

$$F(X, Y) = g(\phi X, Y). \tag{2.8}$$

Then the tensor field F is symmetric (0, 2) tensor field

$$F(X, Y) = F(Y, X), \tag{2.9}$$

and

$$F(X, Y) = (D_X\eta)(Y). \tag{2.10}$$

Also in an LP-Sasakian manifold, the following relation holds,

$$'R(X, Y, Z, \xi) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y), \tag{2.11}$$

and

$$S(X, \xi) = (n-1)\eta(X), \tag{2.12}$$

where $'R$ and S denotes curvature tensor of type (0, 4) and the Ricci tensor of type (0, 2) of M^n respectively.

3. Quarter-Symmetric non-metric connection in an LP-Sasakian manifold

Let (M^n, g) be an LP-Sasakian manifold with Levi-Civita connection D. We define a linear connection $\overline{D}$ on M^n by

$$\overline{D}_X Y = D_X Y + \eta(Y)\phi X + a(X)\phi Y, \tag{3.1}$$

where η and a are 1-forms associated with vector field ξ and A on M^n given by

$$g(X,\ \xi) = \eta(X), \tag{3.2)(a}$$

and

$$g(X, A) = a(X), \tag{3.2)(b}$$

for all vector fields $X \in \chi(M^n)$, where $\chi(M^n)$ is the set of all differentiable vector fields on M^n.

Using (3.1), the torsion tensor $\overline{T}$ of M^n with respect to the connection $\overline{D}$ is given by

$$\overline{T}(X, Y) = \eta(Y)\phi X - \eta(X)\phi Y + a(X)\phi Y - a(Y)\phi X. \tag{3.3}$$

A linear connection satisfying (3.3) is called a quarter-symmetric connection. Further using (3.1), we have

$$(\overline{D}_X g)(Y,Z) = -\eta(Y)g(\phi X,Z) - \eta(Z)g(\phi X,Y) - 2a(X)g(\phi Y,Z). \tag{3.4}$$

A linear connection $\overline{D}$ defined by (3.1) satisfies (3.3) and (3.4) is called a quarter-symmetric non-metric connection.

Let $\overline{D}$ be a linear connection in M^n given by

$$\overline{D}_X Y = D_X Y + H(X,Y). \tag{3.5}$$

Now, we shall determine the tensor field H such that $\overline{D}$ satisfies (3.3) and (3.4).

From (3.5), we have

$$\overline{T}(X,Y) = H(X,Y) - H(Y,X). \tag{3.6}$$

Denote

$$G(X,Y,Z) \equiv (\overline{D}_X g)(Y,Z), \tag{3.7}$$

From (3.5) and (3.7), we have

$$g(H(X,Y),Z) + g(H(X,Z),Y) = -G(X,Y,Z). \tag{3.8}$$

From (3.5), (3.7), (3.8) and (3.4), we have

$$\begin{aligned}
&g(\overline{T}(X,Y),Z) + g(\overline{T}(Z,X),Y) + g(\overline{T}(Z,Y),X)\\
&= g(H(X,Y),Z) - g(H(Y,X),Z) + g(H(Z,X),Y) - g(H(X,Z),Y)\\
&\quad + g(H(Z,Y),X) - g(H(Y,Z),X)\\
&= g(H(X,Y),Z) - g(H(X,Z),Y) - G(Z,X,Y) + G(Y,X,Z)\\
&= 2g(H(X,Y),Z) + G(X,Y,Z) + G(Y,X,Z) - G(Z,X,Y)\\
&= 2g(H(X,Y),Z) - 2\eta(Z)g(\phi X,Y) - 2a(X)g(\phi Y,Z) - 2a(Y)g(\phi X,Z)\\
&\quad + 2a(Z)g(\phi X,Y)
\end{aligned}$$

$$\begin{aligned}
\text{or, } H(X,Y) &= \frac{1}{2}\{\overline{T}(X,Y) + {}'\overline{T}(X,Y) + {}'\overline{T}(Y,X)\} + a(X)\phi Y + a(Y)\phi X\\
&\quad + g(\phi X,Y)\xi - g(\phi X,Y)A,
\end{aligned}$$

where ${}'\overline{T}$ be a tensor field of type (1, 2) defined by

$$g({}'\overline{T}(X,Y),Z) = g(\overline{T}(Z,X),Y).$$

$$\text{or,} \qquad H(X,Y) = \eta(Y)\phi X + a(X)\phi Y.$$

This implies

$$\overline{D}_X Y = D_X Y + \eta(Y)\phi X + a(X)\phi Y.$$

Hence, we can state the following theorem :

Theorem 3.1. *Let* (M^n, g) *be an LP-Sasakian manifold with almost Lorentzian para contact metric structure* (ϕ, ξ, η, g) *admitting a quarter-symmetric non-metric connection* $\overline{D}$ *which satisfies* (3.3) *and* (3.4). *Then the quarter-symmetric non-metric connection is given by*

$$\overline{D}_X Y = D_X Y + \eta(Y)\phi X + a(X)\phi Y.$$

4. Existence of a quarter-symmetric non-metric connection $\bar{D}$ in an LP-Sasakian manifold

Let X, Y, Z be any three vector fields on an LP-Sasakian manifold (M^n, g) with almost Lorentzian para-contact metric structure (ϕ, ξ, η, g). We define a connection $\overline{D}$ by the following equation :

$$\begin{aligned}2g(\overline{D}_X Y, Z) &= Xg(Y,Z) + Yg(Z,X) - Zg(X,Y) + g([X,Y],Z) - g([Y,Z],X)\\ &+g([Z,X],Y) + g(\eta(Y)\phi X - \eta(X)\phi Y + a(X)\phi Y - a(Y)\phi X, Z)\\ &+g(\eta(X)\phi Z - \eta(Z)\phi X + a(Z)\phi X + a(X)\phi Z, Y)\\ &+g(\eta(Y)\phi Z + \eta(Z)\phi Y + a(Y)\phi Z - a(Z)\phi Y, X), \qquad (4.1)\end{aligned}$$

which holds for all vector fields X, Y, $Z \in \chi(M^n)$.

It can be easily verified that the mapping

$$\overline{D} \ : \ (X,Y) \to \overline{D}_X Y$$

satisfies the following equalities :

$$\overline{D}_X(Y+Z) = \overline{D}_X Y + \overline{D}_X Z, \qquad (4.2)$$

$$\overline{D}_{X+Y} Z = \overline{D}_X Z + \overline{D}_Y Z, \qquad (4.3)$$

$$\overline{D}_{fX} Y = f\overline{D}_X Y, \qquad (4.4)$$

and

$$\overline{D}_X(fY) = f\overline{D}_X Y + (Xf)Y, \qquad (4.5)$$

for all X, Y, $Z \in \chi(M^n)$ and for all $f \in F(M^n)$, the set of all differentiable mapping over M^n.

From (4.2), (4.3), (4.4) and (4.5) we can conclude that $\overline{D}$ determines a linear connection on M^n.

Now, from (4.1), we have

$$\begin{aligned}2g(\overline{D}_X Y, Z) - 2g(\overline{D}_Y X, Z) &= 2g([X,Y],Z) + 2\eta(Y)g(\phi X, Z) - 2\eta(X)g(\phi Y, Z)\\ &+2a(X)g(\phi Y, Z) - 2a(Y)g(\phi X, Z). \qquad (4.6)\end{aligned}$$

Hence,

$$\overline{D}_X Y - \overline{D}_Y X - [X,Y] = \eta(Y)\phi X - \eta(X)\phi Y + a(X)\phi Y - a(Y)\phi X.$$

or,

$$\overline{T}(X,Y) = \eta(Y)\phi X - \eta(X)\phi Y + a(X)\phi Y - a(Y)\phi X. \qquad (4.7)$$

Also, we have from (4.1)

$$\begin{aligned}2g(\overline{D}_X Y, Z) + 2g(\overline{D}_X Z, Y) &= 2Xg(Y,Z) + 2\eta(Y)g(\phi X, Z) + 2\eta(Z)g(\phi X, Y)\\ &+4a(X)g(\phi Y, Z),\end{aligned}$$

$$\text{i.e., } (\overline{D}_X g)(Y,Z) = -\eta(Y)g(\phi X, Z) - \eta(Z)g(\phi X, Y) - 2a(X)g(\phi Y, Z). \qquad (4.8)$$

From (4.7) and (4.8) it follows that $\overline{D}$ determines a quarter-symmetric non-metric connection on (M^n, g). It can be easily verified that $\overline{D}$ determines a unique quarter-symmetric non-metric connection on (M^n, g).

Hence, we can state the following theorem :

Theorem 4.1. *Let (M^n, g) be an LP-Sasakian manifold with an almost Lorentzian para-contact metric structure (ϕ, ξ, η, g) on it. Then there exists a unique linear connection $\overline{D}$ satisfying* (3.3) *and* (3.4).

The above theorem proves the existence of a quarter-symmetric non-metric connection in an LP-Sasakian manifold.

5. Curvature tensor of an LP-Sasakian manifold with respect to the quarter-symmetric non-metric connection $\bar{D}$

Let $\overline{R}$ and R be the curvature tensor of the connection $\overline{D}$ and D respectively, then

$$\overline{R}(X,Y)Z = \overline{D}_X\overline{D}_Y Z - \overline{D}_Y\overline{D}_X Z - \overline{D}_{[X,Y]}Z. \tag{5.1}$$

From (3.1) and (5.1), we get

$$\begin{aligned}\overline{R}(X,Y)Z = {}& \overline{D}_X(D_Y Z + \eta(Z)\phi Y + a(Y)\phi Z) - \overline{D}_Y(D_X Z + \eta(Z)\phi X + a(X)\phi Z)\\ & - D_{[X,Y]}Z - \eta(Z)\phi([X,Y]) - a([X,Y])\phi Z,\end{aligned}$$

which gives on simplification,

$$\begin{aligned}\overline{R}(X,Y)Z = {}& R(X,Y)Z + g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X + \eta(Y)\eta(Z)X\\ & -\eta(X)\eta(Z)Y + a(Y)g(X,Z)\xi - a(X)g(Y,Z)\xi\\ & +a(Y)\eta(X)\eta(Z)\xi - a(X)\eta(Y)\eta(Z)\xi + da(X,Y)\phi Z,\end{aligned} \tag{5.2}$$

where $R(X,Y)Z = D_X\, D_Y Z - D_Y D_X Z - D_{[X,Y]}Z$
is the curvature tensor of D with respect to the Riemannian connection.
Contracting (5.2), we find

$$\begin{aligned}\overline{S}(Y,Z) = {}& S(Y,Z) - \psi g(\phi Y,\, Z) + [1 - a(\xi)]g(Y,Z)\\ & +[n - a(\xi)]\eta(Y)\eta(Z) + da(\phi Z,\, Y).\end{aligned} \tag{5.3}$$

and

$$\overline{r} = r - (n-1)a(\xi) + \lambda - \psi^2 \tag{5.4}$$

where $\overline{S}$ and $\overline{r}$ are the Ricci tensor and scalar curvature with respect to $\overline{D}$,

$$\lambda = \operatorname{trace} da\,(\phi Z, Y) \qquad \text{and} \qquad \psi = \operatorname{trace}\phi.$$

Hence, we can state the following theorem :

Theorem 5.1. *The curvature tensor $\overline{R}(X,Y)Z$, the Ricci tensor $\overline{S}(Y,Z)$ and the scalar curvature $\overline{r}$ of an LP-Sasakian manifold with respect to quarter-symmetric non-metric connection is given by* (5.2)*,* (5.3) *and* (5.4) *respectively.*

Let us assume that $\overline{R}(X,Y)Z = 0$ in (5.2) and contracting, we get

$$S(Y,Z) = \psi g(\phi Y,Z) - [1 - a(\xi)]g(Y,Z) - [n - a(\xi)]\eta(Y)\eta(Z) - da(\phi Z,Y),$$

which gives

$$r = (n-1)a(\xi) - \lambda + \psi^2.$$

Hence, we can state the following theorem :

Theorem 5.2. *If an LP-Sasakian manifold M^n admits a quarter-symmetric non-metric connection whose curvature tensor vanishes, then the scalar curvature r is given by $r = (n-1)a(\xi) - \lambda + \psi^2$.*

From (5.2) it follows that

$$'\overline{R}(X,\, Y,\, Z,\, W) + {}'\overline{R}(Y,\, X,\, Z,\, W) = 0. \tag{5.5}$$

$$\begin{aligned} '\overline{R}(X,Y,Z,W) + {}'\overline{R}(X,Y,W,Z) &= \eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(Z)g(Y,W) \\ &+\eta(Y)\eta(W)g(X,Z) - \eta(X)\eta(W)g(Y,Z) + a(Y)\eta(W)g(X,Z) \\ &-a(X)\eta(W)g(Y,Z) + a(Y)\eta(Z)g(X,W) - a(X)\eta(Z)g(Y,W) \\ &+2a(Y)\eta(X)\eta(Z)\eta(W) - 2a(X)\eta(Y)\eta(Z)\eta(W) \\ &+2da(X,Y)g(\phi Z,W). \end{aligned} \tag{5.6}$$

$$\begin{aligned} '\overline{R}(X,Y,Z,W) - {}'\overline{R}(Z,W,X,Y) &= \eta(Y)\eta(Z)g(X,W) - \eta(X)\eta(W)g(Y,Z) \\ &+a(Y)\eta(W)g(X,Z) - a(X)\eta(W)g(Y,Z) + a(Z)\eta(Y)g(X,W) \\ &-a(W)\eta(Y)g(X,Z) + a(Y)\eta(X)\eta(Z)\eta(W) - a(X)\eta(Y)\eta(Z)\eta(W) \\ &+a(Z)\eta(X)\eta(Y)\eta(W) - a(W)\eta(X)\eta(Y)\eta(Z) \\ &+da(X,Y)g(\phi Z,W) - da(Z,W)g(\phi X,Y) \end{aligned} \tag{5.7}$$

and

$$\begin{aligned} &'\overline{R}(X,Y,Z,W) + {}'\overline{R}(Y,Z,X,W) + {}'\overline{R}(Z,X,Y,W) \\ &= da(X,Y)g(\phi Z,W) + da(Y,Z)g(\phi X,W) + da(Z,X)g(\phi Y,W). \end{aligned} \tag{5.8}$$

If the 1-form a is closed then from (5.8) it follows that

$$'\overline{R}(X,Y,Z,W) + {}'\overline{R}(Y,Z,X,W) + {}'\overline{R}(Z,X,Y,W) = 0, \tag{5.9}$$

where

$$'\overline{R}(X,Y,Z,W) = g(\overline{R}(X,Y)Z,W) \text{ and } 'R(X,Y,Z,W) = g(R(X,Y)Z,W).$$

Hence, we can state the following theorem :

Theorem 5.3. *The curvature tensor of an LP-Sasakian manifold with respect to the quarter-symmetric non-metric connection $\overline{D}$, satisfies the relation* (5.5)*,* (5.6)*,* (5.7) *and* (5.8)*. In particular, if the 1-form a is closed, then*

$$'\overline{R}(X,Y,Z,W) + {}'\overline{R}(Y,Z,X,W) + {}'\overline{R}(Z,X,Y,W) = 0.$$

6. Symmetric and skew-symmetric condition of Ricci tensor of $\bar{D}$ in an LP-Sasakian manifold

From (5.3), we have

$$\begin{aligned} \overline{S}(Z,Y) &= S(Z,Y) - \psi g(\phi Z,Y) + [1-a(\xi)]g(Y,Z) \\ &+[n-a(\xi)]\,\eta(Y)\,\eta(Z) + da(\phi Y,Z). \end{aligned} \tag{6.1}$$

From (5.3) and (6.1), we have

$$\overline{S}(Y,Z) - \overline{S}(Z,Y) = da(\phi Z,Y) - da(\phi Y,Z). \tag{6.2}$$

If $\overline{S}(Y,Z)$ is symmetric then the left hand side of (6.2) vanishes, hence we get

$$da(\phi Z,Y) = da(\phi Y,Z). \tag{6.3}$$

Moreover if relation (6.3) holds, then from (6.2), $\overline{S}(Y,Z)$ is symmetric.
Hence, we can state the following theorem :

Theorem 6.1. *The Ricci tensor* $\overline{S}(Y,Z)$ *of the manifold with respect to quarter-symmetric non-metric connection* $\overline{D}$ *in an LP-Sasakian manifold is symmetric if and only if the relation* (6.3) *holds.*

From (5.3) and (6.1), we have

$$\overline{S}(Y,Z)+\overline{S}(Z,Y)=2S(Y,Z)-2\psi g(\phi Y,Z)+2[1-a(\xi)]g(Y,Z)$$

$$+2[n-a(\xi)]\eta(Y)\eta(Z)+da(\phi Y,Z)+da(\phi Z,Y). \tag{6.4}$$

If $\overline{S}(Y,Z)$ is skew-symmetric then the left hand side of (6.4) vanishes, and we get

$$S(Y,Z)=\psi g(\phi Y,Z)-[1-a(\xi)]\,g(Y,Z)$$

$$-[n-a(\xi)]\,\eta(Y)\eta(Z)-\frac{1}{2}[da(\phi Y,Z)+da(\phi Z,Y)]. \tag{6.5}$$

Moreover if $S(Y,Z)$ is given by (6.5), then from (6.4), we get

$$\overline{S}(Y,Z)+\overline{S}(Z,Y)=0.$$

Hence, we can state the following theorem :

Theorem 6.2. *If an LP-Sasakian manifold admits a quarter-symmetric non-metric connection* $\overline{D}$ *then a necessary and sufficient condition for the Ricci tensor of* $\overline{D}$ *to be skew-symmetric is that the Ricci tensor of the Levi-Civita connection* D *is given by* (6.5).

7. Skew- symmetric properties of projective Ricci tensor with respect to quarter-symmetric non-metric connection $\bar{D}$ in an LP-Sasakian manifold

Projective Ricci tensor in a Riemannian manifold is defined by (Chaki and Saha [2]) as follows

$$P(X,Y)=\frac{n}{(n-1)}[S(X,Y)-\frac{r}{n}g(X,Y)]. \tag{7.1}$$

Analogous to this definition, we define projective Ricci tensor with respect to quarter-symmetric non-metric connection $\overline{D}$, given by

$$\overline{P}(X,Y)=\frac{n}{(n-1)}[\overline{S}(X,Y)-\frac{\overline{r}}{n}g(X,Y)]. \tag{7.2}$$

From (5.3), (5.4) and (7.2), we have

$$\overline{P}(X,Y)=\frac{n}{(n-1)}[S(X,Y)-\psi g(\phi X,Y)+\{1-a(\xi)\}g(X,Y)+\{n-a(\xi)\}\eta(X)\eta(Y)$$

$$+da(\phi Y,X)-\left\{\frac{r-(n-1)\,a(\xi)+\lambda-\psi^2}{n}\right\}g(X,Y)]. \tag{7.3}$$

From (7.3), we have

$$\overline{P}(Y,X)=\frac{n}{(n-1)}[S(Y,X)-\psi g(\phi Y,X)+\{1-a(\xi)\}g(Y,X)+\{n-a(\xi)\}\eta(X)\eta(Y)$$

$$+da(\phi X,Y)-\left\{\frac{r-(n-1)\,a(\xi)+\lambda-\psi^2}{n}\right\}g(Y,X)]. \tag{7.4}$$

From equation (7.3) and (7.4), we have

$$\overline{P}(X,Y)+\overline{P}(Y,X)=\frac{n}{(n-1)}[2S(X,Y)-2\psi g(\phi X,Y)+2\{1-a(\xi)\}g(X,Y)$$
$$+2\{n-a(\xi)\}\eta(X)\eta(Y)-2\Big\{\frac{r-(n-1)\,a(\xi)+\lambda-\psi^2}{n}\Big\}\,g(X,Y)$$
$$+da(\phi X,Y)+da(\phi Y,X)]. \tag{7.5}$$

If $\overline{P}(X,Y)$ is skew-symmetric then the left hand side of (7.5) vanishes, and we get

$$S(X,Y)=[\psi g(\phi X,Y)-\{1-a(\xi)\}g(X,Y)-\{n-a(\xi)\}\eta(X)\eta(Y)$$
$$+\Big\{\frac{r-(n-1)\,a(\xi)+\lambda-\psi^2}{n}\Big\}\,g(X,Y)-\frac{1}{2}\{da(\phi X,Y)+da(\phi Y,X)\}]. \tag{7.6}$$

Moreover if $S(X,Y)$ is given by (7.6), then from (7.5), we get

$$\overline{P}(X,Y)+\overline{P}(Y,X)=0,$$

i.e. Projective Ricci tensor of $\overline{D}$ is skew-symmetric.
Hence, we can state the following theorem :

Theorem 7.1. *If an LP-Sasakian manifold admits a quarter-symmetric non-metric connection $\overline{D}$ then a necessary and sufficient condition for the Projective Ricci tensor of $\overline{D}$ to be skew-symmetric is that the Ricci tensor of the Levi-Civita connection D is given by* (7.6).

8. Einstein manifold with respect to quarter-symmetric non-metric connection $\bar{D}$ in an LP-Sasakian manifold

A Riemannian manifold M^n is called an Einstein manifold with respect to Riemannian connection if

$$S(X,Y)=\frac{r}{n}g(X,Y). \tag{8.1}$$

Analogous to this definition, we define Einstein manifold with respect to quarter-symmetric non-metric connection $\overline{D}$ by

$$\overline{S}(X,Y)=\frac{\overline{r}}{n}\,g(X,Y). \tag{8.2}$$

From (5.3), (5.4) and (8.2), we have

$$\overline{S}(X,Y)-\frac{\overline{r}}{n}\,g(X,Y)=S(X,Y)-\frac{r}{n}\,g(X,Y)-\psi g(\phi X,Y)+da(\phi Y,X)$$
$$+\Big[\frac{n+\psi^2-\lambda-a(\xi)}{n}\Big]\,g(X,Y)+[n-a(\xi)]\eta(X)\eta(Y). \tag{8.3}$$

If

$$\psi g(\phi X,Y)+da(X,\phi Y)=\Big[\frac{n+\psi^2-\lambda-a(\xi)}{n}\Big]\,g(X,Y)+[n-a(\xi)]\eta(X)\eta(Y). \tag{8.4}$$

Then from (8.3), we get

$$\overline{S}(X,Y)-\frac{\overline{r}}{n}\,g(X,Y)=S(X,Y)-\frac{r}{n}\,g(X,Y). \tag{8.5}$$

Hence, we can state the following theorem :

Theorem 8.1. *In an LP-Sasakian manifold M^n with quarter-symmetric non-metric connection if the relation* (8.4) *holds, then the manifold is an Einstein manifold for the Riemannian connection if and only if it is an Einstein manifold for the connection* $\overline{D}$.

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